© 2016 IJSRST | Volume 2 | Issue 5 | Print ISSN: 2395-6011 | Online ISSN: 2395-602X
Themed Section: Mathematics

Radiating Charged Black Hole with an Internal Monopole

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ABSTRACT

The study of global monopoles and the space-times associated with them has become highly relevant and has received attention of many research workers. Here, I am going to discuss the field of a radiating charged black hole with an internal monopole and this field is derived by the metric, it becomes the Bonnor-Vaidya metric, also it can be reduces to the metric given by Hong-Wei Yu [1993], which represents the field of a radiating black hole with a global monopole. The null and time-like geodesics for the metric are obtained. The metric in the cosmological backgrounds of the de Sitter universe and the physical aspects of these metrics and the particular cases associated with them are also discussed.

I. INTRODUCTION

Global monopoles are regards as topological defects produced when global symmetry breaking occurs. Kibble [1976] and Vilenkin [1985] have discussed the possibility of creation of global monopoles during the phase transition in the early universe. The study of global monopoles and the space-times associated with them has become highly relevant and has received attention of many investigators. Barnola and Vilenkin [1989] have obtained a metric describing the field of a static black hole with an internal monopole Hiscock [19901 has discussed particle creation through formation of global monopoles in the early universe. Hong-Wei yu [1993] has obtained an exact solution of Einstein field equations representing the field of a radiating black hole with an internal monopole In the absence of monopole his solution reduces to the radiating star solution of Vaidya [1951]. The charged generalization of Vaidya metric is discussed by Bonnor and Vaidya [1970]. It describes the exterior field of radiating charged black hole. Here I am trying to generalize Bonnor-Vaidya metric to include a global monopole .

In recent times considerable attention has been given to the solutions of Einstein equations that represent metrics embedded in a cosmological background. The Scwarzschild exterior metric in the background of de Sitter universe is discussed by Tolman [1934]. Mc Vittee [1933] has derived a metric which represents a mass particle in an expanding universe. Mallet [1985] and Vick [1985] have examined the field produced by a radiating mass in de Sitter universe .Recently Patel and Desai [1997] have discussed the higher dimensional Vaidya metric in Einstein and de Sitter background. Patel and Patel [1999] have generalized this solution to include the electric charge. Patel and Akabari [1979] have obtained Bonnor-Vaidya metric in the background of Einstein universe. Tikekar and Patel [1995] have discussed the field of a radiating black hole with an interior monopole in the backgrounds of Einstein and de Sitter universe. Here I generalize their work to include the electric charge.

II. RADIATING CHARGED BLACK HOLE WITH A GLOBAL MONOPOLE

Let us consider a space-time given by the line element

$$ds^2 = 2dudr + 2Bdu^2 - r^2(d\theta^2 + sin^2\theta d\emptyset^2) \quad (2.1)$$

where, B is a function of r and u and $(x^1, x^2, x^3, x^4) = (r, \theta, \emptyset, u)$. For the metric (2.1) the surviving components of the Einstein tensor

$G_{ik} = R_{ik} - \frac{1}{2}Rg_{ik}$ are given by

$$G_{14} = \frac{2B'}{r} - \frac{1}{r^2}(1 - 2B),$$

$$G_{22} = \frac{G_{33}}{sin^2\theta} - r(2B' + rB'') \ \&$$

$$G_{44} = \frac{2}{r}(2BB' - \dot{B}) - \frac{2B}{r^2}(1 - 2B) \quad (2.2)$$

Here a prime and a dot denote derivatives relative to r and u respectively.

Now, to solve the Einstein-Maxwell equations

$$G_{ik} = -8\pi T_{ik} \quad (2.3)$$

Where, the energy momentum tensor T_{ik} is given by

$$T_{ik} = E_{ik} + \sigma\rho_i\rho_k + T_{ik}(mono), \ \rho_i\rho^i = 0 \quad (2.4)$$

Where

$$E_{ik} = g^{ab}F_{ia}F_{kb} + \frac{1}{4}g_{ik}F_{ab}F^{ab} \quad (2.5)$$

and E_{ik} is the electromagnetic energy tensor satisfying the Maxwell equations

$$F_{ik,n} + F_{kn,i} + F_{ni,k} = 0\ , \frac{\partial}{\partial x^i}\left(\sqrt{-g}F^{ik}\right) = \sqrt{-g}4\pi J^k \quad (2.6)$$

Where F_{ik} is the electromagnetic field tensor and J^i is the 4-current vector. $\sigma\rho_i\rho_k$ is the energy momentum tensor arising out of the flowing null radiation. $T_{ik}(mono)$ is the energy momentum tensor for a global monopole.

