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Interacting Two Fluid Model In Higher Dimensions

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ABSTRACT

In the framework of Kaluza-Klein cosmology, we study the dynamics of two interacting fluids in a spatially flat universe. One of them is the pressure less dark matter (DM) and the other is the dark energy (DE) satisfying a barotropic equation with constant equation of state (EoS) parameter. We consider two separate cases when $\omega_d \neq 0$ and $\omega_d = -1$ i.e. Cosmological constant. We observe that the evolution equations for interacting components and Hubble parameter can be solved completely as a function of scale factor or redshift.

Keywords : Kaluza-Klein Cosmology, Dark Matter, Dark energy, Interacting cosmic fluid, Cosmological parameter.

I. INTRODUCTION

The concept of non-Euclidean geometry proposed by Riemann led to a formalism to describe geometry of arbitrary dimensional manifold. Guided by this idea Clifford, Helmholtz and Hinton in late 1800 speculated a higher dimensional space-time. The concept of extra-dimensions is found important to realize unification of the fundamental forces in nature including quantization of the gravitation. Kaluza and Klein [1, 2] introduced an extra dimension to unify gravity with electromagnetism and the resulting theory of gravity based on a compact space-time is known as Kaluza-Klein theory (in short, KK). This model has been used in many literature for studying the models of cosmology as well as particle physics [3, 4].

Cosmological models are usually constructed taking into account perfect fluids or a mixture of non-interacting perfect fluids [5] and each of them evolves separately according to energy conservation law. However, there is no evidence confirming that this will be the only scenario, hence it is interesting to study cosmological models with an interacting mixture of fluids. The exchange of energy among these fluids might play an important role in the evolution of the universe. Recent observations predict that dark matter and dark energy are the major components of the universe. Due to the lack of knowledge on dark matter and dark energy, most of the investigations in the dark sector rely on the assumption that these two unknown components evolve independently. However, it is possible that the coupling in the dark sector might be responsible for late acceleration of the universe. Wettrich [6] introduces model featuring an interaction between matter and dark energy. Billyard and Coley [7] studied spatially flat isotropic cosmological model in the presence of a mixture of scalar field with an exponential potential and a perfect fluid. They included an interaction term, which helps to understand the transfer of energy from the scalar field to matter component. It is found that the interaction significantly affect the qualitative behaviour of the evolution. The late-time behaviour of these models may be of cosmological interest. Assuming an exponential potential and a linear coupling, Amendola [8] studied the effect of a coupled quintessence field. Farrar and Peebles [9] studied physical processes where the dark matter particles are coupled to scalar field by a Yukawa coupling and other processes are explored in Ref. [10]. A mixture of two interacting fluids is considered here to study cosmological evolution in the frame work of KK Cosmology.

Sharov et al [11] studied the dynamics of a mixture of two barotropic interacting cosmic fluids namely pressure less dark matter (DM) and dark energy (DE) satisfying a barotropic equation $p_d = \omega_d \rho_d$, ω_d Is the equation of state (EoS), in a spatially flat FLRW universe. Here we study the same interacting model between DM and DE in 5d general relativity i.e. in the framework of KK cosmology. This paper is organized as follows: In section II. We set up the Einstein field equation in higher dimensions i.e. KK model. In section III. The analytic solutions for DM and DE with constant equation of state is presented and finally in section IV. We give a brief discussion.

II. KALUZA-KLEIN COSMOLOGY

The Einstein field equation is given by

$$R_{AB} - \frac{1}{2} g_{AB} R = k T_{AB}, \tag{1}$$

where A and B runs from 0 to 4, R_{AB} Is the Ricci tensor. The 5-dimensional space time metric of KK- cosmology is given by [12]

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2) + (1-kr^2)d\psi^2\right], \tag{2}$$

where $a(t)$ Denotes the scale factor and $k = 0, 1, (-1)$ represents the curvature parameter for flat and closed (open) universe. Considering a cosmological model where KK universe filled with a perfect fluid. The Einstein's field equation for the metric given by Eq. (4) becomes

$$\rho = 6\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \tag{3}$$

$$p = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2} \tag{4}$$

For simplicity, we consider a flat universe, i.e. $k = 0$. The above Eqs. (3) and (4) reduces to

$$\rho = 6\frac{\dot{a}^2}{a^2}$$

$$p = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2}$$

The Hubble parameter is defined as $H = \frac{\dot{a}}{a}$. The covariant derivative of five dimensional energy-momentum tensor $T^{AB}_{;B} = 0$ yields the continuity equation:

$$\dot{\rho} + 4H(p + \rho) = 0 \tag{5}$$

Using the equation of state $p = \omega\rho$ in five dimensions, the equation of continuity reduces to

$$\dot{\rho} + 4H\rho(1 + \omega) = 0 \tag{6}$$

We consider two types of interacting fluids with total energy density as $\rho = \rho_m + \rho_d$, where ρ_m and ρ_d represent energy densities for pressure less dark matter (DM) and dark energy (DE). Here in the case of interacting models the following equations results from the continuity equation:

$$\dot{\rho}_m + 4H\rho_m = Q, \tag{7}$$

$$\dot{\rho}_d + 4H\rho_d(1 + \omega_d) = -Q, \tag{8}$$

where Q denotes the interaction between DM and DE.

