

# On Soft Almost Paracompactness in Soft Topological Space

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## ABSTRACT

Recently, the author [25] introduced a new class spaces namely, soft nearly paracompact spaces and established some characterizations of these spaces. In this work, some new notions in soft space such as soft  $\alpha$ -almost regular spaces, soft almost paracompact spaces and soft  $\alpha$ -almost paracompact spaces are introduced. We also investigate some basic properties of these concepts and obtain several interesting results and characterizations of soft nearly compact and nearly paracompact spaces.

**Keywords:** Soft  $\alpha$ -almost regular spaces, soft semi-regular spaces, soft almost paracompact spaces, soft  $\alpha$ -almost paracompact spaces.

## I. INTRODUCTION

The In 1999, the concept of a soft set was introduced by D. Molodtsov [1]. Soft topological Space is considered as Mathematical tools for dealing with uncertainties, and a fuzzy topological space is a special case of the soft topological space. Xiao et al. [18] and Pei and Miao [19] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. So it is important to study the structures of soft sets for information systems. Several theories, such as the theory of fuzzy sets [20] and theory of rough sets [21] can be considered as mathematical tools for dealing with uncertainties. These theories have their inherent difficulties as pointed out in [1]. The reason for these difficulties is, possibly, the inadequency of the parametrization tool of the theories. In the recent years, papers about soft sets theory and their applications in various fields have been writing increasingly. In 1999, Molodtsov [1] initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna

et al.[3], Aygunoglu and Aygun [4] and Hussain et al are continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5]. M.K. Singal and A. Mathur, [22] introduced nearly compact spaces in topological spaces. In 1969, M.K. Singal and S.P. Arya [23] introduced the notion of nearly paracompact Spaces in topological spaces and studied their some properties. F. Lin [24] introduced the notion of soft connected sets and soft paracompact spaces and explored some of their basic properties.

In this work, some new notions in soft space such as soft  $\alpha$ -almost regular spaces, soft almost paracompact spaces and soft  $\alpha$ -almost paracompact spaces are introduced. We also investigate some basic properties of these concepts and obtain several interesting results and characterizations of soft nearly compact and nearly paracompact spaces.

## II. PRELIMINARY NOTE

Throughout the paper, the space  $X$  and  $Y$  stand for soft topological spaces with  $(X, \tau, E)$  and  $(Y, \nu, K)$  assumed unless otherwise stated. Moreover, throughout this paper, a soft mapping  $f : X \rightarrow Y$  stands for a mapping, where  $f : (X, \tau, E) \rightarrow (Y, \nu, K)$ ,  $u : X \rightarrow Y$  and  $p : E \rightarrow K$  are assumed mappings unless otherwise stated.

**Definition: 2.1[1]:** Let  $X$  be an initial universe and  $E$  be a set of parameters. Let  $P(X)$  denotes the power set of  $X$

and  $A$  be a non-empty subset of  $E$ . A pair  $(F, A)$  is called a soft set over  $X$ , where  $F$  is a mapping given by  $F: A \rightarrow P(X)$  defined by  $F(e) \in P(X) \forall e \in A$ . In other words, a soft set over  $X$  is a parameterized family of subsets of the universe  $X$ . For  $e \in A$ ,  $F(e)$  may be considered as the set of  $e$ -approximate elements of the soft set  $(F, A)$ .

**Definition 2.2[11]:** A soft set  $(F, A)$  over  $X$  is called a null soft set, denoted by  $\tilde{\varnothing}$ , if  $e \in A, F(e) = \varnothing$ .

**Definition 2.3[11]:** A soft set  $(F, A)$  over  $X$  is called an absolute soft set, denoted by  $\tilde{A}$ , if  $e \in A, F(e) = X$ . If  $A = E$ , then the  $A$ -universal soft set is called a universal soft set, denoted by  $\tilde{X}$ .

