International Journal of Scientific Research in Science and Technology
© 2019 IJSRST | Volume 6 | Issue 3 | Print ISSN: 2395-6011 | Online ISSN: 2395-602X

Fuzzy Inventory System of Perishable Items with Reliability and Time Varying Demand

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Abstract - The proposed paper develops a deterministic inventory system of perishable items with reliability and time varying exponential demand. Reliability of products is one of the important factor in the success of every real life business organization. So highly reliable products are mostly preferred by the business organizations for optimization of their total cost/profit. A partially backlogging shortage is considered in the proposed inventory system and it is assumed that the unsatisfied demand is a function of waiting time. The objective of the present study is to optimizes the total variable inventory cost both in crisp and fuzzy seance. To support the study, a numerical example is given to show the validity of the developed model and also the behavior of key parameters in the optimal solution.

Keywords : Inventory, Perishability, Partial-backlogging, Reliability and Time Dependent Exponential Demand Rate.
Subject Classification: MSC 2010 (90B05)

INTRODUCTION

All respective parameters of the crisp inventory system are known with their certain values. But in reality, they are not so certain. Hence there is a need to consider the inventory systems in fuzzy environment. Inventory modeling is a particular field of operations research in which, we optimizes some real life problems. In controlling inventory, the following two factors (1) dete- rioration and (2) obsolescence have an important role. Deterioration is a realistic phenomenon and it is the loss of marginal value from the usefulness of original items. Obsolescence is the replacement of present products in the market by the upcoming new and better products.

In the past literature, some scholars were assumed that the demand rate may be constant, time varying increasing, decreasing, advertisement, price and inventory level dependent. Because, the demand of upcoming newly products/government policies such as insurance/GST(goods and service tax), life care vaccinations, fashionable garments, digital products, motor vehicles, smart phones etc. increases first for some time and later it becomes constant. In this area,Ghare and Schrader [1] worked on an inventory model for ex- ponentially decaying items. Zadeh were constructed two inventory models [2, 4]. The model [2] consists the concept of fuzzy set theory in inventory modeling. And the model [4] shows a new approach of decision processes for the analysis of a complex system. Bellan and Zadeh [3] analyz ed a decision policy for an inventory system in the sense of fuzziness. Park [5] focused on the interpretation of EOQ model in fuzzy sense. Hargia [6] worked on an op- timal EOQ model of perishable products with time varying demand. Chang et al. [7] determined a reorder point of an inventory system of perishable products with fuzzy type back order quantity. Yao and Lee [8] studied an in- ventory model with and without back order incorporating trapezoidal fuzzy order quantity. Kao and Hsu [9] determined a reorder point of a lot size in- ventory model for perishable items with fuzzy type demand. Yao and Chang [10] focused on an inventory system without back orders. Jaggi et al. [11] discussed an optimal ordering policy for an inventory system of perishable products incorporating inflation induced demand. Sahoo et al. [12] studied an inventory system of perishable products having constant deterioration, price dependent demand, time dependent holding cost. Jaggi et al. [13] constructed fuzzy inventory system for perishable items allowing shortages and time dependent demand rate. Behra and Tripathy developed two inven- tory models [14, 22]. The model [14] is fuzzy EOQ model having penalty cost and time dependent deterioration. And the model [22] consists the dis- cussion of replenishment policy of perishable products having reliability and time varying demand. Sahoo et al. [15] analyzed fuzzy inventory system for perishable products with time proportional exponential demand. Mona et al. [16] studied a fuzzy constrained probabilistic inventory model with trapezoidal fuzzy type parameters. Hossen et al. [17] established an inven- tory system of perishable products taking price and time dependent demand and fuzzy type inventory costs under inflation. Arora [18] studied inventory models for decaying items and allowing shortage.

Maragatham and Palani [19] worked on an inventory system of perishable products having shortages and lead time varying demand. Sahoo and Tripathy [20] constructed EOQ model for three parameter weibull decaying products with linear trended de- mand, time varying salvage and holding cost. Mohanty and Tripathy [21] constructed fuzzy inventory system of perishable products incorporating ex- ponentially decreasing demand and fuzzy type back order parameter. Naik and Patel [23] analyzed an inventory model consisting an imperfect quality for repairable products taking price and time dependent demand and various deterioration rates. Shah and Naik [24] developed supplier-retailer inventory system consisting coordinated production, ordering, shipment and pricing under trade credit. Kizito et al. [25] studied a stochastic inventory model for global supply chain problem. Maria et al. [26] focused on the implemen- tation of new parameters into a deterministic inventory system and studied streamline effectiveness indicators of the inventory management.