For the metric (2.1) the non-zero $T_{ik}(mono)$ are given by Hong-Wei yu [1993]

$$T_{14}(mono) = \frac{\eta_0^2}{r^2} \quad T_{44}(mono) = 2B\frac{\eta_0^2}{r^2} \quad (2.7)$$

where η_0 is a constant. From Maxwell's equations one can find the only non-zero component F_{41} of F_{ik}. It is given by

$$F_{41} = -F_{14} = \frac{q(u)}{r^2} \quad (2.8)$$

where $q(u)$ is an arbitrary function of u and it represents the charge. The non-zero J^i is given by

$$4\pi J^1 = -\frac{\dot{q}}{r^2} \quad (2.9)$$

Clearly J^i is a null vector. The non zero E_{ik} are given by

$$E_{22} = \frac{E_{33}}{sin^2\theta} = \frac{r^2}{2B}E_{44} = r^2E_{14} = \frac{q^2}{8\pi r^2} \quad (2.10)$$

We take the null vector ρ_i in the form $\rho_i = (0,0,0,1)$ (2.11)

Using the above results in (2.3) we get the differential equations,

$$r^2B'' + 2rB' - \frac{q^2}{r^2} = 0 \quad , \ 2B'r + 2B + \frac{q^2}{r^2} + 8\pi\eta_0^2 - 1 = 0 \quad (2.12)$$

And $$8\pi\sigma = \frac{2\dot{B}}{r} \quad (2.13)$$

The equations (2.12) admit the solution

$$2B = 1 + \frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r} \tag{2.14}$$

where $m(u)$ is an arbitrary function of u and is represents the mass.

The radiation density σ is given by

$$4\pi\sigma = \frac{1}{r^2}\left(\frac{\dot{q}q}{r} - \dot{m}\right) \tag{2.15}$$

The geometry of our solution is described by the line element

$$ds^2 = 2dudr + \left[1 + \frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r}\right] du^2 - r^2(d\theta^2 + sin^2\theta d\emptyset^2) \tag{2.16}$$

The metric (2.16) describes the field of a radiating charged black hole with a global monopole, when $\eta_0 = 0$ we get the Bonnor-Vaidya metric, when $q = 0$ the metric (2.16) reduces to the metric given by Hong-Wei yu [1993]. $\eta_0 = 0 = q$, we recover the usual radiating star solution of Vaidya [1951].

III. THE TIME-LIKE AND NULL GEODESICS

In the previous section we have obtained the metric (2.16) for the field of a charged black hole with a global monopole. The mass $m(u)$ is positive and is some decreasing function of u, The charge $q(u)$ may be either positive or negative, and its magnitude is a decreasing function of u. A particle whose motion is due to the gravitational field is determined by the Lagrangian.

$$L = \frac{1}{2} g_{ik} \frac{dx^i}{d\tau}\frac{dx^k}{d\tau} \tag{3.1}$$

where τ is the proper time. Using (2.16) in (3.1) one can obtains

$$L = \frac{1}{2}\left(1 + \frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r}\right)\dot{u} + \dot{u}\dot{r} - \frac{1}{2}r^2\dot{\theta}^2 - \frac{1}{2}r^2 sin^2\theta\dot{\emptyset}^2 \tag{3.2}$$

Here a dot indicates differentiation by τ . The geodesic equations can be found from

