

Study Approach EElimination Et Choix Traduisant la REalite (ELECTRE) for Dynamic Multi-Criteria Decision Junika Napitupulu, Doli Hasibuan

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ABSTRACT

Decision Support System (DSS) is a model-based system that consists of procedures for processing the data and considerations to assist managers in making decisions. DSS is used to support the calculation of the value data to generate a priority ranking as a decision support. The solution to these problems is utilizing Multi-Criteria Decision Making (MCDM) by a particular method as a decision support system at this stage of the selection with values calculated using the elimination method Traduisant la réalité et Choix (ELECTRE). ELECTRE method selected for use outranking concept which compares the value of each alternative.

Keyword: Alternative and Criteria, Decision Support System, Multi-Criteria, ELECTRE

I. INTRODUCTION

Decision Support System is one part of the system information useful to improve the effectiveness of decision-making [1] [2] [3] [4], the problem that being the object decision support systems there are problems of structured and unstructured [5] [6].

The problem of decision-making is essentially a form of election from the variety of alternative decisions that might be selected whereby the election process a particular mechanism [4] [7], hoping to result in a decision that was best, to avoid subjectivity resulting decision especially on the decision of multi-criteria used a method of EElimination Et Choix Traduisant la réalité (ELECTRE).

ELECTRE methods used the concept of ranking based on alternative and criteria established, ELECTRE methods could be utilized under conditions where alternatives are not appropriate with the criteria will be eliminated, and suitable alternative can be generate.

II. THEORY

Decision Support System is an interactive information system that provides information, modeling, and manipulating data [1]. The system was used to assist decision-making in situations semi-structured and

unstructured situations, where no one knows for sure how the decision should be made [3] [4]. Decision support system is intended to support the management in conducting analytical work in situations that are less structured and less clear criteria [4]. Decision support systems are not designed to automate decision-making, but it provides an interactive tool that allows decision-makers to perform a variety of analyses using models available. Decisions were taken to solve a problem could be seen from the structure that can be divide into [4] [8]:

1. Decision structured (structured decision) that the decision to do repetitive and routine, decision-making procedures are very clear, the decision was mainly carried out at lower levels of management.
2. Semi-structured decision that is a decision that has two properties, some properties can be handled by computers and the others remained to be done by decision-makers, decision-making procedures in the outline is already there, but some things still require a policy of decision maker. Usually, this kind of decisions taken by the mid-level manager in an organization.
3. Decision unstructured (unstructured decision), namely the decision to handle complicated because not recurrent or not always the case, the decision requires experience and a variety of external sources. Such decisions are common in important management.

ELECTRE Method

ELECTRE is one of multiple criteria decision-making method based on the concept of outranking using a pairwise comparison of the alternatives based on any appropriate criteria [2] [9] [10] [11] [12]. ELECTRE methods are used under conditions where the alternative is less in accordance with the criteria eliminated and suitable alternative could be generated [9] [12], in other words, ELECTRE are use for cases with many choices, but only a few criteria involved [9] [12] [10], steps taken to solve the problem using ELECTRE method is as follows:

1. Normalization of Decision Matrix

In this procedure, each attribute is changed to a value comparable. Each normalization from the value r_{ij} can be solved with the formula below:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

R is a matrix that has been normalized, where m stated alternatives, n stated criteria and r_{ij} are the normalized measurements from the alternative i in conjunction with criterion j .

2. Weighting the Normalized Decision Matrix

After normalized, each column of the matrix R multiplied by the weights (w_{ij}) determined by the decision maker. Thus, the normalized weighted matrix is $V = RW$ this can be written as:

$$V = RW \text{ and } \sum_{i=1}^n w_i = 1$$

3. Determine concordance and discordance set

For each pair of alternatives, k and l ($k, l = 1, 2, 3, \dots, m$ and $k \neq l$) a set of criteria is divided into two subsets, concordance, and discordance. If an alternative criteria include in concordance, this can be written as:

$$C_{kl} = \{j, y_{kj} \geq y_{lj}\}, \text{ for } j = 1, 2, 3, \dots, n$$

The corresponding subset is known as the harshness set, and it is portrayed as takes after:

$$D_{kl} = \{j, y_{kj} < y_{lj}\}, \text{ for } j = 1, 2, 3, \dots, n$$

4. Calculate concordance and discordance matrix

To determine the value of the element in concordance matrix is by adding weights included in

a subset of concordance, the formula can be written as:

$$c_{kl} = \sum_{j \in C_w} W_j$$

To determine the value of the element in the matrix discordance is by dividing the maximum difference in the value of the criteria included in the subset discordance with a maximum difference of the value of all the criteria, the formula can be written as:

$$d_{kl} = \frac{\max_{j \in D_{kl}} |y_{kj} - y_{lj}|}{\max_j |y_{kj} - y_{lj}|}$$