Definitions

Fuzzy Set: A fuzzy set Ã on the given universal set X is a set of ordered pairs,
Ã = {(x, $\lambda_{\tilde{A}}$ (x)) : $x \in X$ }
Where, $\lambda_{\tilde{A}} : X \to [0,1]$.

Fuzzy Number: A fuzzy number Ã is a fuzzy set on the real line, if its membership function $\lambda_{\tilde{A}}$ satisfying,

1. $\lambda_{\tilde{A}}(x)$ is upper semi continuous.
2. $\lambda_{\tilde{A}}(x) = 0$.
3. $\exists$ real numbers a_2 and a_3, $a_1 \le a_2 \le a_3 \le a_4$ such that $\lambda_{\tilde{A}}$ (x) is increasing on $[a_1, a_2]$, decreasing on $[a_3, a_4]$, and $\lambda_{\tilde{A}}(x) = 1$, for each x in $[a_2, a_3]$.

Trapezoidal Fuzzy Number: A trapezoidal fuzzy number (TFN) is specified by four tuple (a_1, a_2, a_3, a_4), where $a_1 < a_2 < a_3 < a_4$ and defined by continuous membership function $\lambda_{\tilde{A}} : X \to [0,1]$, as follows,

$$\lambda_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & otherwise. \end{cases}$$

Signed Distance: If $\lambda_{\tilde{A}} = (a, b, c, d)$ is a trapezoidal fuzzy number, the signed distance of $\lambda_{\tilde{A}}$ is given by, $d(\tilde{A}, 0) = \frac{(a+2b+2c+d)}{6}$.

3 Some Notations and Assumptions:

Corresponding to the proposed model, we have following assumptions and notations,

1. Demand R(t) is, $R(t) = ae^{rt}$, $a, r > 0$.
2. r is the reliability parameter.
3. δ is the partially backlogging parameter.

4. θ is constant deterioration.
5. $\tilde{\theta}$ is constant deterioration in fuzzy sense.
6. $\tilde{r}$ is the reliability parameter in fuzzy sense.
7. $\tilde{\delta}$ is the partially backlogging parameter in fuzzy sense.
8. o_C is the ordering cost per order.
9. h_C is the holding cost per unit per unit time.
10. s_C is the shortage cost per unit per cycle.
11. Q is the inventory level at time $t = 0$.
12. T_1 is the time of zero inventory level.
13. The cycle length is T .
14. The replenishment rate is infinite.
15. The lead time is zero.
16. $TC(T_1, T)$ is the total variable inventory cost per cycle.
17. $\tilde{TC}(T_1, T)$ is total variable inventory cost per cycle in fuzzy sense.

4 Mathematical Derivation of the Model:

Graphically, it has been seen that an inventory system contains the maximum inventory level Q in the beginning of each cycle. The maximum inventory level Q decreases due to demand and deterioration in the interval $[0, T_1]$. It becomes zero at time $t = T_1$. In the shortage interval $[T_1, T]$ the unsatisfied demand is backlogged at a rate of
$B(t) = \frac{1}{1+\delta(T-t)}$, where t is the waiting time and δ is the backlogging parameter .
At any time t in $[0, T]$, the instantaneous inventory level is given by the following differential equations,

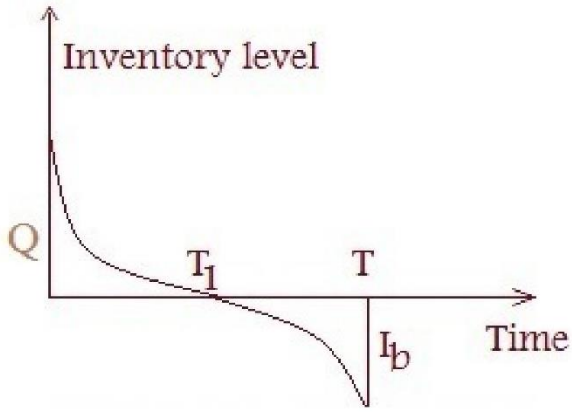

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Figure 1: Inventory Model

$$\frac{dI}{dt} + \theta I = -ae^{rt}, \qquad 0 \leq t \leq T_1 \tag{1}$$

With condition,

$$I(T_1) = 0$$

$$\frac{dI}{dt} = -\left(\frac{ae^{rt}}{1+\delta(T-t)}\right), \qquad T_1 \leq t \leq T \tag{2}$$

With condition,

$$I(T_1) = 0$$

The equations (1) and (2) have the solutions equations (3) and (4) respectively.

