Comparison of L-moments of Probability Distributions for Extreme Value Analysis of Rainfall for Estimation of Peak Flood Discharge for Ungauged Catchments

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ABSTRACT

Estimation of Peak Flood Discharge (PFD) at a desired location on a river is important for planning, design and management of hydraulic structures. For ungauged catchments, rainfall depth becomes an important input in derivation of PFD. So, rainfall depth can be estimated through frequency analysis by fitting of probability distributions to the rainfall data. In this paper, the series of annual 1-day maximum rainfall derived from daily rainfall data recorded at Una district is used to estimate the 1-day maximum rainfall adopting six probability distributions. Method of L-moments is used for determination of parameters of distributions. Goodness-of-Fit tests viz., Chi-square and Kolmogorov-Smirnov are applied for checking the adequacy of fitting of probability distributions to the recorded data. Root Mean Square Error (RMSE) is used for the selection of most suitable probability distribution for estimation of rainfall. Based on GoF test results and RMSE values, the study identifies the Extreme Value Type-1 (EV1) is better suited distribution for rainfall estimation. By applying the procedures, as described in CWC guidelines, the 1-hour value of distributed rainfall is computed from the estimated 1-day maximum rainfall using EV1 distribution and adopted for computation of PFD for ungauged catchments. The study suggests the computed PFD from rational formula could be considered for design of flood protection measures for river Swan and its tributaries joining the Beas river basin, Himachal Pradesh.

Keywords: Chi-square, Extreme Value, Kolmogorov-Smirnov, Mean Square Error, Rainfall, Peak Flood

I. INTRODUCTION

Estimation of Peak Flood Discharge (PFD) at a desired location on a river is important for planning, design and management of hydraulic structures such as dams, bridges, barrages and design of storm water drainage systems. These include different types of flood such as standard project flood, probable maximum flood and design basis flood. In case of large river basins, the hydrological and stream flow series of a significant duration are generally available. However, for ungauged catchments, more data is not available other than rainfall (NIH, 2011). The rainfall data is also of shorter duration and may pertain to a neighbouring basin. Rainfall depth thus becomes an important input in derivation of PFD (Singh et al., 2001). For arriving at such design values, Extreme Value Analysis (EVA) of rainfall is carried out.

Out of number of probability distributions, Exponential (EXP), Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2), Generalized Extreme Value (GEV), Generalized Pareto (GPA), and Normal (NOR) distributions are generally used in EVA of rainfall (Neslihan et al., 2010). Generally, Method of Moments (MoM) is used for determination of parameters of the distributions. But, the MoM is not giving satisfactory results though the method exists for a longer period. It is sometimes difficult to assess exactly what information about the shape of a distribution is conveyed by its moments of third and higher order; the numerical values of sample moments particularly when the sample is small, can be very different from those of the probability distribution from which the sample was drawn; and the estimated parameters of distributions fitted by the MoM are often less accurate than those obtained by other estimation procedures such as maximum likelihood method, method of least squares and probability weighted moments. To overcome this, the alternative approach, namely L-moments (LMO) is discussed in this paper and also used in EVA of rainfall (Hosking, 1990).

In the recent past, number of studies has been carried out by different researchers on adoption of probability distributions for Rainfall Frequency Analysis (RFA). Topaloglu (2002) reported that the frequency analysis of the largest, or the smallest, of a sequence of hydrologic events has long been an essential part of the design of hydraulic structures. Guevara (2003) carried out hydrologic analysis using probabilistic approach to estimate engineering design parameters of storms in Venezuela. Kumar and Chatterjee (2005) employed the LMO to define homogenous regions within 13 gauging sites of the north Brahmaputra region of India. Di Balldassarre et al. (2006) used the LMO for regionalization of annual precipitation in northern central Italy. Eslamian and Feizi (2007) carried out RFA using monthly maximum rainfall for an arid region in Isfahan Province (Iran) through LMO.

