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Propagation of Strong Cylindrical and Spherical Shock Waves in Uniform Medium

Dr. Vishal Saxena
Assistant Professor of Physics, Upadhi (P.G.) College Pilibhit, U.P., India

ABSTRACT- Shock wave, strong pressure wave in any elastic medium such as air, water, or a solid substance, produced by supersonic aircraft, explosions, lightning, or other phenomena that crate violent changes in pressure. Shock waves differ from sound waves in that the wave front, in which compression takes place, is a region of sudden and violent change in stress, density, and temperature. Because of this. Shock waves propagate in a manner different front that of ordinary acoustic waves. In particular, shock waves travel faster than sound, and their speed increases as the amplitude is raised; but the intensity of a shock wave also decreases faster than does that of a sound wave, because some of the energy of the shock wave is expended to heat the medium in which it travels. The amplitude of a strong shock wave, as created in air by an explsion, decreases almost as the inverse square of the distance until the wave has become so weak that it obeys the laws of acoustic waves, Shock waves alter the mechanical, electrical, and thermal properties of solids and, thus, can be used to study the equation of state (a relation between pressure, temperature, and volume) of any material.

In the present paper the propagation of strong cylindrical & spherical shock in a uniform medium has been investigated by CCW method for the freely propagating shock. The Relations for the shock velocity, shock strength & pressure are estimated.

Keywords:- Propagation, Strong, Cylindrical, Spherical, Shock, Waves, Uniform, Medium.

INTRODUCTION- The wave phenomenon is one of the most interesting topic of present day research. It is not only interesting in itself, but has many applications in different branches of modern science and technology such as astrophysics, geophysics, hypersonic flights, explosions, plasma physics, meteorology, earthquake tsunami etc. In particular, the study of shock waves in different medium is of immense importance for the production of very high temperature and pressure. The problem of strong blast waves has been studied by many authos, Sedov (1946), Taylor (1950), Lin (1954), Yadav (1993), Yadav and Tripathi (1995), Yadav et al. (1995, 1996) and many others are among them. Sedov (1946). Taylor (1950) and Lin (1954) sought similar solutions. The similarity method based on the series expansion in powers of inverse square of small mach number, is extremely laborious Recently, Yadav and Tripathi (1994) have studied the proparation of strong cylindrical shock and obtained analytical relations for shock velocity. Yadav et al. (1996) have used Chester (1954), Chisnell (1955) and Whitham (1958) method to study the propagation of sound and temperature behind the strong shock in non-uniform medium for three cases of plane cylindrical and spherical symmetrics. In the CCW approximation, a shock is not affected by disturbances in the flow behind the shock, i.e. CCW method describes the freely propagation of the shock. The significance of effect of overtking disturbances (EOD) on the propagation of shock has been given by Yousaf (1974, 1985) and Yadav (1992a, 1992b).

In this paper, the propagation of strong cylindrical and spherical shock in a uniform medium has been investigated by CCW method for the freely propagating shock. Yadav (1992) treatment has been applied to estimated the effect of overtaking disturbances on the shock propagating freely in the medium of uniform density. When a shock, produced by an intense explosion, moves in a medium of uniform density the disturbances are generated in the from of shock and travel along C, characteristics. These C_+, disturbances are reflected from the sonic line watch exists behind the shock front propagate along C_, characteristics and finally overtake the shock. These overtaking disturbances moves in non-uniform regions created by the shock. It is assumed that the non-uniform regions has density distribution $\rho_0 = \rho' \log r$.

The analytical relations for shock velocity, shock strength, the pressure, the partial velocity behind the shock have been obtained for both cases. The correction percentage in shock velocity, shock strength, the pressure, the particle velocity in shock medium has been estimated.

BASIC EQUATION

Under the assumption that the gas is in viscid and non-conducting of heat, the system of the equations for one-dimensional adiabatic flow, are

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r}\right) = 0 \qquad \ldots(1)$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\rho + \rho\left(\frac{\partial u}{\partial r} + \alpha\frac{\partial u}{r}\right) = 0 \qquad \ldots(2)$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(p\rho^{-1}) = 0 \qquad \ldots(3)$$

Where, $u\ (r,t), p(r,t)$ and $\rho(r,t)$ denote respectively the particle velocity, the pressure and density at distance r from the origin at time t and γ is the specific heat index of the gas.

