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A Simple Hybrid Method for Finding the Root of Nonlinear Equations

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ABSTRACT

In this paper, we proposed a simple modification of McDougall and Wotherspoon [11] method for approximating the root of univariate function. Our modification is based on the approximating the derivative in the corrector step of the proposed McDougall and Wotherspoon Newton like method using secant method. Numerical examples demonstrate the efficiency of the proposed method.
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I. INTRODUCTION

Consider a problem for solving nonlinear equation of the form

$$f(x)=0, \qquad (1)$$

where $f : \mathbf{R} \rightarrow \mathbf{R}$ is continuously differentiable function suppose there is a solution $x^* \in \mathbf{R}$ for which $f(x^*) = 0$. Newton's method is one of the famous and well known method of solving equation (1)[11] Newton's method iteratively produces a sequence$\{x_k\}$ of approximations that converges quadratically to a simple root $x*$ from any given initial point $x_0 \in \mathbf{R}$, in the neighborhood of x^* via:

$$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}, \qquad k=0,1,2,.... \qquad (2)$$

where $q_k=\frac{f(x_k)-f(x_{k-1})}{x_k-x_{k-1}}$.

In an attempt to reduce the computational cost of Newton's method, secant methods have been introduced [5]. These methods approximate the derivative in Newton's method using secant line.

Starting with two estimate of the root x_{-1}, x_0.the iterate formula is given by

$$x_{k+1}=x_k-q_k^{-1}f(x_k), \qquad k=0,1,2,... \qquad (3)$$

The convergence rate of secant method is normally superlinear.

Several modifications of Newton method was given in order to reduce its computational cost or to increase its rate of convergence, see, for example, [2, 6, 8, 10, 13, 14] and reference therein. For determining the root of a nonlinear equation, Weerakoon and Fernando [16], suggest an improvement to the iteration of Newton's method which involves an definite integral of the derivative of the function, and the relevant area is approximated by rectangular rule. Homeier [3], consider a modification of Newton method for finding zero of a univariate function. The modification converges cubically. Per iteration it requires one evaluation of the function and two evaluation of its derivative. Thus, the modification is suitable if the calculation of the derivative has a similar or lower cost than that of the function itself. Ozban [12], present some new variants of Newton method based on harmonic mean and midpoint integration rule. The order of convergence of the proposed method is three. Kou et al. [7], present a new modification of Newton's method for solving nonlinear equation which converges cubically. The modification requires two function evaluation and one first derivative evaluation. Thus, the new method is preferable if the computational cost of the first derivative is equal or more than those of the function itself. Jayakumar [5], presents a new class of Newton's method for solving single nonlinear equation. The method is the generalization of Simpson's integral rule applied on the Newton's theorem.

Wang [15], present a third-order family of Newton-like iteration method for solving nonlinear equations. These methods avoid computation of second-order derivatives and require one evaluation of the function and two evaluations of the first derivative per iteration. Newton's method is said to fail in certain cases leading to oscillation, divergence to increasingly large number, or off-shooting away to another root further from the desired domain or off shooting to an invalid domain where the function may not be defined. Tiruneh et al. [9], argue that a solution is still possible in most of these cases by application of Newton's method alone without resorting to other methods and with the modified formula based on Newton's method has better convergence characteristic than the classical Newton's method. For the root of a nonlinear equation, (1) McDougall and Wotherspoon [11] presents a simple modification to the standard Newton's method for approximating the root of a univariate function. The modified method converges faster with a convergence order $1+\sqrt{2}\approx 2.4$ which makes it super quadratic convergence rate compared with quadratic for the standard Newton's method.

McDougall and Wotherspoon, obtained the following scheme: Given the initial starting point x0,. for k = 0, set

$$x_0^* = x_0$$

then

$$x_1 = x_0 - \frac{f(x_0)}{f'(\frac{1}{2}[x_0 + x_0^*])} = x_0 - \frac{f(x_k)}{f'(x_0)}$$

followed by $(for\ k \geq 1$

$$x_k* = x_k - \frac{f(x_k)}{f'(\frac{1}{2}[x_{k-1} + x_{k-1}^*])}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(\frac{1}{2}[x_k + x_k^*])}$$

Our aim here is to present a hybrid method for solving nonlinear equations of one variable. This is achieved by avoiding the computation of derivative in the corrector step of the McDougall and Wotherspoon [11]

The rest of this paper is organized as follows. Section 2 is designed for the description and the algorithm of the new method. In section 3 computational experiment of the proposed method compared to the existing classical method is given, and finally the conclusion comes in section 4.

