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Modern Portfolio Theory: A Review of the Work Done on Performance Measures and their Role in Portfolio Construction

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ABSTRACT

This paper reviews the literature on The Modern Portfolio Theory starting from the contribution of Markowitz. It discusses the merits and de merits of the theory as reviewed through empirical findings, theoretical developments and modeling works. The review combines all the works which proved that the statistical approach to portfolio management is the best alternative to fundamental and technical analysis. Many authors have developed methodologies which reduce the computational complications of the Markowitz's model. The review mainly highlights the productive aspects of the contributions made by Elton et al to the literature of Portfolio management. The study is done with an idea to enhance the portfolio management criteria through the use of time varying betas.
Keywords: Review, Portfolio Management, Performance Measures, Sharpes Ratio, Alpha, Beta, Expected Return, Risk Measures.

I. INTRODUCTION

The fundamental analysis and technical analysis are the ways of handling risk in financial markets. The fundamental analysis aims to identify the causes of changes in stock price by studying the relevant company, the industry behind it and the economy around it. This is very hard and it is almost impossible to predict the stock prices based on this study. The technical analysis recognizes the patterns in pricing and volume. It looks for clues for the buying action based on these patterns. This analysis cannot be used as a foundation to the risk management system. A system which emerged from the statistical approach to chance if used effectively can answer the flaws of both fundamental and technical analysis. This system called the "Modern Portfolio Theory" which emerged in 1960's is considered in this review. Even though the theory faced a lot of criticism, it still remains the principal foundation for the financial theory. The aim of this review is to examine whether Modern Portfolio Theory is an effective tool for portfolio management.

II. METHODS AND MATERIAL

2. Review of the Modern Portfolio Theory

In 1900 the French mathematician, Louis Bachelier, studied financial markets. Bachelier argued that prices will go up or down with equal probability and that their volatility is measurable. He argued that the distribution of price movements is bell-shaped with very large changes assumed to be extremely rare. It was Markowitz who took the first step in applying Bachelier's ideas. Others followed in their footsteps, making Modern Portfolio Models simpler and more usable for investors and portfolio managers.

In the 1950's the investment community talked about risk but there was no measurable specification for the term. However, investors were eager to quantify their risk variable. Markowitz(1952) showed that the variance of the rate of return was an important measure of risk under a reasonable set of assumptions and came forward with the formulas for computing the variance of the portfolio. The use of this formula revealed the importance of diversification in reducing the risk and also provided guidance on how to diversify effectively.

Markowitz rejected the notion that investors should maximize discounted returns and choose their portfolio accordingly. His view was that this rule failed to imply diversification, no matter how the anticipated returns were formed. He also rejected the law of large numbers in portfolios made up of securities, objecting to the claim that it would result in both maximum expected returns and minimum variance, and pointing out that returns from securities are too inter correlated for all variance to be eliminated with diversification. Markowitz also pointed out that a portfolio with maximum expected returns is not necessarily the one with the minimum variance. Based on these observations he presented the "expected returns-variance of returns" rule (Markowitz, 1952). Markowitz's idea was that investors should hold mean-variance efficient portfolios.

Markowitz concluded that the expected return-variance of return rule not only revealed the benefits of diversification but it also pointed towards the right type of diversification for the right reason. It is not enough to diversify by simply increasing the number of securities held. If most of the firms in the portfolio are within the same industry they are more likely to do poorly at the same time than firms in separate industries. In the same way it is not enough to make variance small to invest in large number of securities. It should be avoided to invest in securities with high covariance among themselves and it is obvious that firms in different industries have lower covariance than firms within the same industry (Markowitz, 1952). Markowitz concluded that by mixing stocks that flip tail and those that flip heads you can lower the risk of your overall portfolio.

William Sharpe (1963) studied and worked on Markowitz's model, trying to simplify the calculations in order to make it more practical for use. According to Sharpe, evidence showed that using comparatively few parameters could lead to almost the same results as obtained by using much larger sets of relationships among securities. Sharpe was confident that his model enabled analysis at low cost and therefore was an initial practical application of the Markowitz technique (Sharpe W. F., A Simplified Model for Portfolio Analysis, 1963).