**Theorem 2.4[2]:** Let  $Y$  be a non-empty subset of  $X$ , then  $\tilde{Y}$  denotes the soft set  $(Y, E)$  over  $X$  for which  $Y(e) = Y$ , for all  $e \in E$ .

**Definition 2.5 [11]:** The union of two soft sets  $(F, A)$  and  $(G, B)$  over the common universe  $X$  is the soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We write  $(F, A) \cup (G, B) = (H, C)$

**Definition 2.6 [11]:** The intersection  $(H, C)$  of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $X$ , denoted by  $(F, A) \cap (G, B)$ , is defined as  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for all  $e \in C$ .

**Definition 2.7[2]:** Let  $(F, A)$  be a soft set over a soft topological space  $(X, \tau, E)$ . We say that  $x \in (F, E)$  read as  $x$  belongs to the set  $(F, E)$  whenever  $x \in F(e)$  for all  $e \in E$ . Note that for any  $x \in X$ ,  $x \notin (F, E)$ , if  $x \notin F(e)$  for some  $e \in E$ .

**Definition 2.8 [11]:** Let  $(F, A)$  and  $(G, B)$  be two soft sets over a common universe  $X$ . Then  $(F, A) \subseteq (G, B)$  if  $A \subseteq B$ , and  $F(e) \subseteq G(e)$  for all  $e \in A$ .

**Definition 2.9 [2]:** Let  $\tau$  be the collection of soft sets over  $X$ , then  $\tau$  is said to be a soft topology on  $X$  if it satisfies the following axioms.

- (1)  $\tilde{\varnothing}, \tilde{X}$  belong to  $\tau$ .
- (2) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
- (3) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . Let  $(X, \tau, E)$  be a soft topological space over  $X$ , then the members of  $\tau$  are said to be soft open sets in  $X$ . A

soft set  $(F, A)$  over  $X$  is said to be a soft closed set in  $X$ , if its relative complement  $(F, A)^c$  belongs to  $\tau$ .

**Definition 2.10[12]:** For a soft set  $(F, A)$  over  $X$ , the relative complement of  $(F, A)$  is denoted by  $(F, A)^c$  and is defined by  $(F, A)^c = (F^c, A)$ , where  $F^c: A \rightarrow P(X)$  is a mapping given by  $F^c(e) = X - F(e)$ , for all  $e \in A$ .

**Definition 2.11[2]:** Let  $X$  be an initial universal set and  $A$  be the non-empty set of parameters. Then a soft set  $(P, A)$  over  $X$  is said to be soft point if there is exactly one  $\lambda \in A$ , such that  $P(\lambda) = \{x\}$  for some  $x \in X$  and  $P(\mu) = \varnothing, \forall \mu \in A \setminus \{\lambda\}$ . It will be denoted by  $P_\lambda^x$ .

**Definition 2.12:** A soft set  $(F, A)$  in a soft topological space  $X$  is called

- (i) soft regular open (resp. soft regular closed) set [13] if  $(F, A) = \text{Int}(\text{Cl}(F, A))$  [resp.  $(F, A) = \text{Cl}(\text{Int}(F, A))$ ].
- (ii) soft semi-open (resp. soft semi-closed) set [5] if  $(F, A) \subseteq \text{Cl}(\text{Int}(F, A))$  [resp.  $\text{Int}(\text{Cl}(F, A)) \subseteq (F, A)$ ].
- (iii) soft pre-open (resp. soft pre-closed) [13] if  $(F, A) \subseteq \text{Int}(\text{Cl}(F, A))$  [resp.  $\text{Cl}(\text{Int}(F, A)) \subseteq (F, A)$ ].
- (iv) soft  $\alpha$ -open (resp. soft  $\alpha$ -closed) [13] if  $(F, A) \subseteq \text{Int}(\text{Cl}(\text{Int}(F, A)))$  [resp.  $\text{Cl}(\text{Int}(\text{Cl}(F, A))) \subseteq (F, A)$ ].
- (v) soft  $\beta$ -open (resp. soft  $\beta$ -closed) set [13] if  $(F, A) \subseteq \text{Cl}(\text{Int}(\text{Cl}(F, A)))$  [resp.  $\text{Int}(\text{Cl}(\text{Int}(F, A))) \subseteq (F, A)$ ].
- (vi) soft  $\gamma$ -open (resp. soft  $\gamma$ -closed) set [6] if  $(F, A) \subseteq [\text{Int}(\text{Cl}(F, A)) \cup \text{Cl}(\text{Int}(F, A))]$  [resp.  $\text{Int}(\text{Cl}(F, A)) \cap \text{Cl}(\text{Int}(F, A)) \subseteq (F, A)$ ].