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{x}^i}\right) = \frac{\partial L}{\partial x^i} \tag{3.3}$$

$$\frac{d}{d\tau}\left[\left(1 + \frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r}\right)\dot{u} + \dot{r}\right] = \left(-\frac{1}{r}\frac{dm}{du} + \frac{q}{r^2}\frac{dq}{du}\right)\dot{u}^2 \tag{3.4}$$

$$\frac{d^2u}{d\tau^2} = \left(\frac{m}{r^2} - \frac{q^2}{r^3}\right)\dot{u}^2 - r\dot{\theta}^2 - r sin^2\theta\dot{\emptyset}^2 \tag{3.5}$$

$$\frac{d}{d\tau}\left(r^2\dot{\theta}\right) = r^2 sin\theta cos\theta\dot{\emptyset}^2 \tag{3.6}$$

$$\frac{d}{d\tau}\left(r^2 sin^2\theta\dot{\emptyset}^2\right) = 0 \tag{3.7}$$

From the equations (3.6) and (3.7), it is clear that the total angular momentum is conserved and hence the motion of the test body will be in a plane. The plane of motion is chosen to be $\theta = \frac{\pi}{2}$ = constant.

The equations of motion then become

$$\frac{dv}{d\tau} = \frac{L_m}{r} - \frac{qL_q}{r^2} \tag{3.8}$$

$$\frac{d^2u}{d\tau^2} = \left(\frac{m}{r^2} - \frac{q^2}{r^3}\right)\dot{u}^2 - \frac{l^2}{r^3} \tag{3.9}$$

$$\frac{dl}{d\tau} = 0 \tag{3.10}$$

where,
$$v = \left(1 + \frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r}\right)\dot{u} + \dot{r} \tag{3.11}$$

$$L_m = -\frac{dm}{du}\dot{u}^2 \tag{3.12}$$

$$L_q = -\frac{dq}{du}\dot{u}^2 \text{ , } l = r^2\dot{\emptyset} \tag{3.13}$$

From time-like geodesics, the Langrangian has the normalization

$$2L = (v + \dot{r})\dot{u} - \frac{l^2}{r^2} = 1 \tag{3.14}$$

and hence (3.14) implies $\dot{u} = \frac{1+\frac{l^2}{r^2}}{v+\dot{r}}$ (3.15)

From (3.11) we get $\dot{u} = \frac{(v-\dot{r})}{\left(1+\frac{q^2(u)}{r^2}-8\pi\eta_0^2-\frac{2m(u)}{r}\right)}$ (3.16)

From (3.15) and (3.16) one can obtained

$$v^2 = \dot{r}^2 + \left(1+\frac{l^2}{r^2}\right)\left(1+\frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r}\right) \quad (3.17)$$

The differentiation of equation (3.17)

$$\frac{dv}{d\tau} = \dot{r}\frac{d\dot{r}}{d\tau} + \dot{r}\frac{m}{r^2} - \dot{r}\frac{q^2}{r^3} - \dot{r}\left(8\pi\eta_0^2 - 1\right)\frac{l^2}{r^3} + \dot{r}\frac{3ml^2}{r^4} - \dot{r}\frac{2q^2l^2}{r^5} + \left(\frac{L_m}{r} - \frac{qL_q}{r^2}\right)\frac{\left(1+\frac{l^2}{r^2}\right)}{\dot{u}} \quad (3.18)$$

From (3.14) and (3.8) implies $v = \frac{\left(1+\frac{l^2}{r^2}\right)}{\dot{u}} - \dot{r}$ and $\frac{dv}{d\tau} = \frac{L_m}{r} - \frac{qL_q}{r^2}$.

Therefore, $v\frac{dv}{d\tau} = \left(\frac{L_m}{r} - \frac{qL_q}{r^2}\right)\left(\frac{\left(1+\frac{l^2}{r^2}\right)}{\dot{u}} - \dot{r}\right)$ (3.19)

From (3.18) and (3.19) we obtained

$$\frac{d^2r}{d\tau^2} = -\frac{L_m}{r} - \frac{m-qL_q}{r^2} + \frac{q^2+(8\pi\eta_0^2-1)l^2}{r^3} - \frac{3ml^2}{r^4} - \frac{2q^2l^2}{r^5} \quad (3.20)$$