In the securities literature, Sharpe (1964) and Lintner quickly realized that since the SIM beta measured the contribution that an individual stock made to the riskiness of a well diversified portfolio, the risk premium required for a particular stock would be proportional to its beta.

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The development of market models in the 1960's (Sharpe, 1963, 1964; Lintner, 1965; Mossin,1966) greatly simplified the mean-variance optimization process originally propounded by Markowitz (1952, 1959). Rather than employing an analytically cumbersome process of calculating co variances via a "pairs" analysis of relationships between pairs of securities, these models measured covariance of security returns relative to a single factor: the broad market. Thus, the significant analytical advancement afforded by the single-period mean variance portfolio model was a drastic reduction in the number of inputs and computing time compared with the Markowitz method, which at that time, was practically infeasible for larger portfolios.
Sharpe (1963, 1964), Lintner (1965), Mossin (1966) and Treynor (1965) posited a single factor model using the S&P500 Index as a proxy of broad market returns as being a sufficient model of covariance, despite the fact that this index did not include the returns from dividends and re-invested income (i.e. it was a price – rather than total return – index). Elton and Gruber in 1973 proved though subsequent testing of more convoluted models that the predictive capacity of single index models was no worse than the Markowitz model.

In 1976, Elton, Gruber and Padberg pointed out the difficulty in estimating the type of input data that was necessary, especially in terms of correlation matrices. They also highlighted that construction of efficient portfolios consume more time and cost. In addition to these points they talked about the difficulty of the portfolio managers to manage risk-return trade off. According to Elton and his co-writers, part of these problems had been solved with the use of the Single Index Model to generate variance-covariance structures.
Phillips & Seagle (1975) compared the two approaches as follows: Markowitz's approach for deriving efficient sets was that every security can be viewed as being related to an index unique to itself. Sharpe's approach, however, with his single-index model, was that every security is related to the same index. The returns on various securities in Sharpe's model are assumed to be

related only through common correlations with the market return (Phillips & Seagle, 1975).

In a report by Frankfurter et al. (1976) comparing the Sharpe portfolio selection model and Markowitz's model, it was found that under conditions of uncertainty the Sharpe approach was more than just a shortcut computational scheme.

Frankfurter, Phillips, & Seagle,(1976) in their article titled "Performance of the Sharpe Portfolio Selection Model: A Comparison(1976)" established that the Sharpe approach outperformed the standard Markowitz approach in selecting efficient portfolios, which means that the Sharpe approach has potential advantages over the Markowitz approach. The fact that the Sharpe model uses fewer, and different, estimators to summarize past history was thought to be one of the main reasons for this.

Sharpe W. F(1967) says that Until Markowitz suggested this approach to portfolio analysis no full and specific basis existed to justify diversification in portfolio selection. With his approach a framework to address the issues and an algorithm for employing that framework for practical problems was provided. Markowitz did not, however, suggest a preferred technique for security analysis or a suitable method for portfolio selection. He concentrated on providing a general structure for the whole process and providing an algorithm for performing the task of portfolio analysis..

According to Sharpe, Alexander, & Bailey(1999), Markowitz's theory though not an entirely new concept, mean-variance optimization was not a widely used strategy at the time. In his paper, Markowitz formally presented his view that although investors want to maximize returns on securities they also want to minimize uncertainty, or risk. These are conflicting objectives which must be balanced against each other when the investor makes his or her decision. Markowitz asserts that investors should base their portfolio decisions only on expected returns, i.e. the measure of potential rewards in any portfolio, and standard deviation, the measure of risk. The investor should estimate the expected returns and standard deviation of each portfolio and then choose the best one on the grounds of the relative magnitudes of these two parameters.