**Definition 2.13 [25]:** A space  $(X, \tau, E)$  is said to be soft almost normal (briefly, SAN) iff every pair of disjoint sets  $(F, A)$  and  $(G, A)$ , one of which is soft closed and the other is soft regular closed, there exist disjoint soft open sets  $(U, A)$  and  $(V, A)$  such that  $(F, A) \subseteq (U, A)$ ,  $(G, A) \subseteq (V, A)$ .

**Definition 2.14 [25]:** For any soft topological space  $(X, \tau, E)$ , the family of all soft open sets which can be written as the union of an arbitrary number of soft regularly open sets of  $X$  is a topology on  $(X, \tau, E)$ , weaker than the initial topology  $\tau$  on  $X$  and denoted by  $\tau_s$ . This  $\tau_s$  is called soft semi-regular topology and members of  $\tau_s$  are called soft semi-regular open set.

**Definition 2.15 [25]:** Let  $(X, \tau, E)$  be a soft topological space and  $X$  will be called soft nearly compact (in short, soft N-compact) iff every soft open covering of  $X$  has finite subcollection, the soft interiors of soft closures of which covers  $X$ .

### III. Soft almost regular and soft almost paracompact spaces

In this section soft  $\alpha$ -almost regular, soft almost paracompact space and soft  $\alpha$ -almost paracompact space are defined. Their characterizations and relationships among these notions are also established.

**Definition 3.1:** A soft subset  $(G, A)$  of a space  $X$  is soft  $\alpha$ -almost regular iff for any soft point  $P_\lambda^x \in (G, A)$  and any soft regularly open set  $(U, A)$  containing  $P_\lambda^x$ , there exists an open set  $(V, A)$  such that  $P_\lambda^x \in (V, A) \subseteq \text{SCI}(V, A) \subseteq (U, A)$ .

Or, equivalently, for any soft regularly closed set  $(F, A)$  of a space  $X$  and any soft point  $P_\lambda^x \in (G, A)$  such that  $X \in X \setminus (F, A)$ , there exists disjoint soft open neighborhoods of  $P_\lambda^x$  and  $(F, A)$  respectively.

**Definition 3.2 [25]:** A soft topological space  $X$  is called soft nearly paracompact (briefly, SNP) if every soft regularly open covering admits a locally finite soft open refinement. A subset  $(U, A)$  is called soft nearly paracompact if the relative topology defined on it is soft nearly paracompact.

**Definition 3.3 [25]:** Let  $(X, \tau, E)$  be a soft topological space and  $(U, A)$  be a soft subset  $X$ . The soft set  $(U, A)$  is soft  $\alpha$ -nearly paracompact (in short, soft  $\alpha$ -N-paracompact) iff every soft regular open covering of  $(U, A)$ , has an soft open refinement which covers  $(U, A)$  and is soft locally finite for every point in  $(X, \tau, E)$ . The subset  $(U, A)$  is soft nearly paracompact (in short, soft N-paracompact) iff  $(U, A)$  is soft nearly paracompact (in short, soft N-paracompact) as a subspace.

