

# Weak Spherical Divergence Shock Through Exponentially Increasing Density Medium

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## ABSTRACT

Propagation of weak spherical diverging shock wave through in ideal gas having solid body rotation have been investigated by Chisnell-Chester-Whitham method. The medium becomes inhomogeneous on account of solid body rotation assuming in initial density distribution law as :  $\rho_0 = \rho^l \exp(\lambda r/r_0)$ , where  $\rho^l$  is the density at the axis of symmetry,  $\lambda$  is a constant and  $r_0$  is a non-dimension constant, the analytical expression for shock velocity and shock strength have been obtained. Finally, the expression for the pressure, the density and the particle velocity immediately behind the shock have also been derived. It is observed that the smaller value of the propagation distance  $r$ , the shock velocity must decreases with shock propagation for the large value of  $r$ , however is must increase with further advancement of the shock.

**Introduction-** In recent years many techniques have been used to tackle propagation of shock wave. Using similarity method the propagation of spherical shock waves through a rotating gas has received a considerable attention in the recent past [1-2] since the similarity method is based on a serious expression in powers of inverse square of mach Number, which for small value represent strong shock, the conclusion drawn from these investigation should be reliable for only strong shock. In this paper, the effort is made to investigate the propagation diverging spherical weak shock through a rotating gas using Chisnell-Chester-Whitman (6, 7, 8) method. Assuming an initial density at the axis of symmetry,

$$\rho_0 = \rho^l \exp(\lambda r/r_0),$$

where  $\rho^l$  is the density at the axis of symmetry,  $\lambda$  is a constant and  $r_0$  is non dimensional constant, the analytical expression for shock velocity and shock strength have been derived.

Finally, the expression for the pressure the density and the particle velocity immediate behind the shock have also been obtained.

**Analytical Expression For Shock Velocity And Shock Strength** - The basic equations governing the spherical symmetric flow of a gas enclosed by the shock front are written as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{v^2}{r} = 0 \quad \dots\dots(a)$$

$$\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \cdot (vr) = 0 \quad \dots\dots(b)$$

$$\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \rho + \left( \frac{\partial u}{\partial r} + \frac{2u}{r} \right) \rho = 0 \quad \dots\dots(c) \quad [1]$$

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r}\right) \cdot (P \cdot \rho^{-\gamma}) = 0 \quad \dots\dots(d)$$

Where u, v, are the radial and azimuthally components of particle velocity, P, ρ and γ respectively the pressure, the density and adiabatic of the gas.

If, P<sub>0</sub> and ρ<sub>0</sub> denoted the undisturbed value of pressure and density in front of the shock wave, the boundary conditions for weak shock can be written as :

$$P = P_0 \left\{ 1 + \frac{4\gamma}{\gamma+1} \varepsilon \right\} \quad \dots\dots(a)$$

$$\rho = \rho_0 \left\{ 1 + \frac{4\varepsilon}{\gamma+1} \right\} \quad \dots\dots(b)$$

$$u = \frac{4 a_0}{\gamma+1} \varepsilon \quad \dots\dots(c) \quad [ 2 ]$$

$$U = a_0 M \quad \dots\dots(d)$$

Where, a<sub>0</sub> is the sound velocity in undisturbed medium and we take Mach number as :  $M = 1 + \varepsilon$   
[3]

Where ε is a parameter and negligible in comparison to unity (i.e., <<1).

For diverging shock the characteristic form of system of equation (1) (i.e., the form in which equation contains derivatives in only one direction in (r, t plane)) is,

$$dp + \rho c du + 2\rho \frac{C^2 u}{u+c} \frac{dr}{r} - \frac{\rho C v^2}{u+c} \frac{dr}{r} = 0, \quad [ 4 ]$$

Where, C (=  $\sqrt{\gamma P / \rho}$ ) is the local sound velocity immediately behind the shock.

The equilibrium state of the gas is assumed to be specified by the conditions  $\frac{\partial}{\partial t} = 0$  and  $v = r\Omega_0$  where Ω<sub>0</sub> is the constant angular velocity. Consequently, the equilibrium condition prevailing in front of the shock can be written as

$$\frac{1}{\rho_0} \frac{dP_0}{dr} - r \Omega_0^2 = 0, \quad [ 5 ]$$

Assuming the initial density distribution law as

$$\rho_0 = \rho^I \exp(\lambda r / r_0), \quad [ 6 ]$$

$$P_0 = \frac{\Omega_0^2}{\lambda / r_0} (r - r_0 / \lambda) \rho^I \exp(\lambda r / r_0), \quad [ 7 ]$$

$$a_0 = \Omega_0 [(r - r_0 / \lambda) \cdot \lambda r_0 / \lambda]^{1/2} \quad [ 8 ]$$