$$I = -at + aT_1 + \frac{a(\theta - r)}{2}t^2 + \frac{a(\theta + r)}{2}T_1^2 + \frac{ar\theta}{2}t^3 \qquad 0 \leq t \leq T_1 \tag{3}$$

(for the first order approximation of $e^{\theta t}$ and e^{rt})

$$\begin{aligned} I = -at + aT_1 - \frac{a(\delta + r)}{2}t^2 + \frac{a(\delta + r)}{2}T_1^2 + a\delta Tt - a\delta TT_1 + \frac{ar\delta}{2}Tt^2 \\ - \frac{ar\delta}{2}TT_1^2 - \frac{ar\delta}{3}t^3 + \frac{ar\delta}{3}T_1^3 \qquad T_1 \leq t \leq T \end{aligned} \tag{4}$$

(for the first order approximation of $e^{\theta t}$ and e^{rt})
Putting t=0 in equation (3), we obtain the initial order quantity Q, i.e

$$Q = aT_1 + \frac{a(\theta + r)}{2} T_1^2 \tag{5}$$

Putting t=T in equation (4),we obtain the back-order quantity I_B, i.e

$$\begin{aligned} I_B = &-aT + aT_1 - \frac{a(r-\delta)}{2}T^2 + \frac{a(r+\delta)}{2}T_1^2 - a\delta T T_1 \\ &+ \frac{ar\delta}{6}T^3 + \frac{ar\delta}{3}T_1^3 - \frac{ar\delta}{2}TT_1^2 \end{aligned} \tag{6}$$

Per cycle the total inventory cost is given by the equation (7)

$$TC(T_1, T) = \left(\frac{1}{T}\right)[O_C + H_C + D_C + S_C] \tag{7}$$

Where O_C, H_C, D_C and S_C are the corresponding ordering, inventory carrying , deterioration and shortage costs.
Per cycle, the ordering cost is,

$$O_C = \frac{o_c}{T} \tag{8}$$

Per cycle, the inventory carrying cost is,

$$H_C = \frac{h_c}{T}\int_0^{T_1} I(t)dt$$

or

$$H_C = \frac{h_c}{T}\left[\frac{a}{2}T_1^2 + \frac{2a(\theta - r)}{3}T_1^3 + \frac{ar\theta}{8}T_1^4\right] \tag{9}$$

Per cycle, the deterioration cost is,

$$D_C = \frac{d_c}{T}\left[Q - \int_0^{T_1} I(t)dt\right]$$

or

$$D_C = \frac{d_c}{T}\left[aT_1 + \frac{a(\theta + r - 1)}{2}T_1^2 - \frac{2a(\theta - r)}{3}T_1^3 - \frac{ar\theta}{8}T_1^4\right] \tag{10}$$

Per cycle, the shortage cost is,

$$S_C = -\frac{s_c}{T}\int_{T_1}^{T} I(t)dt$$

or

$$S_C = -\frac{as_c}{T}\left[-T + T_1 + \frac{(\delta - r)}{2}T^2 + \frac{(r+\delta)}{2}T_1^2 - \delta T T_1 + \frac{r\delta}{6}T^3 + \frac{r\delta}{3}T_1^3 - \frac{r\delta}{2}TT_1^2\right] \tag{11}$$

Putting these values of O_C, H_C, D_C and S_C in equation (7), we obtain,

$$\begin{aligned} TC(T_1,T) = &\frac{1}{T}\Big[o_c + a(d_c - s_c)T_1 + as_cT + \frac{a}{2}\left\{h_c + d_c(\theta + r - 1) - s_c(r+\delta)\right\}T_1^2 - \\ &\frac{a(\delta - r)s_c}{2}T^2 + a\delta s_c T T_1 + \frac{a}{3}\left\{2(h_c - d_c)(\theta - r) - r\delta s_c\right\}T_1^3 - \\ &\frac{ar\delta s_c}{6}T^3 + \frac{ar\delta s_c}{2}TT_1^2 + \frac{ar\theta(h_c - d_c)}{8}T_1^4\Big] \end{aligned} \tag{12}$$

The necessary conditions for the minimum of total cost $TC(T_1, T)$ are,

$$\frac{\partial TC(T_1,T)}{\partial T_1} = 0 \qquad \text{and} \qquad \frac{\partial TC(T_1,T)}{\partial T} = 0 \tag{13}$$

On solving the equations in equation (13), we obtain the optimum values of T_1 and T for which the total cost is minimum.