Steinbach (2001) in his review appreciates the parametric optimization model of Markowitz that was both sufficiently general to be applicable to a significant range of practical situations and simple enough to be usable for theoretical analysis. Steinbach further mentions that Markowitz's work in the 1950's probably raised more questions than it answered. Indeed, it spurred a tremendous amount of related research.

Amenc & Le Sourd(2003) point out that the difficulty in computing the variance-covariance matrix was an obstacle to the implementation of the Markowitz model. While talking about Sharpe's model they point out that Sharpe's single index model, or empirical market model as it is sometimes called, has no theoretical basis; it only proposes a simplified view. Based on this review it is understood that Markowitz created a theory of portfolio choice in the uncertain future. He quantified the difference between the risk that was taken on individual assets and the aggregated risk of the portfolio. He showed that the portfolio risk came from covariance of the assets which made up the portfolio. The marginal contribution of a security to the portfolio return variance is therefore measured by the covariance between the return of the security and the return of the portfolio but not by the variance of the security itself. Markowitz's argument that the risk of a portfolio is less than the risk of each asset in the portfolio taken individually, provides quantitative evidence of the merits of diversification.

Mandelbrot.B (2004) argues that to construct efficient portfolios good forecasts of earnings, share prices and volatility for possibly thousands of stocks are needed. Also it is necessary to calculate its covariance with every other stock, which requires extensive calculations. Last but not least, the exercise needs constant repetition because of changes in the price of stocks. The article proved that If the investments are spread across unrelated stocks the potential profit will maximize whether the economy is slowing down or growing. If more and more stocks in different combinations are added, an efficient portfolio emerges.

Elton and Gruber came forward with even more simplified techniques for devising optimal portfolios which also gave a better understanding of the choice of securities to be included in a portfolio. These techniques were based on the Single Index Model but the calculations in the model were easy to compute and led to very similar results as those Markowitz‘s model would produce using the complete matrix. According to Elton et al., the one reason security returns are correlated is that there is a common response to market changes; therefore, a useful measure of this correlation might be obtained by relating the return of a single security to the return of a market index. The key assumption of the single index model is that e_i (the random or uncertain element of *ai* in stock *i*) is independent of e_j (same element in stock *j*) for all values of *i* and *j*. This indicates that the only reason stocks vary together, systematically, is their common co-movement with the market. In other words, that no other effects beyond the market explain for co-movements among securities (Elton, Gruber, Brown, & Goetzmann, 2007).

The Single Index Model, therefore, uses the market index as a proxy for the common factor. A market index, gives us a considerable amount of past data which we can use to estimate systematic risk. Because the index model is linear it is possible to estimate the beta coefficient, or sensitivity, of a security on the index using a single-variable regression. The excess return of a security is regressed on the excess return of the index. In order to estimate the regression, historical samples of paired observations, i.e. the returns of the security and the returns of the market at the same point in time, are collected (Bodie, Kane, & Marcus, 2009). They further acknowledged Markowitz work which identified the efficient set of portfolios. The principal idea behind the frontier set of risky portfolios is that the investor should only be interested in the portfolio which gives the highest expected return for any given risk level. Also, the frontier is a set of portfolios that minimizes the variance for any target expected return. They conclude that the Markowitz’s model requires a huge number of estimates to construct the covariance matrix and extensive calculations to construct the efficient frontier. They further add that, the Markowitz model does not provide any guideline as to the forecasting of the security risk premiums that are necessary to compute the efficient frontier of risky assets.

3. Review of the Perfomance measures used:

The following are the reviews regarding the parameters used in the construction of the portfolio.Alpha, or Jensen‘s alpha (1968), in connection to constructing optimal portfolio is a risk-adjusted performance measure that adjusts expected or average returns for beta risk. (Nielsen & Vassalou, 2004) Alpha in the regression equation is, put in simple terms, a return a portfolio is attaining over a comparing investment, an index, taking risk also into consideration. Alpha is the active components of an investment and typically represents either market timing or security selection (Scott, 2009). Alpha of a security is therefore the component of a securities return that is independent of the market‘s performance, or a random variable. In other words it represents that component of return insensitive to the return on the market (Elton, Gruber, Brown, & Goetzmann, 2007).