5. Determine the dominant matrix of concordance and discordance

Concordance dominant matrix can be built with the help of value threshold, which is by comparing each value of the matrix element concordance with the threshold value $C_{kl} \geq \underline{C}$, with a threshold value (\underline{C}), are:

$$\underline{C} = \frac{\sum_{k=1}^n \sum_{l=1}^n c_{kl}}{m * (m - 1)}$$

The value of each element of the matrix F as the dominant matrix of concordance is determining as:

$$f_{kl} = 1, \text{ if } c_{kl} \geq \underline{C} \text{ and } f_{kl} = 0, \text{ if } c_{kl} < \underline{C}$$

To build a dominant matrix discordance also use the help of a threshold value, which is:

$$\underline{d} = \frac{\sum_{k=1}^n \sum_{l=1}^n d_{kl}}{m * (m - 1)}$$

The value of each element matrix G as the dominant matrix discordance can be written as:

$$g_{kl} = 0, \text{ if } c_{kl} \geq \underline{d} \text{ and } g_{kl} = 1, \text{ if } c_{kl} < \underline{d}$$

6. Determining aggregate matrix dominance

The next step is to determine dominance aggregate matrix as a matrix E , which each element is the element matrix multiplication between the matrix elements F with G , written as:

$$e_{kl} = f_{kl} * g_{kl}$$

7. Elimination of the less favorable alternative

Matrix E gives the order of selection of each alternative, i.e., when $e_{kl} = 1$ then the alternative A_k is a better option than A_l . So that the rows in a matrix E which has some at least $e_{kl} = 1$ can be eliminated, thus the best alternative is that dominates other alternatives.

III. RESULT AND DISCUSSION

The required data in the calculation process ELECTRE method is data that used to the selection criteria and weights, as well as alternative data that would be selected. The criteria utilized for the assessment can be seen in the below table:

TABLE I Criteria

No	Criteria
1	(C1)
2	(C2)
3	(C3)
4	(C4)
5	(C5)

Criteria weights using a five level of importance as follows:

TABLE II Weighted Criteria

No	Weighted
1	Very Low
2	Low
3	Enough
4	High
5	Very High

ELECTRE method testing process conducted by determining alternatives and the value of each criterion for each alternative, such as the table below:

TABLE III Alternative Value for Each Criteria

No	Alternative	C1	C2	C3	C4	C5
1	A1	14	4	18	14	6
2	A2	12	3	20	12	5
3	A3	14	6	16	14	5

Decision makers give preference weights following criteria:

$$W = (5, 3, 4, 4, 2)$$

The matrix are formed from the alternative value table is as follows:

$$X = \begin{bmatrix} 14 & 4 & 18 & 14 & 6 \\ 12 & 3 & 20 & 12 & 5 \\ 14 & 6 & 16 & 14 & 5 \end{bmatrix}$$

Next is the calculation method of ELECTRE gradually:

1. Normalization of Matrix

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

$$|X_1| = \sqrt{14^2 + 12^2 + 14^2} = 23.1517$$

$$r_{11} = \frac{x_{11}}{|x_1|} = \frac{14}{23.1517} = 0.6047$$

$$r_{21} = \frac{x_{21}}{|x_1|} = \frac{12}{23.1517} = 0.5183$$

$$r_{31} = \frac{x_{31}}{|x_1|} = \frac{14}{23.1517} = 0.6047$$

The way calculates for each column in table 3 can be used the above methods, so we get the matrix R.

$$R = \begin{bmatrix} 0.6047 & 0.5122 & 0.5750 & 0.6047 & 0.6470 \\ 0.5183 & 0.3841 & 0.6389 & 0.5183 & 0.5392 \\ 0.6047 & 0.7682 & 0.5111 & 0.6047 & 0.5392 \end{bmatrix}$$

2. Weighting the Normalized Decision Matrix

The next step multiplies the value of the matrix R by weighting each criteria with equation $v_{ij} = w_i r_{ij}$, for example is the first row in the matrix V is calculated as follows:

$$v_{11} = 0.6047 \times 5 = 3.0235$$

$$v_{12} = 0.5122 \times 5 = 1.5366$$

$$v_{13} = 0.5750 \times 5 = 2.3000$$

$$v_{14} = 0.6047 \times 5 = 2.4188$$

$$v_{15} = 0.6470 \times 5 = 1.2940$$

After doing the calculations for all rows of the matrix R showed the following results:

$$V = \begin{bmatrix} 3.0235 & 1.5366 & 2.3000 & 2.4188 & 1.2940 \\ 2.5915 & 1.1523 & 2.5556 & 2.0732 & 1.0784 \\ 3.0235 & 2.3046 & 2.0444 & 2.0444 & 1.0784 \end{bmatrix}$$