**Definition 3.4:** A soft topological space  $X$  is called soft almost paracompact (briefly, S $\alpha$ AP) iff for every soft open covering  $\mathcal{U} = \{(U, A)_\alpha : \alpha \in J\}$  of  $X$  there exists a locally finite soft open family  $\mathcal{V}$  which refines  $\mathcal{U} = \{(V, A)_\alpha : \alpha \in J\}$  such that which covers  $X = \bigcup_\alpha \{\text{SCI}(V, A)_\alpha : (V, A)_\alpha \in \mathcal{U}, \alpha \in J\}$

**Definition 3.5:** A soft topological space  $X$  is called soft  $\alpha$ -almost paracompact (briefly, S $\alpha$ AP) if every soft open covering admits a locally finite soft open refinement which covers  $X$ . A soft topological space  $(U, A)$  is called soft  $\alpha$ -almost paracompact (briefly, S $\alpha$ AP) if every soft

open covering admits a locally finite soft open refinement which covers  $(U, A)$ .

**Definition 3.6:** A space  $X$  is soft locally nearly paracompact (soft locally almost paracompact) iff for each soft point there exists an soft open neighborhoods  $(U, A)$  such that  $\text{SCI}(U, A)$  is soft  $\alpha$ -nearly paracompact (soft  $\alpha$ -almost paracompact).

**Theorem 3.7.** Let  $(X, \tau, E)$  be a space in which there is a soft dense  $\alpha$ -nearly paracompact subset  $(G, A)$ . Then  $(X, \tau, E)$  is soft almost paracompact.

**Proof.** Let  $\mathcal{U} = \{(U, A)_\alpha : \alpha \in I, \text{infinite set}\}$  be any soft regularly open covering of  $X$ . There exists a soft dense  $\alpha$ -nearly paracompact subset  $(G, A)$  of  $X$ . Since  $(G, A)$  is soft  $\alpha$ -nearly paracompact, there exists an soft open  $X$ -locally finite family  $\mathcal{V} = \{(V, A)_\alpha : \alpha \in I\}$  which refines  $\mathcal{U}$  such that  $(G, A) \subseteq \bigcup_\alpha \{(V, A)_\alpha : \alpha \in J, \text{a finite set}\}$ . Then,

$\tilde{X} = (G, A) \subseteq \text{SCI}\{\bigcup_\alpha \{(V, A)_\alpha : \alpha \in J\}\} = \bigcup_\alpha \{\text{SCI}(V, A)_\alpha : \alpha \in J\}$  which for every soft regularly open covering of the space, there exists a locally finite family of soft open sets which refines it and the soft closures of whose members cover the space, hence,  $X$  is soft almost paracompact space.

**Theorem 3.8.** Let  $(G, A)$  be an  $v$  soft  $\alpha$ -almost regular  $\alpha$ -almost paracompact subset of a space  $X$ . Then for every soft regularly open cover  $\mathcal{U}$  of  $(G, A)$  there exists an  $X$ -locally finite family of soft regularly closed sets which refines  $\mathcal{U}$  and covers  $A$ .

**Proof.** Let  $\mathcal{U} = \{(U, A)_\alpha : \alpha \in I, \text{infinite set}\}$  be any soft regularly open covering of  $(G, A)$ . For each,  $P_\lambda^x \in (G, A)$ , there exists a soft regularly open set  $(V, A)_x$  such that  $P_\lambda^x \in (V, A)_x$ ,

$\subseteq (U, A)_{\alpha(x)}$ , for some,  $(U, A)_{\alpha(x)} \in \mathcal{U}$ . Now,  $\mathcal{V} = \{(V, A)_x : P_\lambda^x \in (G, A)\}$  is a soft regularly open covering of  $(G, A)$ . There exists an  $X$ -locally finite family of soft open sets which refines  $\mathcal{V}$  and the soft closures of whose members cover  $(G, A)$ . The family  $\text{SCI}(\mathcal{C}) = \{\text{SCI}(C, A) : (C, A) \in \mathcal{C}\}$  is then an  $X$ -locally finite family of soft regularly closed sets which refines  $\mathcal{U}$  and covers  $(G, A)$ .