BOUNDARY CONDITION

Let p_0 and ρ_0 denote the undisturbed values of pressure and density in front of the shock wave and u_1 and ρ_1 be the values respective quantities at any point immediately after the passage of the shock, then the well know Rankine-Hugoniot conditions permit us to express u_1, p_1 and ρ_1 in term of the undisturbed values of these quantities by means of the following equations,

$$p_1 = \rho_0 a_0^2\left[\frac{2m^2}{\gamma+1} - \frac{(\gamma-1)}{\gamma(\gamma+1)}\right]$$

$$\rho_1 = \rho_0\left[\frac{(\gamma+1)m^2}{(\gamma-1)m^2+2}\right] \qquad \ldots(.4)$$

$$u_1 = \frac{2a_0}{\gamma+1}\left[m - \frac{1}{m}\right]$$

Where, $m = U/a_0$, U being the shock velocity a_0 is the sound velocity in undisturbed medium and m is mach member.

FOR STRONG SHOCK

We take $U \gg a_0$ the boundary conditions (4) reduces to,

$$\rho = \rho_0\left(\frac{\gamma+1}{\gamma-1}\right)$$

$$p = \frac{2\rho_0}{\gamma+1}U^2 \qquad \ldots(5)$$

$$u = \frac{2U}{\gamma+1}$$

THEORY

The characteristic form of the system of the equations (2) - (3) is

$$dp + \rho a du + \frac{\alpha\rho a^2 u}{u+a}\ \frac{dr}{r} = 0 \qquad \ldots(6)$$

FROM STRONG SHOCK WAVE

Freely propagation of shock

From conditions (5), we haves

$$p = \frac{2\rho_0 U^2}{\gamma + 1}$$

$$dp = \frac{2\rho_0 dU^2}{\gamma + 1} + \frac{2U^2 d\rho_0}{\gamma + 1}$$

$$u = \frac{2U}{\gamma + 1}$$

$$du = \frac{2dU}{= \gamma + 1}$$

$$\rho a du = \rho_0 s. U \frac{2}{\gamma - 1} dU = \rho_0 s \frac{2}{\gamma - 1} UdU$$

$$\frac{\rho a^2 u}{u + a}\ \frac{dr}{r} = \rho_0 \frac{\gamma + 1}{\gamma - 1} s^2 U^2 \frac{(\gamma - 1)}{(\gamma + 1)} \frac{2U}{\gamma + 1} \Big/ \frac{2U}{\gamma + 1} + sU \frac{\gamma - 1}{\gamma + 1}$$

Putting these values in equation (6) and simplifying, we get

$$\frac{2\rho_0 dU^2}{\gamma+1} + \frac{2U^2 d\rho_0}{\gamma+1} + r_0 s \frac{2U}{\gamma+1} + \frac{\alpha\rho_0 s^2 U^2 \left(\frac{\gamma-1}{\gamma+1}\right) U}{\frac{U}{\gamma+1}[2+s(\gamma-1]^r}\ \frac{dr}{r} = 0 \qquad \ldots(7)$$

Here, ρ_0 is the constant for uniform medium here $dp_0 = 0$

$$\frac{z}{\gamma + 1}\left[\rho_0 dU^2 + \rho_0 sUdU + \frac{\alpha\rho_0 s^2 U(\gamma - 1)}{2 + s(\gamma - 1)}\ \frac{dr}{r}\right] = 0$$

$$dU^2 + sUdU + \frac{\alpha s^2 U^2(\gamma - 1)}{\{2 + s(\gamma - 1\}} \frac{dr}{r} = 0$$

$$2UdU + sUdU + \frac{\alpha s^2 U^2(\gamma - 1)}{[7 + s(\gamma - 1)]} \frac{dr}{r} 20$$

$$dU(2 + s) + \frac{\alpha s^2 U(\gamma - 1)}{\{2 + s(\gamma - 1\}} \frac{dr}{r} = 0$$

$$\frac{dU}{U} + \frac{\alpha s^2(\gamma-1)}{(2-s)[2+s(\gamma-1]} \frac{dr}{r} = 0 \qquad \ldots(8)$$