III. EQUATION OF STATE PARAMETER AND EVOLUTION

In this framework we consider the following form of interaction term [12] defined as

$$Q = \alpha H[\rho_m' + \rho_d'], \tag{9}$$

where α is the dimensionless coupling parameter. Here prime denotes the differentiation with respect to $x = \ln(a/a_0)$, a_0 is the present value of the scale factor which is taken as $a_0 = 1$in the present work. This interaction term can also be written as

$$Q = -4\alpha H(\rho_m + \rho_d + \omega_d \rho_d) \tag{10}$$

Using the expression of Q in equation (9), the conservation equations (7) and (8) give

$$(1-\alpha)\rho_m' + 4\rho_m - \alpha\rho_m' = 0 \tag{11}$$

$$(1+\alpha)\rho_d' + 3(1+\omega_d) + \alpha\rho_d' = 0 \tag{12}$$

with EoS for pressure less DM (ω_m^{eff}) and DE (ω_d^{eff}) as

$$\omega_m^{eff} = -\frac{Q}{4H\rho_m} = \frac{\alpha}{r}[r + 1 + \omega_d] \tag{13}$$

$$\omega_d^{eff} = \omega_d - \frac{Q}{4H\rho_d} = \omega_d - \alpha[r + 1 + \omega_d], \tag{14}$$

where $r = \frac{\rho_m}{\rho_d}$ is called coincidence parameter.

So the effective EoS parameter due to interaction is given by

$$\omega^{eff} = \omega_m^{eff} + \omega_d^{eff} = \omega_d + \frac{\alpha}{r}(1-r)[r + 1 + \omega_d] \tag{15}$$

The effective EoS parameter (ω^{eff}) for the interacting system becomes variable in nature, even if the same of the component fluids are constant.

If the energy transfers from DE to DM due to interaction i.e., in case of $Q > 0$, $\omega_m^{eff} < 0$, which means that the effective EoS of the pressure less dark matter may behave like a candidate of dark energy (EoS $<\frac{1}{4}$, if $Q > H\rho_m$). On the other hand the effective EoS for dark energy satisfies $\omega_d < \omega_d^{eff} < \infty$.

Again if < 0 , i.e., if energy transfer from DM to DE, $\omega_m^{eff} > 0$ and $\omega_d^{eff} < \omega_d$.

However, introducing the total energy density of the universe, $\rho = \rho_m + \rho_d$, ρ_m And ρ_d can be expressed as

$$\rho_d = -\frac{\rho' + 4\rho}{4\omega_d} \tag{16}$$

$$\rho_m = \frac{\rho' + 4(1+\omega_d)\rho}{4\omega_d} \tag{17}$$

We also obtain the following second order differential equation for the total energy density ρ that can describe the possible dynamics of the universe and this is given by

$$\rho'' + 4\rho'\left[2 + (1-\alpha)\omega_d - \frac{\omega_d'}{4\omega_d}\right] + 16\rho\left[(1+\omega_d) - \frac{\omega_d'}{4\omega_d}\right] = 0 \tag{18}$$

IV. INTERACTION DYNAMICS

Now we will consider the following cases –

When ω_d is independent of time: in this case Eq. (18) becomes

$$\rho'' + 4\rho'[2 + (1-\alpha)\omega_d] + 16\rho[(1+\omega_d)] = 0, \tag{19}$$

which solves for ρ as

$$\rho = \rho_1 e^{m_1 x} + \rho_2 e^{m_2 x} = \rho_1 a^{m_1} + \rho_2 a^{m_2}. \tag{20}$$