**Remark 3.9.** In any soft space there is no proper soft regularly open soft  $\alpha$ -almost regular  $\alpha$ -nearly paracompact subset i.e. every soft regularly open soft  $\alpha$ -

nearly paracompact soft  $\alpha$ -almost soft regular open subset is soft closed.

If  $(G,A)$  is a soft dense  $\alpha$ -nearly paracompact soft  $\alpha$ -almost regular subset of a soft space  $X$ , then every soft regularly open cover of  $(G,A)$  is an soft open cover of  $X$ , hence every regularly open covering of  $X$  has a locally finite soft regularly closed refinement.

**Lemma 3.10.** Let  $(G,A)$  be any soft  $\alpha$ -almost regular subset of a space  $X$  such that for every soft regularly open cover  $\mathcal{U}$  of  $(G,A)$  there exists an  $X$ -locally finite family  $\mathcal{V}$  which refines  $\mathcal{U}$  and covers  $(G,A)$ , then for every soft regularly open covering  $\mathcal{U}$  of  $(G,A)$  there exists an  $X$ -locally finite family  $\mathcal{V}$  of soft closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ .

**Proof.** Let  $\mathcal{U} = \{(U,A)_i : i \in I, \text{infinite set}\}$  be any soft regularly open covering of  $(G,A)$ . Then, for each soft point  $P_\lambda^x \in (G,A)$  there exists a soft regularly open set  $(V,A)_x$  such that  $P_\lambda^x \in (V,A)_x, \tilde{\subseteq} \text{SCI}\{(V,A)_x \tilde{\subseteq} (U,A)_{i(x)}, \text{for some, } i(x) \in I.$

Let  $\mathcal{V} = \{(V,A)_x : P_\lambda^x \in (G,A)\}$ . Since,  $\mathcal{V}$  is a soft regularly open covering of  $(G,A)$ , there exists an  $X$ -locally finite family  $\mathcal{C} = \{(P,A)_j : j \in J\}$  of soft open sets which refines  $\mathcal{V}$  and covers  $(G,A)$ .  $\{\text{SCI}(P,A)_j : j \in J\}$  is then  $X$ -locally finite family of soft closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ .

**Remark 3.11.** If  $(G,A)$  is a soft dense soft  $\alpha$ -almost regular subset of a space  $X$  with the properties as in lemma 3.10, then every soft regularly open covering of  $(G,A)$  is a soft regularly open covering of  $X$  and every  $X$ -locally finite soft closed covering of  $(G,A)$  is soft closed covering of  $X$ . Then, it follows that if for every soft regularly open covering  $\mathcal{U}$  of  $(G,A)$ , there exists an  $X$ -locally finite family  $\mathcal{V}$  which refines  $\mathcal{U}$  and covers  $(G,A)$ , then there exists an  $X$ -locally finite family  $\mathcal{V}$  of soft regularly closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$  (i.e. covers  $X$ ).

**Lemma 3.12.** Let  $(G,A)$  be a soft dense subset of a space  $X$  such that for every soft regularly open covering  $\mathcal{U}$  of  $(G,A)$ , there exists an  $X$ -locally finite family of soft regularly closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ , then  $X$  is soft nearly paracompact space.

**Proof.** It follows that every soft regularly open covering of  $X$  has a locally finite soft regularly closed refinement, hence,  $X$  is soft nearly paracompact space.

**Lemma 3.13.** Let,  $(G,A)$  be any soft dense subset of a space  $X$  such that every soft regularly open covering of  $A$  is a soft regularly open covering of  $X$ . If  $X$  is soft nearly paracompact then  $(G,A)$  is a soft  $\alpha$ -nearly paracompact space.