3. Determining the set of concordance and discordance index

Concordance index is determined by the equation:

$$C_{kl} = \{j, y_{kj} \geq y_{lj}\}, \text{ for } j = 1, 2, 3, \dots, n$$

Discordance index is determined by the equation:

$$D_{kl} = \{j, y_{kj} < y_{lj}\}, \text{ for } j = 1, 2, 3, \dots, n$$

TABLE IV Concordance and Discordance Index

Concordance Index	Discordance Index
$C_{12} = \{1,2,4,5\}$	$D_{12} = \{3\}$
$C_{13} = \{1,3,4,5\}$	$D_{13} = \{2\}$
$C_{21} = \{3\}$	$D_{21} = \{1,2,4,5\}$
$C_{12} = \{3,5\}$	$D_{12} = \{1,2,4\}$
$C_{12} = \{1,2,4\}$	$D_{12} = \{3,5\}$
$C_{12} = \{1,2,4,5\}$	$D_{12} = \{3\}$

4. Calculate concordance and discordance matrix

a. Calculate the matrix concordance

To calculate the concordance matrix using the following equation:

$$c_{kl} = \sum_{j \in C_w} W_j$$

$$C_{12} = w_1 + w_2 + w_4 + w_5 = 5 + 3 + 4 + 2 = 14$$

$$C_{13} = w_1 + w_3 + w_4 + w_5 = 5 + 4 + 4 + 2 = 15$$

$$C_{21} = w_3 = 4$$

$$C_{23} = w_3 + w_5 = 4 + 2 = 6$$

$$C_{31} = w_1 + w_2 + w_4 = 5 + 3 + 4 = 12$$

$$C_{32} = w_1 + w_2 + w_4 + w_5 = 5 + 3 + 4 + 2 = 14$$

From the above results obtained following concordance matrix:

$$C = \begin{bmatrix} - & 14 & 15 \\ 4 & - & 6 \\ 12 & 14 & 0 \end{bmatrix}$$

b. Calculate the matrix discordance

To calculate the discordance matrix using the following equation:

$$d_{kl} = \frac{\max_{j \in D_{kl}} |y_{kj} - y_{lj}|}{\max_j |y_{kj} - y_{lj}|}$$

$$d_{12} = \frac{\max\{|2.3000 - 2.5556\}}{\max\{|3.0235 - 2.5915\}; \{1.5366 - 1.1523\}; \{2.3000 - 2.5556\}; \{2.4188 - 2.0732\}; \{1.2940 - 1.0784\}}$$

$$d_{12} = \frac{\max\{0.2556\}}{\max\{0.4320; 0.3843; 0.2556; 0.3456; 0.2156\}}$$

$$d_{12} = \frac{0.2556}{0.4320} = 0.5917$$

$$d_{13} = \frac{\max\{|1.5366 - 2.3046\}}{\max\{|3.0235 - 3.0235\}; \{1.5366 - 2.3046\}; \{2.3000 - 2.0444\}; \{2.4188 - 2.4188\}; \{1.2940 - 1.0784\}}$$

$$d_{13} = \frac{\max\{0.7680\}}{\max\{0, 0.7680; 0.2556; 0; 0.2156\}}$$

$$d_{13} = \frac{0.7680}{0.7680} = 1$$

$$d_{21} = \frac{\max\{|2.5915 - 3.0235\}; \{1.1523 - 1.5366\}; \{2.0732 - 2.4188\}; \{1.0784 - 1.2940\}}{\max\{|2.5915 - 3.0235\}; \{1.1523 - 1.5366\}; \{2.5556 - 2.0444\}; \{2.0732 - 2.4188\}; \{1.0784 - 1.0784\}}$$

$$d_{21} = \frac{\max\{0.4320; 1.1523; 0.3456\}}{\max\{0.4320; 1.1523; 0.5112; 0.3456; 0\}}$$

$$d_{21} = \frac{1.1523}{1.1523} = 1$$

$$d_{23} = \frac{\max\{|2.5915 - 3.0235\}; \{1.1523 - 1.5366\}; \{2.0732 - 2.4188\}}{\max\{|2.5915 - 3.0235\}; \{1.1523 - 1.5366\}; \{2.5556 - 2.3000\}; \{2.0732 - 2.4188\}; \{1.0784 - 1.2940\}}$$