The total cost $TC(T_1, T)$ will also be optimum, if the following sufficient conditions are satisfied,

$$\left(\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TC(T_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(T_1,T)}{\partial T_1 \partial T}\right)^2 > 0 \text{ and } \left(\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}\right) > 0.$$

After differentiating the equation (12), we obtain the following equations,

$$\begin{aligned}\frac{\partial TC(T_1,T)}{\partial T_1} =&\frac{1}{T}\left[a(d_c-s_c)+a\left\{h_c+d_c(\theta+r-1)-s_c(r+\delta)\right\}T_1+\right.\\&a\delta s_cT+a\left\{2(h_c-d_c)(\theta-r)-r\delta s_c\right\}T_1^2+ar\delta s_cTT_1+\\&\left.\frac{ar\theta(h_c-d_c)}{2}T_1^3\right]\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial TC(T_1,T)}{\partial T} =&\frac{1}{T}\left[as_c-a(\delta-r)s_cT+a\delta s_cT_1-\frac{ar\delta s_c}{2}T^2+\right.\\&\left.\frac{ar\delta s_c}{2}T_1^2\right]-\frac{1}{T^2}\left[o_c+a(d_c-s_c)T_1+as_cT+\right.\\&\frac{a}{2}\left\{h_c+d_c(\theta+r-1)-s_c(r+\delta)\right\}T_1^2-\frac{a(\delta-r)s_c}{2}T^2+\\&a\delta s_cTT_1+\frac{a}{3}\left\{2(h_c-d_c)(\theta-r)-r\delta s_c\right\}T_1^3-\frac{ar\delta s_c}{6}T^3+\\&\left.\frac{ar\delta s_c}{2}TT_1^2+\frac{ar\theta(h_c-d_c)}{8}T_1^4\right]\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial^2 TC(T_1,T)}{\partial T_1^2} =&\frac{1}{T}\left[a\left\{h_c+d_c(\theta+r-1)-s_c(r+\delta)\right\}+ar\delta s_cT\right.\\&\left.2a\left\{2(h_c-d_c)(\theta-r)-r\delta s_c\right\}T_1+\frac{3ar\theta(h_c-d_c)}{2}T_1^2\right]\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial^2 TC(T_1,T)}{\partial T\partial T_1} =&\frac{1}{T}\left[a\delta s_c+ar\delta s_cT_1\right]-\frac{1}{T^2}\left[a(d_c-s_c)+\right.\\&a\left\{h_c+d_c(\theta+r-1)-s_c(r+\delta)\right\}T_1+a\delta s_cT+\\&\left.a\left\{2(h_c-d_c)(\theta-r)-r\delta s_c\right\}T_1^2+ar\delta s_cTT_1+\frac{ar\theta(h_c-d_c)}{2}T_1^3\right]\end{aligned} \tag{17}$$

$$\frac{\partial^2 TC(T_1,T)}{\partial T^2} = \frac{1}{T}\left[-a(\delta-r)s_c - ar\delta s_c T\right] - \frac{2}{T^2}\Big[as_c - a(\delta-r)s_c T + a\delta s_c T_1 - \frac{ar\delta s_c}{2}T^2 + \frac{ar\delta s_c}{2}T_1^2\Big] + \frac{2}{T^3}\Big[o_c + a(d_c-s_c)TT_1 + as_c T + \frac{a}{2}\left\{h_c + d_c(\theta+r-1) - s_c(r+\delta)\right\}T_1^2 - \frac{a(\delta-r)s_c}{2}T^2 \; a\delta s_c TT_1 + \frac{a}{3}\left\{2(h_c-d_c)(\theta-r) - r\delta s_c\right\}T_1^3 - \frac{ar\delta s_c}{6}T^3 + \frac{ar\delta s_c}{2}TT_1^2 + \frac{ar\theta(h_c-d_c)}{8}T_1^4\Big] \tag{18}$$

Fuzzy Model: The entire market is filled with uncertainty. The uncertainty is handled by fuzzy set theory. So in fuzzy environment, let us consider the uncertainty in parameters θ, r and δ. Because fuzziness is a closed possible approach to reality. Let $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$, $r = (r_1, r_2, r_3, r_4)$ and $\delta = (\delta_1, \delta_2, \delta_3, \delta_4)$ be the trapezoidal fuzzy numbers. The total variable inventory cost per cycle is given by the signed distance method as follows,

$$\tilde{TC}(T_1,T) = \frac{1}{6T}\left[\tilde{TC}_1(T_1,T) + 2\tilde{TC}_2(T_1,T) + 2\tilde{TC}_3(T_1,T) + \tilde{TC}_4(T_1,T)\right] \tag{19}$$