**Proof.** Obvious and omitted.

**Theorem 3.14.** Let  $(G,A)$  be any soft dense  $\alpha$ -almost regularly open subset of a space  $X$  such that every soft regularly open covering of  $(G,A)$  is a soft regularly open covering of  $X$ . Then, the followings are equivalent:

- (a)  $X$  is soft nearly paracompact space.
- (b)  $(G,A)$  is soft  $\alpha$ -nearly paracompact space.
- (c)  $(G,A)$  is soft  $\alpha$ -almost paracompact.
- (d) For every soft regularly open covering  $\mathcal{U}$  of  $(G,A)$  there exists an  $X$ -locally finite family which refines  $\mathcal{U}$  and covers  $(G,A)$ .
- (e) For every soft regularly open covering  $\mathcal{U}$  of a there exists an  $X$ -locally finite family  $\mathcal{V}$  of soft closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ .
- (f) For every soft regularly open covering  $\mathcal{U}$  of  $(G,A)$  there exists an  $X$ -locally finite family  $\mathcal{V}$  of soft regularly closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ .

**Proof:** (a)  $\Rightarrow$  (b): It follows from lemma 4.7.

(b)  $\Rightarrow$  (a): It follows from the fact that "If  $(G,A)$  is soft  $\alpha$ -almost regular soft  $\alpha$ -nearly paracompact subset of a space  $X$ , then  $(G,A)$  is soft  $\alpha$ -nearly paracompact space".

(b)  $\Rightarrow$  (c): Easy and omitted.

(c)  $\Rightarrow$  (d): It follows from lemma 3.8.

(d)  $\Rightarrow$  (e): It follows from lemma 3.10.

(e)  $\Rightarrow$  (f): It follows from remark 3.11.

(f)  $\Rightarrow$  (a): It follows from lemma 3.13.

**Theorem 3.15.** Let  $(G,A)$  be any soft dense  $\alpha$ -almost regular subset of a space  $(X,\tau,E)$  such that every soft regularly open covering of  $(G,A)$  is a soft regularly open covering of  $X$ . If  $X$  is soft almost paracompact, then  $X$  is soft nearly paracompact space.

**Proof:** Let  $\mathcal{U} = \{(U,A)_i : i \in I, \text{infinite set}\}$  be any soft regularly open covering of  $(X,\tau,E)$ . For each soft point  $P_\lambda^x \in (G,A)$ , there exists a soft regularly open set  $(V,A)_x$  such that  $P_\lambda^x \in (V,A)_x, \tilde{\subseteq} \text{SCI}\{(V,A)_x \tilde{\subseteq} (U,A)_i, \text{for some, } i \in I.$

Now,  $\mathcal{V} = \{(V,A)_x : P_\lambda^x \in X\}$  is a soft regularly open covering of  $(G,A)$  i.e. of  $X$ . Since  $X$  is soft almost paracompact, there exists a soft open locally finite family  $\mathcal{C} = \{(C,A)_j : j \in J\}$  which refines  $\mathcal{V}$  such that  $X =$

$\text{SCI}\{\bigcup_j \{(C,A)_j: j \in J\} = \bigcup_j \{\text{SCI}(C,A)_j: j \in J\}$ . Now for each  $j \in J$ , there exists  $x(j) \in (G,A)$  such that  $(C,A)_j \subseteq \text{SCI}\{(V,A)_{x(j)} \subseteq (U,A)_{i(x)}$ , for some  $x(i) \in I$ . Hence, we have,  $\text{SCI}((C,A)_j) \subseteq \text{SCI}\{(V,A)_{x(j)} \subseteq (U,A)_{i(x)}$ . Now,  $\{\text{SCI}((C,A)_j): j \in J\}$  is locally finite family of soft regular closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ , hence  $(G,A)$  is soft  $\alpha$ -nearly paracompact i.e.  $(X, \tau, E)$  is soft nearly paracompact space.