Integrating, we have

$$\log U + \frac{\alpha s^2(\gamma - 1)}{(2 - s)[2 + s(\gamma - 1)]} \log r = \log k$$

$$\log U = \log k^{r - \alpha s^2(\gamma-1)/(2-s)\{2+s(\gamma-1)\}}$$

Therefore, expressions for shock velocity and shock strength for freely propagating shock can be written as shock velocity

$$U = kr^{-\alpha s^2(\gamma-1)/(2-s)\{2+s(\gamma-1)\}} \qquad \ldots(9)$$

Shock Strength

$$\frac{U}{a_0} = k'\gamma^{-\frac{1}{2}} r^{-\alpha s^2(\gamma-1)/(2-s)(2+s(\gamma-1)} \qquad \ldots(10)$$

Where, k is the constant of integration constant

CORRECTION DUE TO OVERTAKING DISTURBANCES

For overtaking disturbances, we have taken differential equation

$$dp - \rho a du + \frac{\alpha \rho a^2 u}{u - c}\frac{dr}{r} = 0$$

Now from the conditions (5), we have

$$p = \frac{2\rho_0 U^2}{\gamma + 1}$$

$$dp = \frac{2\rho_0 dU^2}{\gamma + 1} + \frac{2U^2 d\rho_0}{\gamma + 1}$$

$$u = \frac{2u}{\gamma + 1}$$

$$du = \frac{2dU}{\gamma + 1}$$

$$\rho a du = \left(\frac{2\gamma}{\gamma + 1}\right)^{½} \rho_0 \frac{dU^2}{(\gamma - 1)}$$

$$\frac{\rho a^2 u}{u - a} = \frac{4\gamma U^2 \rho_0}{(\gamma + 1)[2 - \{2\gamma(\gamma - 1\}]}\frac{dr}{r}$$

Putting these values of equation (10) and simplifying, we get

$$\frac{2\rho_0 dU^2}{\gamma+1} + \frac{2U^2}{\gamma+1} d\rho_0 - \left(\frac{2\gamma}{\gamma-1}\right)^{½} \rho_0 \frac{dU^2}{\gamma+1} + \frac{4\gamma\alpha U^2 \rho_0}{(\gamma+1)[2-\{2y(\gamma-1\}^{½}]}\frac{dr}{r} = 0 \quad \ldots(11)$$

Here it is supposed that when shock propagates, the medium perturbs and becomes non-uniform whose density distributions is given by $\rho_0 = \rho' \log r$. Under this assumption equation (11) reduces to,

$$(2 - s)dU^2 + \frac{2}{2 - s}\frac{drU^2}{r \log r} + \frac{4\alpha\gamma U^2}{(2 - s)[2 - \{2\gamma(\gamma - 1\}]^2}\frac{dr}{r} = 0$$

$$\frac{dU^2}{U^2} + \frac{2}{(2-s)}\frac{dr}{r \log r} + \frac{2\alpha\gamma}{(2-s)[2-\{2\gamma(\gamma-1\}]^2}\frac{dr}{r} = 0 \quad \ldots(12)$$

Where, $B_2 = \frac{2}{(2-s)}. C_2 = \frac{4\gamma}{(2-s)[2-\{2\gamma(\gamma-1]^2}$

Integration of equation (12) gives

$$\log U + B_2 \log r + C_2 \alpha \log r = \log K$$

$$\log U = \log(K - B_2 - \alpha C_2)$$

$$\therefore U = Kr^{\alpha C_2}(\log r)^{B_2}$$

Now from equation (5)

$$u - \frac{2U}{\gamma + 1}$$

$$du_+ = \frac{2dU}{\gamma+1} \quad \ldots(13)$$

Substituting these in equation (9) and simplifying

$$du_+ = \frac{2}{\gamma + 1}\frac{d}{dr}\left[Kr^{\ \alpha s^2(\gamma-1)/(2-s)\{2+2s[\gamma-1)}\right]$$