Where,

$$\tilde{TC}_1(T_1,T) = \frac{1}{T}\Big[o_c + a(d_c-s_c)T_1 + as_c T + \frac{a}{2}\left\{h_c + d_c(\theta_1+r_1-1) - s_c(r_1+\delta_1)\right\}T_1^2 - \frac{a(\delta_1-r_1)s_c}{2}T^2 + a\delta_1 s_c TT_1 + \frac{a}{3}\left\{2(h_c-d_c)(\theta_1-r_1) - r_1\delta_1 s_c\right\}T_1^2 - \frac{ar_1\delta_1 s_c}{6}T^3 + \frac{ar_1\delta_1 s_c}{2}TT_1^2 + \frac{ar_1\theta_1(h_c-d_c)}{8}T_1^4\Big] \tag{20}$$

$$
\begin{aligned}
\tilde{TC_2}(T_1,T) =& \frac{1}{T}\left[o_c + a(d_c - s_c)T_1 + as_cT + \right.\\
& \frac{a}{2}\left\{h_c + d_c(\theta_2 + r_2 - 1) - s_c(r_2+\delta_2)\right\}T_1^2 - \frac{a(\delta_2 - r_2)s_c}{2}T^2 + \\
& a\delta_2 s_c TT_1 + \frac{a}{3}\left\{2(h_c - d_c)(\theta_2 - r_2) - r_2\delta_2 s_c\right\}T_1^2 - \frac{ar_2\delta_2 s_c}{6}T^3 + \\
& \left.\frac{ar_2\delta_2 s_c}{2}TT_1^2 + \frac{ar_2\theta_2(h_c - d_c)}{8}T_1^4\right]
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\tilde{TC_3}(T_1,T) =& \frac{1}{T}\left[o_c + a(d_c - s_c)T_1 + as_cT + \right.\\
& \frac{a}{2}\left\{h_c + d_c(\theta_3 + r_3 - 1) - s_c(r_3+\delta_3)\right\}T_1^2 - \frac{a(\delta_3 - r_3)s_c}{2}T^2 + \\
& a\delta_3 s_c TT_1 + \frac{a}{3}\left\{2(h_c - d_c)(\theta_3 - r_3) - r_3\delta_3 s_c\right\}T_1^2 - \frac{ar_3\delta_3 s_c}{6}T^3 + \\
& \left.\frac{ar_3\delta_3 s_c}{2}TT_1^2 + \frac{ar_3\theta_3(h_c - d_c)}{8}T_1^4\right]
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\tilde{TC_4}(T_1,T) =& \frac{1}{T}\left[o_c + a(d_c - s_c)T_1 + as_cT + \right.\\
& \frac{a}{2}\left\{h_c + d_c(\theta_4 + r_4 - 1) - s_c(r_4+\delta_4)\right\}T_1^2 - \frac{a(\delta_4 - r_4)s_c}{2}T^2 + \\
& a\delta_4 s_c TT_1 + \frac{a}{3}\left\{2(h_c - d_c)(\theta_4 - r_4) - r_4\delta_4 s_c\right\}T_1^2 - \frac{ar_4\delta_4 s_c}{6}T^3 + \\
& \left.\frac{ar_4\delta_4 s_c}{2}TT_1^2 + \frac{ar_4\theta_4(h_c - d_c)}{8}T_1^4\right]
\end{aligned} \tag{23}
$$

Putting these values of $\tilde{TC_1}(T_1,T)$, $\tilde{TC_2}(T_1,T)$, $\tilde{TC_3}(T_1,T)$ and $\tilde{TC_4}(T_1,T)$ in equation (18), we obtain the equation (24),

$$
\begin{aligned}
\tilde{TC}(T_1,T) = & \frac{1}{T}[6o_c + 6a(d_c - s_c)T_1 + 6as_cT \\
& + \frac{a}{2}[6h_c + d_c(\theta_1 + 2\theta_2 + 2\theta_3 + \theta_4 + r_1 + 2r_2 + 2r_3 + r_4 - 6) \\
& - s_c(r_1 + 2r_2 + 2r_3 + r_4 + \delta_1 + 2\delta_2 + \delta_3 + \delta_4)]]
\end{aligned} \tag{24}
$$

$$\frac{\partial \tilde{TC}(T_1,T)}{\partial T} = \frac{1}{4T}[6a(d_c - s_c) + a[6h_c + d_c(\theta_1 + 2\theta_2 + 2\theta_3 + \theta_4 + r_1 + 2r_2 + 2r_3 + r_4 - 6) - s_c(\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4 + r_1 + 2r_2 + 2r_3 + r_4)]T_1 + as_c(\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4)T + a[2(h_c - d_c)(\theta_1 + 2\theta_2 + 2\theta_3 + \theta_4 - r_1 + 2r_2 + 2r_3 + r_4) - s_c(r_1\delta_1 + 2r_2\delta_2 + 2r_3\delta_3 + r_4\delta_4)]T_1^2 + as_c(r_1\delta_1 + 2r_2\delta_2 + 2r_3\delta_3 + r_4\delta_4]TT_1 + \frac{a(h_c - d_c}{2}(r_1\theta_1 + 2r_2\theta_2 + 2r_3\theta_3 + r_4\theta_4)T_1^3] \tag{25}$$