**Theorem 3.16.** Let  $(G,A)$  be any soft  $\alpha$ -almost regular open subset of a space  $X$  Then, the followings are equivalent:

- (a)  $(G,A)$  is soft  $\alpha$ -nearly paracompact space.
- (b)  $(G,A)$  is soft  $\alpha$ -almost paracompact.
- (c) For every soft regularly open covering  $\mathcal{U}$  of  $(G,A)$ , there exists an  $X$ -locally finite family of soft regularly closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ .

**Proof:** (a)  $\Rightarrow$  (b): Easy and omitted.

(b)  $\Rightarrow$  (c): It follows from lemma 4.8.

(c)  $\Rightarrow$  (a): Let  $\mathcal{U} = \{(U,A)_i: i \in I, \text{ infinite set}\}$  be any soft regularly open covering of  $(G,A)$ . Then there exists an  $X$ -locally finite family  $\mathcal{V}$  of soft regularly closed sets which refines  $\mathcal{U}$  and covers  $(G,A)$ . Since,  $\mathcal{V}$  is  $X$ -locally finite, for each soft point  $P_\lambda^x \in X$ , there exists a soft open set  $(H,A)_x$  such that  $P_\lambda^x \in (H,A)_x$  and  $(H,A)_x$  intersects finitely many members of  $\mathcal{V}$ . Then  $\alpha((H,A)_x)$  is soft regularly open set containing  $P_\lambda^x$  which intersects finitely many members of  $\mathcal{V}$ . Let,  $\mathcal{G} = \{\text{SInt}(\text{SCI}((H,A)_x)): P_\lambda^x \in X\}$ . It is soft regularly open cover of  $(G,A)$ , hence there exists an  $X$ -locally finite family  $\mathcal{B}$  of soft regularly closed sets which refines  $\mathcal{G}$  and covers  $(G,A)$ . Now for each  $(V,A) \in \mathcal{V}$ , let  $(V,A)^* = (X \setminus \bigcup \{(B,A): (B,A) \in \mathcal{B}, (B,A) \tilde{\cap} (V,A) = \tilde{\varphi}\}) \tilde{\cap} (G,A)$ . Clearly,  $(V,A)^*$  is soft open set containing  $(B,A) \tilde{\cap} (V,A)$  and  $(B,A) \tilde{\cap} (V,A)^* \neq \tilde{\varphi}$  if and only if  $(B,A) \tilde{\cap} ((V,A) \tilde{\cap} (G,A)) \neq \tilde{\varphi}$ . Since,  $\mathcal{V}$  refines  $\mathcal{U}$ , then for each  $(V,A) \in \mathcal{V}$ , there exists  $(U,A)_v \in \mathcal{U}$  such that  $(V,A) \subseteq (U,A)_v$ . Let  $\mathcal{H} = \{(V,A)^* \tilde{\cap} (U,A)_v: (V,A) \in \mathcal{V}\}$ . Then  $(G,A)$  is a soft open  $X$ -locally finite family which refines  $\mathcal{U}$  and covers  $(G,A)$ . Hence,  $(G,A)$  is soft  $\alpha$ -nearly paracompact space.

**Lemma 3.17.** If  $(F,A)$  is a soft regularly closed soft  $\alpha$ -almost paracompact subset of a space  $X$ , then  $(F,A)$  is almost paracompact set.