II. METHODS AND MATERIAL

Description of the New Method

In this section, we present our new scheme. Our method is based on approximating the derivative in the corrector step using secant method approach.

Trevor and Simon(2014)[11] suggest a predictor-corrector rule of the form ; given x0, for k = 0

set

$$x_0^* = x_0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(\frac{1}{2}[x_0 + x_0^*])} = x_0 - \frac{f(x_k)}{f'(x_0)}$$

$for\ k \geq 1$

$$x_k* = x_k - \frac{f(x_k)}{f'(\frac{1}{2}[x_{k-1} + x_{k-1}^*])}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(\frac{1}{2}[x_k + x_k^*])}.$$

Here, we approximate the derivative in each iteration (not including the first iteration) using secant method. Thus, we obtain the following modified scheme f or k = 0

$$x^* = x_0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$for\ k \geq 1$

$$x_k^* = x_k - \frac{f(x_k)}{f'[\frac{1}{2}(x_{k-1} + x_{k-1}^*)]}$$

$$x_{k+1} = x_k - \sigma_k^{-1} f(x_k)$$

where

$$\sigma_k = \frac{f(x_k^*) - f(x_k)}{x_k^* - x_k}$$

Algorithm for the Propose Method

Step 0: Given $x_0 \in \mathbb{R}$, stoping tolerance ϵ for $k = 0$ set $x_0^* = x_0$
Step 1: Compute $f(x_0)$ and $f'(x_0)$
Step 2: Compute $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
Step 3: Checking the stoping condition, i.e if $|x_1 - x_0| \leq \epsilon$ stop.
Step 4: For $k \geq 1$ compute $x_k^* = x_k - \frac{f(x_k)}{f'[\frac{1}{2}(x_{k-1}+x_{k-1}^*)]}$
Step 5: Compute $x_{k+1} = x_k - \sigma_k^{-1} f(x_k)$, where $\sigma_k = \frac{f(x_k^*)-f(x_k)}{x_k^*-x_k}$
Step 6: Check the stoping condition, if $|x_{k+1} - x_k| \leq \epsilon$ stop.
Step 7: Set $k = k+1$ and goto $step$ 4

III. RESULT AND DISCUSSION

Numerical Results

Computational test of the method were performed using MATLAB R2010a. All computations were carried out on a PC with Intel COREi3 processor with 4GB of RAM. We compare the performance of McDougall and Wotherspoon method (TSM) and the proposed method (PM) using 20 test functions also considered [11]. The stopping criteria used for these comparisons are $| x_{n+1} - x_n | < \delta$ where $\delta = 1.0e - 27$. For generating complex Mathematical Equation that is third party window. It display math questions in GUI but in backend it generate XML file so that later we can parse the equations and can evaluate.