This is the effective central force acting on the test particle and agrees with the result obtained by Koberlein and Mallett [1995] with the addition of monopole charge. For null geodesics, the normalization of the Lagrangian is

$$2L = (v+\dot{r})\dot{u} - \frac{l^2}{r^2} = 0 \quad (3.21)$$

and consequently we get $\dot{u} = \frac{l^2/r^2}{v+\dot{r}}$ (3.22)

With (3.16) and (3.22), the normalization condition on the Lagrangian may be written as

$$v^2 = \dot{r}^2 + \frac{l^2}{r^2}\left(1+\frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r}\right) \quad (3.23)$$

Differentiation of the above equation gives

$$\frac{dv}{d\tau} = \dot{r}\frac{d\dot{r}}{d\tau} - \dot{r}\left(8\pi\eta_0^2 - 1\right)\frac{l^2}{r^3} + \dot{r}\frac{3ml^2}{r^4} - \dot{r}\frac{2q^2l^2}{r^5} + \left(\frac{L_m}{r} - \frac{qL_q}{r^2}\right)\frac{l^2}{r^2\dot{u}} \quad (3.24)$$

From (3.21) and (3.8) one can derive,

$$v = \frac{l^2}{r^2\dot{u}} - \dot{r} \quad \text{and} \quad \frac{dv}{d\tau} = \frac{L_m}{r} - \frac{qL_q}{r^2}$$

Therefore, $$v\frac{dv}{d\tau} = \frac{l^2}{r^2\dot{u}}\left(\frac{L_m}{r} - \frac{qL_q}{r^2}\right) - \dot{r}\frac{L_m}{r} + \dot{r}\frac{qL_q}{r^2} \quad (3.25)$$

From (3.24) and (3.25) one can obtained,

$$\frac{d^2r}{d\tau^2} = -\frac{L_m}{r} + \frac{qL_q}{r^2} + \frac{(8\pi\eta_0^2-1)l^2}{r^3} - \frac{3ml^2}{r^4} - \frac{2q^2l^2}{r^5} \quad (3.26)$$

Here τ is not the proper time but a linearly related parameter.

Multiplying (3.26) by $2\frac{dr}{d\tau}$ and integrating over *r,* we can derive

$$\left(\frac{dr}{d\tau}\right)^2 = -2L_m logr - \frac{2qL_q}{r} + \frac{(8\pi\eta_0^2-1)l^2}{r^2} + \frac{2ml^2}{r^3} - \frac{q^2l^2}{r^4} + k \quad (3.27)$$

Where k is an integration constant. Let us take $y = \frac{1}{r}$ then $\frac{dr}{d\tau} = -l\frac{dy}{d\emptyset}$

Therefore the equation (3.27) becomes

$$\left(\frac{dy}{d\emptyset}\right)^2 = \frac{2L_m logy}{l^2} - \frac{2qL_{qy}}{r^2} + \left(8\pi\eta_0^2 - 1\right)y^2 + 2my^3 - q^2y^4 + \frac{k}{l^2}$$

Differentiating the above equation with respect to $\emptyset$, then we have

$$\frac{d^2y}{d\emptyset^2} - \left(8\pi\eta_0^2 - 1\right)y = 3my^2 - 2q^2y^3 + \frac{L_m}{l^2 y} - \frac{qL_q}{l^2} \tag{3.28}$$

Equation (3.28) is the well-known equation for light deflection, with additional terms due to charge, radiation of mass and charge and monopole.

Koberlein and Mallett [1995] have performed a detailed study of the geodesics in the field of a radiating charged black hole. The presence of monopole does not give any qualitative change in the nature of time-like and null geodesics. So we omit the other details regarding the geodesics.