Similarly overtaking disturbances the fluid velocity increment may be expressed as

$$u_- = \frac{2U}{\gamma+1}$$

$$du_- = \frac{2dU}{\gamma+1} \qquad \ldots(14)$$

Thus, velocity increment due to overtaking disturbances can be written as

$$du_- = \frac{2}{\gamma-1}\frac{d}{dr}[Kr^{\alpha C_2(\log r)^{-B_2}}]$$

In presence of overtaking disturbances, the fluid velocity increment will be

$$du_+ + du_- = \frac{2}{\gamma+1}dU^+$$

Substituting values of du_+, du_- and simplifying, we get

$$\frac{2}{\gamma+1}\frac{d}{dr}\left[kr^{-\alpha s^2(\gamma-1)/(2-s)\{2+s(\gamma-1)\}}\right]$$

$$+\frac{2}{\gamma+1}\frac{dr}{r}\,[kr^{\alpha c_2}(\log r)^{-B_2}] = \frac{2}{\gamma+1}dU^* \qquad \ldots(15)$$

Integrating, we have the shock velocity

$$U^* = k\left[r^{-\alpha s^2(\gamma-1)/(2-5)\{2+s(\gamma-1)\}} + r^{\alpha C_2}(\log r)_2^{-B_2} + 1\right] \qquad \ldots(16)$$

Shock strength

$$\frac{U*}{\alpha_0} = k'\left[r^{-\alpha s^2(\gamma-1)/(2-5)\{2+s(\gamma-1)\}} + r^{\alpha C_2}(\log r)_2^{-B_2} + 1\right]\log r \qquad \ldots(17)$$

RESULT AND DISCUSSION

The relation (9) derived for freely propagation shock velocity, shows that shock decays asymptotically with propagation distance r. In presence of overtaking disturbances shock velocity modified to relation (16), whose variation also depends on adiabatic index γ, and propagation distances r, the variations of freely propagating shock velocity and modified by overtaking disturbances have been numerically obtained by taking initially $U/a_0 = 16$, at $r = 2$ for $\gamma = 1.33$ for cylindrical spherical shock.

It is found that shock velocity continuously decreases as shock advances for both freely propagating shock as well as with the inclusion (EOD). Similar variations in shock velocity for cylindrical shock have reported by Tripathi (1995). An increase in γ leads to increase in shock velocity for both FP and EOD.

Finally, the expressions for the pressure and the particle velocity behind the shock for freely propagating shock (p, u) and with the inclusion of EOD (p^*, u^*) are obtained. They are

$$p = \frac{2}{(\gamma+1)}k'\gamma^{-1/2}\,r^{-\alpha s^2}(\gamma-1)/(2-s)\{2+s(\gamma-1)\}$$

$$u = \frac{2}{\gamma+1}kr^{-\alpha s^2}\ (\gamma-1)/(2-s)\{2+s(\gamma-1)\}$$

and

$$p* = \frac{2}{\gamma^{1/2}(\gamma+1)}k'[r^{-\alpha s^2(\gamma-1)/(2-s)\{(2+s(\gamma-1)\}]} + r^{-\alpha C_2}\big(\log r)^{B_2} + 1]\log r$$

$$u^* = \frac{2}{(\gamma+1)}k\big[r^{-\alpha s^2(\gamma-)/(2-s)\{2+s(\gamma-1)\}} + r^{-\alpha C^2}(\log r)^{-B_2} + 1\big]$$

Thus, it is conclude that the results obtained by CCW method may be improved to some ended if effect of overtaking disturbances is taken into account.

REFERENCE :

1. Benger, A and Holt, M . : (1970), J.H.S. Lee, ALAAJ. 8 271
2. Chaturani, P. : (1976), Astrophysic, J. 209, 424.
3. Chester, W : (1954) Philos, Mag, 45 (7), 123.
4. Eddington : (1930) The International Constitution of Stars, Cambride University Press.
5. Grasso. F nd Pirozzoli S . : (2000) Phy. Fluid, 22(1), 205.