ρ_1 and ρ_2 are two arbitrary constants and

$m_1 = 2[-\{2 + (1 - \omega_d)\} + \sqrt{(1-\alpha)^2 \omega_d{}^2 - 4\alpha\omega_d}]$
$m_2 = 2[-\{2 + (1 - \omega_d)\} - \sqrt{(1-\alpha)^2 \omega_d{}^2 - 4\alpha\omega_d}]$.
Here m_1 and m_2 are real, as $\omega_d < -1/3$.
In terms of density parameters Eq. (20) can be written as

$$\left(\frac{H}{H_0}\right)^2 = \Omega_1 (1+z)^{-m_1} + \Omega_2 (1+z)^{-m_2}, \tag{21}$$

where $\Omega_1 = \frac{\rho_1}{\rho_{c0}}$, $\Omega_2 = \frac{\rho_2}{\rho_{c0}}$ and $\rho_{c0} = \frac{6H_0^2}{8\pi G}$. Again the energy densities of DM and DE can be written as

$$\rho_d = -\frac{\rho_1(1+z)^{-m_1}(m_1+4) + \rho_2(1+z)^{-m_2}(m_2+4)}{4\omega_d} \tag{22}$$

and

$$\rho_m = -\frac{\rho_1(1+z)^{-m_1}(m_1+4) + \rho_2(1+z)^{-m_2}(m_2+4)}{4\omega_d} \tag{23}$$

Hence at present time,

$$\rho_{d0} = -\frac{\rho_1(m_1+4) + \rho_2(m_2+4)}{4\omega_{d0}} = -\frac{\rho_1(m_1+4) + \rho_2(m_2+4)}{4\omega_d}$$

and

$$\rho_{m0} = \frac{\rho_1\{m_1+4(1+\omega_{d0})\} + \rho_2\{m_2+4(1+\omega_{d0})\}}{4\omega_{d0}} = \frac{\rho_1\{m_1+4(1+\omega_{d0})\} + \rho_2\{m_2+4(1+\omega_{d0})\}}{4\omega_d},$$

where ω_{d0} Is the present value of EoS of DE, but here $\omega_{d0} = \omega_d$. Moreover the density parameters for DE and DM at present age can be obtained as

$$\Omega_{d0} = -\frac{\Omega_1(m_1+4) + \Omega_2(m_2+4)}{4\omega_{d0}} \tag{24}$$

and

$$\Omega_{m0} = \frac{\Omega_1\{m_1+4(1+\omega_{d0})\} + \Omega_2\{m_2+4(1+\omega_{d0})\}}{4\omega_{d0}} \tag{25}$$

Now Ω_1 And Ω_2 Can be solved as

$$\Omega_1 = \frac{\Omega_{d0}\{\{m_2+4(1+\omega_{d0})\} + \Omega_{m0}(m_2+4)}{(m_2-m_1)} \text{ and } \Omega_2 = \frac{\Omega_{d0}\{\{m_1+4(1+\omega_{d0})\} + \Omega_{m0}(m_1+4)}{(m_2-m_1)}$$

When DE is the cosmological constant: in this case we have $\omega_d = -1$, hence $m_1 = 0$ and $m_2 = -4(1+\alpha)$. So the explicit analytic expressions for DE and DM densities become

$$\rho_d = \rho_1 - \rho_2 \alpha (1+z)^{4(1+\alpha)} \tag{26}$$

and

$$\rho_m = \rho_1 - \rho_2 (1+\alpha)(1+z)^{4(1+\alpha)}, \tag{27}$$

with total density $\rho = \rho_1 - \rho_2(1+z)^{4(1+\alpha)}$.
So at present time, the energy densities can be written as

$$\rho_d = \rho_1 - \rho_2 \alpha \tag{28}$$

and

$$\rho_m = \rho_1 - \rho_2(1+\alpha). \tag{29}$$

Moreover, we get the present day density parameters for DM and DE as follows:
$\Omega_{d0} = \Omega_1 - \alpha\Omega_2$ and $\Omega_{m0} = (1+\alpha)\Omega_2$. Also we get $\Omega_2 = \frac{\Omega_{m0}}{1+\alpha}$ and $\Omega_1 = \Omega_{d0} + \frac{\alpha\Omega_{m0}}{1+\alpha}$
Again the Eq. (21) reduces to

$$\left(\frac{H}{H_0}\right)^2 = \Omega_1 + \Omega_2 (1+z)^{-4(1+\alpha)} \tag{30}$$

V. DISCUSSION

In this work, we have described the dynamics of two interacting fluids, one is the pressureless dark matter and the other one is dark energy satisfying a barotropic equation of state with constant EoS parameter in the framework of Kaluza and Klein. We consider $5D$ spatially flat universe for our present work. Also we look at a such interacting scenario where DE interacts with pressureless DM through a non-gravitational interaction which is characterized by a single coupling parameter α. We have described two different cases where $\omega_d \neq 0$ and $\omega_d = -1$. It is noted that the equations describing the evolution of both fluids can be analytically solved as a function of scale factor or red-shift parameter.In our present work it can be observed that due to the introduction of 5D general relativity, the possible dynamics of the flat universe does not demand any notable variation in comparison to that observed in spatially flat FLRW universe. It should be remembered here that we have not considered the models for variable EoS in DE. Moreover the model parameters have not been constrained using the current astronomical observations.

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