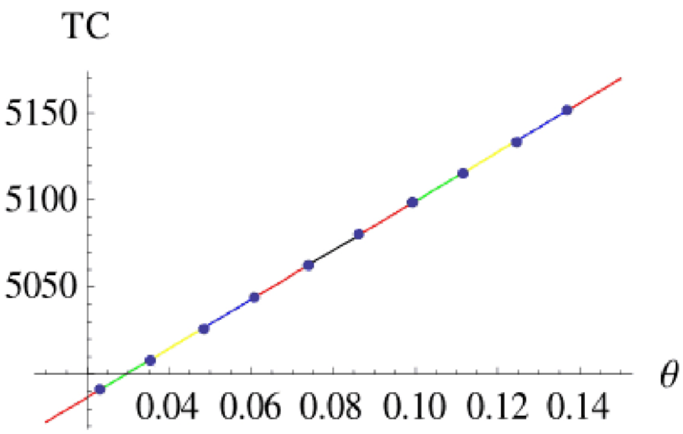
$$\frac{\partial \tilde{TC}(T_1,T)}{\partial T} = \frac{1}{6T}[6as_c - as_c(\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4 - r_1 + 2r_2 + 2r_3 + r_4)T + as_c(\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4)T_1 - \frac{as_c}{2}(r_1\delta_1 + 2r_2\delta_2 + 2r_3\delta_3 + r_4\delta_4)T^2 + \frac{as_c}{2}(r_1\delta_1 + 2r_2\delta_2 + 2r_3\delta_3 + r_4\delta_4)T_1^2] - \frac{1}{6T^2}[6o_c + 6a(d_c - s_c)T_1 + 6as_cT + \frac{a}{2}[6h_c + d_c(\theta_1 + 2\theta_2 + 2\theta_3 + \theta_4 + r_1 + 2r_2 + 2r_3 + r_4 - 6) - s_c(\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4 + r_1 + 2r_2 + 2r_3 + r_4)]T_1^2 - \frac{as_c}{2}(\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4 - r_1 - 2r_2 - 2r_3 - r_4)T^2 + as_c(\delta_1 + 2\delta_2 + 2\delta_3 + \delta_4)TT_1 + \frac{a}{3}[2(h_c - d_c)(\theta_1 + 2\theta_2 + 2\theta_3 + \theta_4 - r_1 - 2r_2 - 2r_3 - r_4) - s_c(r_1\delta_1 + 2r_2\delta_2 + 2r_3\delta_3 + r_4\delta_4)]T_1^3 - \frac{as_c}{6}(r_1\delta_1 + 2r_2\delta_2 + 2r_3\delta_3 + r_4\delta_4)T^3 + \frac{as_c}{2}(r_1\delta_1 + 2r_2\delta_2 + 2r_3\delta_3 + r_4\delta_4)TT_1^2 + \frac{a(h_c - d_c)}{8}(r_1\theta_1 + 2r_2\theta_2 + 2r_3\theta_3 + r_4\theta_4)] \tag{26}$$

Example: Let us assume the numerical data for corresponding parameters in appropriate units as follows,
$a = 1000,\ \theta = 0.01,\ r = 0.1,\ \delta = 1,\ o_c = 50,\ h_c = 5,\ d_c = 3$ and $s_c = 8$

Table 1: variation in $TC(T_1, T)$ w.r.to deterioration parameter θ

θ	T_1	T	$TC(T_1, T)$
0.01	0.8318	1.2907	4972.5017
0.05	0.7588	1.2074	5024.4693
0.08	0.7172	1.1593	5058.9035
0.10	0.6935	1.1317	5080.1998
0.15	0.6444	1.0738	5128.8448

The table 1, shows that as we increase the parameter θ, the total cost $TC(T_1, T)$ is increased. The reason is that the inventory carrying cost is increased. The values of on hand inventory T_1 and cycle length T are decreased.



t.png

Figure 2: variation in TC and θ

Table 2: variation in $TC(T_1, T)$ w.r.to backlogging parameter δ

δ	T_1	T	$TC(T_1, T)$
1	0.8318	1.2907	4972.5017
2	0.6614	0.9044	4531.0603
3	0.1339	0.4281	7211.2714
4	0.0354	0.3433	7857.4241
5	0.0197	0.3297	8049.2914
6	0.0155	0.3260	8107.3129