Investor’s goal is getting better return on their investment selection than they would get investing in an index, also taking risk into consideration. Getting positive alpha means that you are beating the market (Gupta & Straatman, 2005). Alpha is therefore a measure of whether or not an asset beats the market on risk adjusted basis (Gorman & Weigand, 2007).

Alpha is the return associated with an asset for exposure to non-systematic (idiosyncratic) risk. When looking at alpha we are interested in the average value of the firms return net of the impact of market movements (Bodie, Kane, & Marcus, 2009).

It has to be stressed that alpha alone does not say how much the investor should optimally invest in a stock or fund. The variance that is distinctive to the stock or fund also matters. There the connection between alpha, beta and Sharpe Ratio for example, can be seen, because if the investor puts too large fraction of his wealth into particular stock then the distinctive risk may result in a Sharpe Ratio and a lower expected utility (Nielsen & Vassalou, 2004).

Beta in the return of the security equation, that is beta of the security, is a constant that measures the expected change in the security given the change in the return of the market index, measured in standard deviations. In other words the beta of the security measures how sensitive a security‘s return is to the return of the market (Elton, Gruber, Brown, & Goetzmann, 2007).

Beta is a statistical coefficient estimated via linear regression that describes how a particular assets return is influenced by the return associated with a systematic risk factor. Beta is a scaled measure of the correlation of returns between the asset and the systematic risk factor (Gorman & Weigand, 2007).

The use of a Single Index Model demands an estimate of the beta of each stock which is under consideration to be included in a portfolio. Therefore, the beta estimates for individual securities determine, in part at least, which securities will be selected for inclusion in investment portfolios when an optimization algorithm such as the Sharpe approach is used There is evidence, according to Elton and co-writers, that those historical betas provide useful information about future betas. Firms should therefore, at least to start with, use the best estimates of beta available from historical data and various techniques are available to do that (Elton, Gruber, Brown, & Goetzmann, 2007).

To summarize the alpha and beta coverage, the alpha is unrelated to the market movements and positive alpha is a sign of a good choice of investment. Beta, on the other hand, is related to market movements and the higher the beta is the more volatile the stock is to market movements. However, beta and alpha are related since the estimation of the beta will have an effect on the value of alpha, for example, overestimating alpha will underestimate beta (Tofallis, 2008). Because of this it can be speculated that market movements are indirectly related to alpha through beta.

The Sharpe Ratio, or reward-to-variability ratio, is a measure of risk-adjusted performance that uses a benchmark based on the (ex post) capital market line (CML). The ratio measures return relative to the total risk of the portfolio, where the total risk is the standard deviation of portfolio returns. If investors face an exclusive choice among a number of funds, then they can rank them on the basis of their Sharpe ratios. Nielson & Vassalou (2004) have proved that a stock with higher Sharpe ratio will enable investors to achieve a higher expected utility.

Elton, Gruber and Padberg (1979) also constructed multi-factor models in attempts to more accurately attribute residual variances according to extra-market influences on the assumption that residual risk was company-specific risk which was not uncorrelated but rather can be explained by industry influences or other broad economic influences.

4. Review of the Studies on Time varying Betas

Beta has occupied center stage in both risk measurement and risk management since the concept was introduced by Markowitz (1959). It is one of the most widely used measures of risk among practitioners and financial economists. Beta has wide ranging applications in financial economics including testing of asset pricing theories, estimation of the cost of capital, evaluation of portfolio performance and calculation of hedge ratios for index derivatives, etc. Hence improvements in the measurement of beta would have useful ramifications for all these areas.