IV. A RADIATING CHARGED BLACK HOLE WITH A MONOPOLE IN de SITTER UNIVERSE

In this section we wish to generalize the solution of section-2 to include the cosmological constant $\wedge$. So take the metric in the form (2.1) and use the field equations.

$$G_{ik} = -8\pi T_{ik} - \wedge g_{ik} \tag{4.1}$$

where T_{ik} are given by (2.4). Using the field equations (4.1) and relevant results of section-2 we get the differential equations

$$\frac{2B'}{r} - \frac{1}{r^2}(1-2B) + \frac{q^2}{r^4} + \wedge + \frac{8\pi\eta_0^2}{r^2} = 0 \tag{4.2}$$

$$B'' + \frac{2B'}{r} - \frac{q^2}{r^4} + \wedge = 0 \quad \text{and} \quad \frac{8\pi\eta_0^2}{r^2} = \frac{2B''}{r} \tag{4.3}$$

The non zero j^i is given by the equation (2.9) The equations (4.2) admit the solution

$$2B = 1 + \frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r} - \frac{\wedge}{3}r^2 \tag{4.4}$$

where m and q are function of u only. The radiating density is given by

$$8\pi\sigma = \frac{1}{r^2}\left(\frac{q\,\dot{q}}{r} - \dot{m}\right) \tag{4.5}$$

The result (4.5) shows that the cosmological constant has no effect on density σ.

The explicit form of the line element of this solution is

$$ds^2 = 2dudr + \left[1 + \frac{q^2(u)}{r^2} - 8\pi\eta_0^2 - \frac{2m(u)}{r} - \frac{\wedge}{3}r^2\right]du^2 - r^2(d\theta^2 + sin^2\theta d\emptyset^2) \tag{4.6}$$

The metric (4.6) represents the Bonnor-Vaidya solution with a global monopole in the background de Sitter universe.

When $\eta_0 = 0$ and $\wedge = 0$, the metric (4.6) reduces to the Bonnor-Vaidya metric. When m and q are constants, we recover the Reissner-Nordstrom de Sitter metric with global monopole. When, $m = q = \eta_0 = 0$, metric (4.6) gives us tie well known de Sitter metric. Thus the metric (4.6) describes the field of a radiating charged black hole with a monopole embedded in de Sitter universe. On the same way we can discussed a radiating charged black hole with a monopole in Einstein Universe but its leave for readers.

V. CONCLUDING REMARKS

Dadhich and Patel [1999] have presented a method to incorporate the field of a global monopole in any spherically symmetric solution of Einstein field equations. As an application of this method they have derived the metrics describing the fields of Mc Vittee particle with a global monopole in expanding universe and Vaidya radiating star with a global monopole. By their method the solutions discussed by us can also be obtained. The particle motion for the solutions of sections 4 can be discussed. But, for the sake of brevity we shall not enter into these details here.

VI. REFERENCES

1. Barriola, M. and Vilenkin, m Phys. Rev. Lett., 63, 341 [1989].
2. Bonner, W.B. and Vaidya, P.c. Gen. Rel. Grav., 1, 127 [1970].
3. Dhadhich, N. and Patel, L.K., To be Published [1999].
4. Hiscock, W.A. , Phys. Lett., B 234, 469 [1990].
5. Hong-Wei, Yu., Phys. Left, A182, 353 [1993].
6. Kibble, TW B., J. Phys., A9, 1347 [1976].
7. Kinnersley, W., Phys. Rev., 186, 1364 [1969].
8. Koberlein. B D , ard Mallet, R. L., J. Maths Phys.,838, [1995].
9. Mallet, R L., Phys. Rev., D31, 416. [1989]
10. Mc Vlttee, G.C., Mon. Not. R. Astron. Soc, 93, 325 [1933].
11. Patel, L.K., and Akabari R.P., J. Phys. (Math. Gen), A12, 223 [1979].
12. Patel, L.K. and Desai, L.S., Pramana-J. Phys. 48, 819 [1997].
13. Patel, L.K. and Patel, H.M., Hadronic J. 22, 127 [1999].
14. Tikekar, R. and Patel, L.K., Maths. Today, 13, 3 [19951.
15. Tolman, R.C., Relativity, Thermodynamics and cosmology (Oxf. Uni. Press, London) [1934].
16. Vaidya, P.c„ Proc. Indian. Acad. sci., A33, 264 [1951].
17. Vick, D. Left. Nuovo. Cimento, 44 137 [1985].
18. Vilenkin, A. Phys. Repo 121, 263 [1985].