**Proof:** Since  $A$  is soft regularly closed, then  $(F,A) = \text{SCI}(G,A)$ , where,  $(G,A)$  is an soft open subset of  $X$ . Let,  $\{\text{SCI}(G,A) \tilde{\cap} (U,A)_i: i \in I\}$  be any soft open covering of  $(G,A)$ . Then  $\mathcal{U} = \{(U,A)_i: i \in I\}$  is an soft open covering of  $(F,A)$ , hence there exists an  $X$ -locally finite family  $\mathcal{V} = \{(V,A)_j: j \in J\}$  of soft open subsets of  $X$  which refines  $\mathcal{U}$  and is such that  $(F,A) \subseteq \text{SCI}\{\bigcup_j \{(V,A)_j: j \in J\}$ . Now, since  $(G,A)$  is soft open therefore,  $\text{SCI}\{(G,A) \tilde{\cap} \bigcup_j (V,A)_j\} = \text{SCI}\{(G,A) \tilde{\cap} (\text{SCI}(\bigcup_j (V,A)_j))\}$ . Also since  $\{(V,A)_j: j \in J\}$  is locally finite, therefore,  $(\text{SCI}(\bigcup_j (V,A)_j)) = \bigcup_j \text{SCI}(V,A)_j$ . Thus  $\text{SCI}\{(G,A) = \text{SCI}\{(G,A) \tilde{\cap} \bigcup_j (V,A)_j\} = \bigcup_j \text{SCI}\{(G,A) \tilde{\cap} (V,A)_j\} \subseteq \bigcup_j \text{SCI}\{\text{SCI}(G,A) \tilde{\cap} (V,A)_j\}$ . Hence,  $\{(F,A) \tilde{\cap} (V,A)_j: j \in J\}$  is an  $X$ -locally finite family of soft open sets of  $(F,A)$  which refines  $\{(F,A) \tilde{\cap} (U,A)_i: i \in I\}$  and the soft closures of whose members in  $(F,A)$  covers  $(F,A)$ . Hence it follows that  $(F,A)$  is soft almost paracompact space.

**Definition 3.18.** The family of soft regularly open sets in  $(X, \tau, E)$  forms a soft base for a soft semi regular topology  $\tau_s$  on  $X$  called soft semi regularization of  $\tau$ . In this paper by  $X_s$  we shall denote the soft topological space  $(X, \tau_s, E)$ .

**Theorem 3.19.** Every soft almost regular soft locally almost paracompact space is soft locally nearly paracompact.

**Proof:** Let  $X$  be any soft almost regular soft locally almost paracompact space and let  $P_\lambda^x \in X$ . Then there exists a soft regularly open neighborhood  $(U,A)$  of  $P_\lambda^x$  such that  $\text{SCI}(U,A)$  is soft  $\alpha$ -almost paracompact. Thus,  $X_s$  is a soft regular space that for each point  $P_\lambda^x \in X$ , there exists an soft open set (in  $X_s$ )  $(U,A)$  such that  $\text{SCI}((U,A)_\tau) = \text{SCI}((U,A)_{\tau_s})$  is soft  $\alpha$ -almost paracompact in  $X_s$ .

Then  $\text{SCI}((U,A)_{\tau_s})$  is soft almost paracompact in  $X_s$ . The subset  $\text{SCI}((U,A)_{\tau_s})$  is a soft regular soft almost paracompact space. Hence,  $\text{SCI}((U,A)_{\tau_s})$  is soft paracompact in  $X_s$  (since, every soft regular soft almost paracompact space is soft paracompact). It follows that every soft point has an soft open neighbourhood  $(U,A)$  in  $X_s$  such that  $\text{SCI}((U,A)_{\tau_s})$  is soft paracompact. Since,

$X_s$  is regular it follows that every soft point has an soft open neighbourhood  $(V, A)$  (in  $X_s$ ) such that  $SCI((V, A)_{\tau_s})$  is soft  $\alpha$ -paracompact in  $X_s$ , hence  $SCI((V, A)_{\tau}) = SCI((V, A)_{\tau_s})$  is soft  $\alpha$ -nearly paracompact in  $X$ . Hence,  $X$  is soft locally nearly paracompact space.

#### IV. CONCLUSION

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. The soft set theory proposed by Molodtsov [1] offers a general mathematical tool for dealing with uncertain or vague objects. It is shown that soft sets are special type of information system known as single valued information system. In this work, some new notions in soft space such as soft  $\alpha$ -almost regular spaces, soft almost paracompact spaces and soft  $\alpha$ -almost paracompact spaces are introduced. We also investigate some basic properties of these concepts and obtain several interesting results and characterizations of soft nearly compact and nearly paracompact spaces. Hope that the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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