Beta is commonly estimated using ordinary least square regression (OLS) of stock returns on market returns.1 The OLS estimator is biased on stability of the market{model relationship over the estimation interval. The stability of beta has been a matter of intense debate among researchers for the last three decades (Blume 1971, Vasicek 1973, Alexander & Chervani 1980, Brooks et al. 1998). There are sound economic reasons that suggest that beta may be time varying:

- Beta is linked to the leverage of the firm (Hamada 1972, Mandelker & Rhee(1984), hence changes in leverage would give a change in beta (Black 1976, Braun et al. 1995). Fluctuations in stock prices lead to changes in leverage, hence we may expect frequent changes in beta.
- Beta is a measure of risk of an asset as compared to the market. Any news that will not affect market and stock returns uniformly will change beta of the stock. If an event increases variance of the market returns but fails to increase the variance of a security, then occurrence of that event will reduce the beta of that security (Rosenberg & Guy 1976).
- Galai & Masulis (1976) interpret equity as a call option on the assets of the firm. They show that the beta of a stock is related to the beta of the firm’s

assets through a factor that depends on the level of risk free interest rate.

The large body of research efforts confirmed that adjusted betas offered greater predictive power than purely historical (or “raw”) regression betas (Blume, 1971; Vasicek, 1973). Rosenberg (1985) noted that historical (or “raw”) beta values are not “true” betas because they only measure the relationship between a stock and the broad market over a specific measurement window.

In addition to these studies, the usage of Kalman filters added more quality to the research done in this area. It helps to model beta as a time series process, and hence provides a way to test for beta constancy (Bos& Newbold 1984). The standard Kalman filter is applicable to the problem of the market model with a time varying beta, under the assumptions of normality and homoscedasticity of the error. A series of papers has conducted these tests in various countries: USA (Ohlson & Rosenberg 1982), Sweden (Wells 1996) etc. All these papers reject beta constancy. The second strategy uses a bivariate garch model (Bollerslev et al. 1988).

Here, stock returns and index returns are assumed to follow garch processes, and the time varying covariance parameter is identified. This is directly relevant for the problem of computing hedge ratios for using index derivatives.

The traditional Kalman filter assumes that the market model residual is Gaussian and homoscedastic. This is inconsistent with the considerable evidence which has accumulated about heteroscedasticity of financial returns (Bollerslev et al. 1988, Ng & Lilian 1991, Bollerslev et al. 1992). Harvey et al. (1992) derive the modified Kalman filter, which is quasi optimal when errors show conditional heteroscedasticity.

Hence it is understood that ever since Markowitz came forward with his mean-variance portfolio selection model numerous researchers have developed algorithms to produce solutions based on the model, as well as introduced simplifying assumptions in attempts to operate the model. The main results of these efforts have been the diagonal model and linear programming approximation of Sharpe (1963, 1973), the multi-index models of Cohen and Pogue (1967), Jacob‘s limited diversification model (1974) and the simple criteria of Elton et al. (1976).

III. CONCLUSION

The purpose of this article is to provide an insight into the current state of research in the area of portfolio management. Since Markowitz model is the base on which many theories have emerged, all the works which highlighted the merits and demerits of Markowitz’s work are considered for this review. Also the constructive work done by Elton & Gruber and their co-authors are given more priority in this review. Sharpe’s ratio, Treynor,s ratio and beta measures decide the quality of the portfolio and hence the review works done on these measures are included in this study. Only papers which use a combination of empirical, theoretical and modeling approaches are considered in this review.

IV. FURTHER RESEARCH

The decision making process in portfolio optimization is a very complex problem. The selection of performance measures and the method of using these measures in portfolio formation is the key issue which has added to a very strong literature in this field. Better performance indicators should be used to assess the performance of the models. We have tried to identify some of the possible directions of future research in the portfolio management field. The idea of using raw regression betas has become an old idea and the current research works concentrate on using Blume betas and true betas. Betas which have intervening heteroscadastic effects may be tested in this process. Also the use of Bayesian betas may open new areas of research in this field. Better performance indicators should be used to assess the performance of the models. We have tried to identify some of the possible directions of future research in the portfolio management field. It is evident that this field presents intense challenges and significant research opportunities.

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