The table 2, shows that the increment in backlogging parameter δ reduces the total cost. The total cost $TC(T_1, T)$ is first decreases and then increases. The reason is that the purchasing cost is simultaneously decreases and increases. The values of T_1 and T are decreased.

d.png

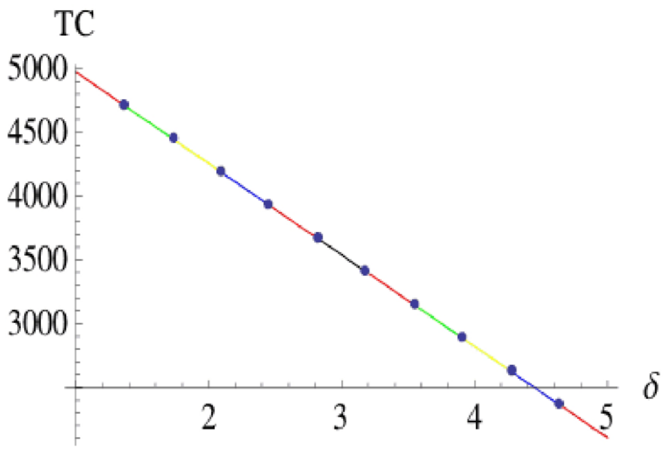


Figure 3: variation in TC and δ

Table 3: variation in $TC(T_1, T)$ w.r.to reliability parameter r

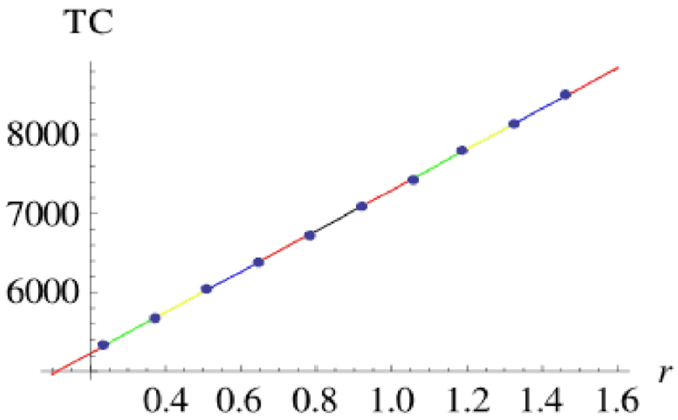
r	T_1	T	$TC(T_1, T)$
0.1	0.8318	1.2907	4972.5017
0.4	0.7041	1.2649	5793.7190
0.8	0.3759	0.9704	6893.0312
1.0	0.2602	0.8594	7398.1719
1.2	0.1718	0.7752	7871.9534
1.6	0.0432	0.6593	8740.2189

From the table 3, we observe that the increment in reliability parameter r also reduces the total cost. The total cost $TC(T_1, T)$ is increased. The reason is that either the ordering or the purchasing cost is increased. The values of on hand inventory T_1 and cycle length T are decreased.

Table 4: variation in $TC(T_1, T)$ w.r.to demand parameter a

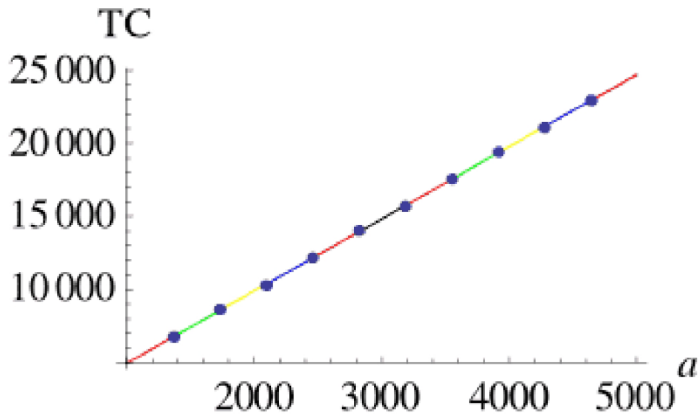
a	T_1	T	$TC(T_1, T)$
1000	0.8318	1.2907	4972.5017
2000	0.8202	1.2808	9906.1143
3000	0.8163	1.2775	14839.6268
4000	0.8144	1.2758	19773.1142
5000	0.8132	1.2748	24706.5914

The table 4, shows that as we increase the demand parameter a, the total cost $TC(T_1, T)$ is increased. The reason is that either the deterioration or the holding cost is increased. The values of T_1 and T are decreased.



r.png

Figure 4: variation in TC and r



a.png

Figure 5: variation in TC and a

Fuzzy cases:

1: When $\theta = (\theta_1,\ \theta_2,\ \theta_3,\ \theta_4)$, $r = (r_1,\ r_2,\ r_3,\ r_4)$ and $\delta = (\delta_1,\ \delta_2,\ \delta_3,\ \delta_4)$ having trapezoidal fuzzy sense, then the variation in total cost has been shown by the following table 5,

Table 5: variation in total cost, when θ, r and δ are trapezoidal fuzzy numbers

$\theta_1 = (\theta_1, \theta_2, \theta_3, \theta_4), r = (r_1, r_2, r_3, r_4), \delta = (\delta_1, \delta_2, \delta_3, \delta_4)$	T_1	T	$TC(T_1, T)$
$\theta = (0.01, 0.02, 0.03, 0.04), r = (0.1, 0.2, 0.3, 0.4), \delta = (1, 2, 3, 4)$	0.4187	0.6178	4660.7266
$\theta = (0.02, 0.03, 0.04, 0.05), r = (0.2, 0.3, 0.4, 0.5), \delta = (2, 3, 4, 5)$	0.3480	0.4961	4584.2206
$\theta = (0.03, 0.04, 0.05, 0.06), r = (0.3, 0.4, 0.5, 0.6), \delta = (3, 4, 5, 6)$	0.3018	0.4199	4539.6193
$\theta = (0.04, 0.05, 0.06, 0.07), r = (0.4, 0.5, 0.6, 0.7), \delta = (4, 5, 6, 7)$	0.2689	0.3669	4512.1778

From the table 5, we conclude that the jointly increment in θ, r and δ, slightly reduces the $TC(T_1, T)$. Also the on hand inventory of T_1 and cycle length T are decreased.

2: When $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ and $r = (r_1, r_2, r_3, r_4)$ are trapezoidal fuzzy numbers and δ is fixed, then the variation in total cost is given by table 6,

Table 6: variation in total cost, when θ and r are trapezoidal fuzzy numbers

$\theta = (\theta_1, \theta_2, \theta_3, \theta_4), r = (r_1, r_2, r_3, r_4)$	T_1	T	$TC(T_1, T)$
$\theta = (0.01, 0.02, 0.03, 0.04), r = (0.1, 0.2, 0.3, 0.4)$	0.4187	0.6178	4660.7266
$\theta = (0.02, 0.03, 0.04, 0.05), r = (0.2, 0.3, 0.4, 0.5)$	0.3775	0.5802	4820.0124
$\theta = (0.03, 0.04, 0.05, 0.06), r = (0.3, 0.4, 0.5, 0.6)$	0.3451	0.5508	4974.0581
$\theta = (0.04, 0.05, 0.06, 0.07), r = (0.4, 0.5, 0.6, 0.7)$	0.3178	0.5259	5123.7693

The table 6, shows that the jointly increase in θ and r, increases the $TC(T_1, T)$ and decreases the values of T_1 and T.

3: When only $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ is a trapezoidal fuzzy number and r and δ are fixed trapezoidal fuzzy numbers, the variation in total cost is given by the table 7,

Table 7: variation in total cost, when θ is a trapezoidal fuzzy number

$\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$	T	T_1	$TC(T_1, T)$
$\theta = (0.01, 0.02, 0.03, 0.04)$	0.4187	0.6178	4660.7266
$\theta = (0.02, 0.03, 0.04, 0.05)$	0.4156	0.6143	4668.3161
$\theta = (0.03, 0.04, 0.05, 0.06)$	0.4126	0.6108	4675.7701
$\theta = (0.04, 0.05, 0.06, 0.07)$	0.4096	0.6074	4683.1469

The table 7, shows that increment in θ increases the total cost and decreases the T_1 and T.

CONCLUSION

In this study, we constructed a deterministic model for perishable products with reliability and time varying exponential demand. From the numerical observations performed on various parameters, we conclude that parameters δ, r and a are more sensitive than the parameter θ. Therefore, the total cost is deeply affected by these parameters δ, r and a. The reason is that the demand of highly reliable products increases continuously and after on hand inventory, products are backlogged at an increased purchasing cost. There- fore, the wholesaler/retailer can reduces their total cost either by backlogging the minimum number of units of seasonal/periodical products that are re- quired for completing the unsatisfied demand of customers in a given period of time or controlling their deterioration. Because after a certain period of time the deterioration or backlogging of the seasonal/periodical products will increases slowly. In future, scholars can generalize it by considering differ- ent assumptions and conditions on advertising, premium facility, advance payment, delay in payment, inventory costs, selling price, demand rate and backlogging rate etc..

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