$$d_{23} = \frac{\max\{0.4320; 0.3843; 0.3456; 0.2156\}}{\max\{0.4320; 0.3843; 0.2556; 0.3456; 0.2156\}}$$

$$d_{23} = \frac{0.4320}{0.4320} = 1$$

$$d_{31} = \frac{\max\{|2.0444 - 2.3000\}; \{1.0784 - 1.2940\}}{\max\{|3.0235 - 3.0235\}; \{2.3046 - 1.5366\}; \{2.0444 - 2.3000\}; \{2.4188 - 2.4188\}; \{1.0784 - 1.2940\}}$$

$$d_{31} = \frac{\max\{0.2556; 0.2156\}}{\max\{0; 0.7680; 0.2556; 0; 0.2156\}}$$

$$d_{31} = \frac{0.2556}{0.7680} = 0.3328$$

$$d_{32} = \frac{\max\{|2.0444 - 2.5556\}}{\max\{|3.0235 - 2.5915\}; \{2.3046 - 1.1523\}; \{2.0444 - 2.5556\}; \{2.4188 - 2.0732\}; \{1.0784 - 1.0784\}}$$

$$d_{32} = \frac{\max\{0.2556; 0.2156\}}{\max\{0.4320; 1.1523; 0.5112; 0.3456; 0\}}$$

$$d_{32} = \frac{0.5112}{1.1523} = 0.4436$$

From the above results obtained discordance matrix below:

$$D = \begin{bmatrix} - & 0.5917 & 1 \\ 1 & - & 1 \\ 0.3328 & 0.4436 & - \end{bmatrix}$$

5. Determine the dominant matrix of concordance and discordance

a. Calculate the dominant matrix concordance

First, determine the threshold value (threshold) by the equation:

$$\underline{c} = \frac{\sum_{k=1}^n \sum_{l=1}^n c_{kl}}{m * (m - 1)}$$

$$\underline{c} = \frac{14 + 15 + 4 + 6 + 12 + 14}{3(3 - 1)} = 10.8333$$

The equation determines f matrix element:
 $f_{kl} = 1, \text{ if } c_{kl} \geq \underline{c} \text{ and } f_{kl} = 0, \text{ if } c_{kl} < \underline{c}$
 So dominant concordance matrix are:

$$F = \begin{bmatrix} - & 1 & 1 \\ 0 & - & 0 \\ 1 & 1 & - \end{bmatrix}$$

- b. Calculate the dominant matrix discordance
 First, determine the threshold value by the equation:

$$\underline{d} = \frac{\sum_{k=1}^n \sum_{l=1}^n d_{kl}}{m * (m - 1)}$$

$$\underline{d} = \frac{0.5917 + 1 + 1 + 1 + 0.3328 + 0.4436}{3(3 - 1)} = 0.7280$$

The equation determines g matrix element:
 $g_{kl} = 0, \text{ if } c_{kl} \geq \underline{d} \text{ and } g_{kl} = 1, \text{ if } c_{kl} < \underline{d}$

So dominant concordance matrix are:

$$G = \begin{bmatrix} - & 0 & 1 \\ 1 & - & 1 \\ 0 & 0 & - \end{bmatrix}$$

6. Determining aggregate matrix dominance

Aggregation of the dominant matrix shows the partial preference order of alternatives, to get this matrix using the following equation:

$$e_{kl} = f_{kl} \times g_{kl}$$

So the calculation is as follows:

$$e_{12} = 1 \times 0 = 0$$

$$e_{13} = 1 \times 1 = 1$$

$$e_{21} = 0 \times 1 = 0$$

$$e_{23} = 0 \times 1 = 0$$

$$e_{31} = 1 \times 0 = 0$$

$$e_{32} = 1 \times 0 = 0$$

So that the dominant aggregation matrix is:

$$E = \begin{bmatrix} - & 0 & 1 \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix}$$

7. Elimination of the less favorable alternative
 Matrix provides order of preference of each alternative, i.e., when $e_{kl} = 1$ then the alternative A_k is a better alternative than A_l , so that line in the matrix which has at least $e_{kl} = 1$ can eliminate. Thus the second and third row can be eliminated and the remaining first line. The value $e_{13} = 1$ indicates that the first alternative is better than a third alternative, so the best alternative is the first alternative.

IV. CONCLUSION

ELECTRE methods apply the concept of elimination of the various alternatives and criteria that do not meet the concordance, and the idea of using ELECTRE method can applied to different cases by following the calculation results described in this study.

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