function	x_0	TSM	$\lvert f(x_{k+1})\rvert$	PM	$\lvert f(x_{k+1})\rvert$	Root
$f(x) = x^3 + 4x^2 - 10$	0.5	11	0	142	0	1.3652
	1	4	0	4	0	1.3652
$f(x) = (x-1)^6 - 1$	2.5	6	0	6	0	2.0000
	3.5	8	0	8	0	2.0000
$f(x) = \sin x \exp x + \ln(x^2+1)$	-0.8	4	0	4	0	-0.6032
	-0.65	4	0	4	0	-0.6032
$f(x) = (x-2)(x+2)^4$	-3	100	3.98e-59	98	2.48e-60	-2.0000
	1.4	8	0	8	0	2.0000
$f(x) = x - 3\ln(x)$	2	fails	fails	5	0	1.8572
	0.5	6	0	7	0	1.8572
$f(x) = x^2 - \exp x - 3x + 2$	2	4	0	4	0	0.2575
	3	5	0	5	0	0.2575
$f(x) = x\exp^{x^2} - (\sin(x))^2 + 3\cos x + 5$	-2	300	2.66e-15	7	2.66e-15	-1.2076
	-0.5	fails	fails	fails	fails	fails
$f(x) = \exp(x) - 4x^2$	0	fails	fails	fails	fails	fails
	1	fails	fails	4	4.4e-16	0.7148
$f(x) = \arctan(x)$	3	7	0	6	0	0.0000
	-3	7	0	6	0	0.0000
$f(x) = -x^4 + 3x^2 + 2$	1	fails	fails	fails	fails	1.8872
	0.5	fails	fails	fails	fails	1.8872
$f(x) = \log(x)$	3	fails	fails	fails	fails	1.0000
	2	5	0	5	0	1.0000
$f(x) = x^5 - x + 1$	2	37	6.66e-16	47	6.66e-16	-1.6730
	3	fails	fails	27	0	-1.6730
$f(x) = 0.5x^3 - 6x^2 + 21.5(x) - 22$	3	6	0	2	0	4.0000
	5	6	0	2	0	4.0000
$f(x) = x^{\frac{1}{3}}$	1	fails	fails	fails	fails	fails
	-1	fails	fails	fails	fails	fails
$f(x) = 10x\exp(-x)^2 - 1$	3	fails	fails	fails	fails	fails
	-1	fails	fails	fails	fails	fails
$f(x) = \sin(x) - \frac{x}{2}$	0.25	4	0	3	0	-1.8955
	1	7	0	5	0	-1.8955
$f(x) = \ln(x^2 + x + 2) - x - 1$	3	6	0	5	0	-0.4382
	2	4	0	4	0	-0.4382
$f(x) = x^2\sin^2(x) + \exp^{x^2\cos x \sin x} - 28$	3.5	5	2.84e -14	5	2.84e -14	3.4375
	4.5	6	2.84e-14	6	2.84e-14	4.6220
$f(x) = \exp(x^2 - 7x - 30) - 1$	4	16	0	16	0	3.0000
	4.5	24	0	23	0	3.0000
$f(x) = \cos(x) - x$	1.7	5	0	4	0	0.7391
	0	5	0	5	0	0.7391

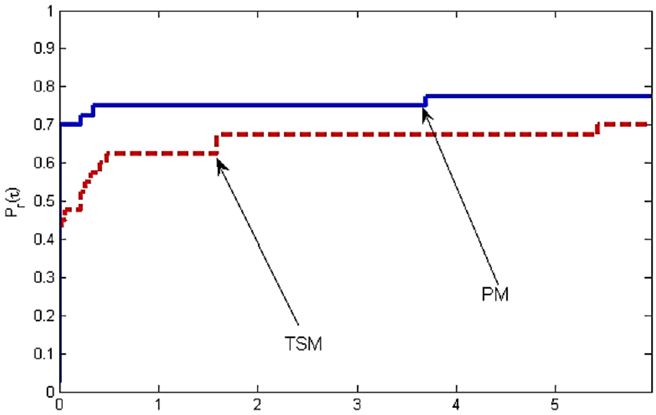

Figure 1: Performance profile of TSM and PM methods with respect to number of iterations for problem 1-20

We compare the performance among the tested methods based on the performance profile presented by Dolan and More[1]. For ns solvers and np problems, the performance profile P : R→[0,1] is defined as follows: Let ρ and S be the set of problems and set of solvers respectively. For each problem $p \in \rho$ and for each solver $s \in S$ we define $t_{p,s}$:=(number of iterations required to solve problem p by solver s). The performance ratio is given by $r_{p,s} := \frac{t_{p,s}}{\min t_{p,s} | s \in S}$. Then the performance profile is defined by $P(\tau) := \frac{1}{n_p}$ size $p \in \rho | r_{p,s} \leqslant \tau$, $\forall \tau \in R$, where $P(\tau)$ is the probability for solver $s \in S$ that a performance ratio $r_{p,s}$ is within a factor $\tau \in R$ of the best possible ratio.

Using all the twenty functions and their two starting point x_0 making a total of 40 test cases. Our method outperformed the method of [11] in most cases, therefore our method is a good improvement to the method of [11].

IV. CONCLUSION

In this paper, we proposed some modification of Newton-like method by using secant method for solving nonlinear equation. The proposed method approximates the derivative in the corrector step in [11]. Numerical result shows that our proposed methods exhibit a good performance.

Finally, we can claim that our proposed method is a good approach for solving nonlinear equations.

The method presented in the research work does not behave well for some choice of x_0, and in some cases if the derivative is zero in the predictor step the method fails. At this point we recommend a future research to overcome these drawbacks.

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