Now substituting the shock conditions, (2) in relation and respective values of various qualities, we have

$$\frac{d\varepsilon}{\varepsilon} + \frac{dr}{r} + \frac{3}{4} \frac{dr}{(r - r_0 / \lambda)} + \frac{\lambda}{2r_0} dr = 0 \quad [ 9 ]$$

On integration equation (9) yields,

$$\varepsilon = K r^{-1} \left(r - \frac{r_0}{\lambda}\right)^{-3/4} \exp(-\lambda r / 2r_0) \quad [ 10 ]$$

Where K is a constant of integration. From equation (2), the expression for the shock velocity and shock strength can be written as :

### Shock Velocity

$$U = B_1 \Omega_0 \left(r - \frac{r_0}{\lambda}\right)^{1/2} + B_2 \Omega_0 r^{-1} \left(r - \frac{r_0}{\lambda}\right)^{-1/4} \exp(\lambda r / 2r_0) \quad [11]$$

Where  $B_1 = (\gamma r_0 / \lambda)^{1/2}$  and  $B_2 = KB_1$

### Shock Strength

$$\frac{U}{a_0} = 1 + K r^{-1} \left(r - \frac{r_0}{\lambda}\right)^{-3/4} \exp(-\lambda r / 2r_0) \quad [12]$$

**DISCUSSION :** Expression (11) for shock velocity, representing the propagation of a diverging spherical shock wave through a rotating gas, contains two types of terms involving the propagation distance  $r$ - one with positive power and other with negative power of  $r$ . Therefore, for low values of  $r$  the one with negative power happens to be the dominant term, whereas for large values of  $r$  it is the other term which primarily determines the shock velocity. Consequently, the shock velocity initially decrease as the shock propagates and attains a minimum value for certain propagation distance  $r_{Umin}$  given by the equation

$$r_{Umin} = r_0 [2(B_1 - B_2) \pm \sqrt{(2B_1 - B_2)^2 + 16 B_2} / 4\lambda B_2] \quad [13]$$

thereafter it increases. This agrees with earlier results (1, 8) for strong spherical shock in a rotating gas. Similar variation in shock velocity for weak spherical shock through self gravitation gas has also been reported by (9). The occurrence of exponential term in the expression permits the parameters governing the propagation to attain theoretically infinite values.

It is also noted from expression (11) that shock velocity increases with increase in  $\Omega_0$ . Expression (12) shows that shock strength continuously decreases as shock propagates.

Finally the expression for the pressure, the density and the particle velocity immediately behind the shock can be written as :

$$P = \rho^I \frac{r_0}{\lambda} \left(r - \frac{r_0}{\lambda}\right) \Omega_0^2 \exp(-\lambda r / 2r_0) \cdot \left[1 + \frac{4\gamma K}{\gamma + 1} r^{-1} \left(r - \frac{r_0}{\lambda}\right)^{-3/4} \exp(-\lambda r / 2r_0)\right] \quad [14]$$

$$\rho = \rho^I \exp(-\lambda r / 2r_0) \cdot \left[1 + \frac{4\gamma K}{\gamma + 1} r^{-1} \left(r - \frac{r_0}{\lambda}\right)^{-3/4} \exp(-\lambda r / 2r_0)\right] \quad [15]$$

And

$$U = \frac{4K}{\gamma + 1} \left(\frac{\gamma r_0}{\lambda}\right)^{1/2} \Omega_0^2 r^{-1} \left(r - \frac{r_0}{\lambda}\right)^{-1/4} \exp(-\lambda r / 2r_0) \quad [16]$$

### REFERENCE :-

1. Chaturani, P., Appl. Sci. Rep. 23, pp-197 (1970)
2. Kumar, S. and Chaturani, P., Proc. Ind. Acad. Sci. 79 (2), pp-82
3. Kumar, S., Lal, K and Prakash, H., Pror. Nat. Conf. Fluid Mech Fluid Power, 8(2), pp-164 (1978)
4. Kumar, S. and Shaturani, P., IF Nuovo Cimento, 50, pp-1997 (1980)
5. Kumar, S., Saxena, D.K. and Chaturani, P., Proc. Nat. Conf. Fluid Mech. Fluid Power, 10 (B6) pp-93 (1981)
6. Chisnell, R.F., Proc. Roy. Soc. A 232, pp-350 (1955)
7. Chester W., Phil. Mag. 45(7), pp-1293 (1954)
8. Whitham, G.B., J. Fluid Mech., 4, pp- 337 (1958)
9. Hirascher, T and Steiner, H. J., Fluid Dynamic Research, pp-32 (3), 61 (2003)