Gonzalez and Valdes (2008) applied LMO for regionalization of monthly rainfall in the Jucar River basin. Yurekli et al. (2009) found GEV and 3-parameter Log-Normal (LN3) distributions (using LMO) as the regional distribution functions for the daily maximum rainfall of Cekerec watershed, Turkey. Gubareva and Gartsman (2010) analysed the extreme hydrometeorological characteristics adopting GEV, GPA, LN3 and Pearson distributions through LMO. Badreldin and Feng (2012) carried out the regional RFA for the Luanhe Basin, Hebei-China by using LMO and Cluster Techniques. But there is no general agreement in applying particular distribution for RFA for different region or country. Moreover, when different distributional models are used for modelling of rainfall data series, a common problem that arises is how to determine which model fits best for a given set of data. This can be answered by formal statistical procedures involving Goodness-of-Fit (GoF) tests and diagnostic index; and the results are quantifiable and reliable.

Qualitative assessment is made from the plots of the recorded and estimated rainfall. For quantitative assessment on rainfall within in the recorded range, Chi-square (χ^2) and Kolmogorov-Smirnov (KS) tests are applied. A diagnostic index, say Root Mean Square Error (RMSE) is used for the selection of most suitable probability distribution for estimation of rainfall. The objective of the paper is to compute PFD using rational formula for ungauged catchments of Beas River Basin (BRB) upstream of river Swan. The methodology adopted in EVA of rainfall using six probability distributions, estimation of PFD using rational formula, computation of GoF tests statistic and diagnostic index are briefly described in the ensuing sections.

II. METHODOLOGY

LMO is analogous to the conventional moments but can be estimated linear combination of order statistics, i.e., by L-statistics. LMO is less subject to bias in estimation and approximate their asymptotic normal distribution more closely in finite samples.

Theoretical Description of LMO

Method of LMO is a modification of the probability weighted moments method explored by Hosking and Wallis (1993). Parameters of the distribution are estimated by equating the sample LMO (l_r) with the distribution of LMO (b_r). In practice, LMO must be estimated from a finite sample. Let $R_{1N} \leq R_{2N \leq \ldots \leq} R_{NN}$ be the ordered sample of size N.

The sample LMO is given by:
$$l_{r+1} = \sum_{k=0}^{r} \frac{(-1)^{r-k}(r+k)!}{(k!)^2(r-k)!} b_k \qquad \ldots (1)$$

where, l_{r+1} is the $r+1^{th}$ sample moment and b_k is an unbiased estimator of β_k with

$$b_k = N^{-1} \sum_{i=k+1}^{N} \frac{(i-1)(i-2)\ldots(i-k)}{(N-1)(N-2)\ldots(N-k)} R_{iN} \qquad \ldots (2)$$

The first two sample LMOs are expressed by:
$$l_1 = b_0 \text{ and } l_2 = 2b_1 - b_0 \qquad \ldots (3)$$

Table 1 gives the details of quantile function and parameters six probability distributions considered in the study (Hosking and Wallis, 1997).

TABLE 1
QUANTILE FUNCTION AND PARAMETERS OF SIX PROBABILITY DISTRIBUTIONS

S.No.	Distribution	Quantile function (R_T)	Parameters by LMO
1	EXP	$R_T = \psi - \mu \log(1 - F)$	ψ (known); $\mu = l_1$
2	EV1	$R_T = \xi - \alpha \log(-\log F)$	$\xi = l_1 - 0.5772157\,\alpha$; $\alpha = l_2/\log 2$
3	EV2	$R_T = \alpha e^{(-\ln(-\ln(F))/k)}$	By using the logarithmic transformation of the recorded data, parameters of EV1 are initially obtained by LMO; and used to determine the parameters of EV2 from $\alpha = e^{\xi}$ and k=1/(scale parameter of EV1).
4	GEV	$R_T = \xi + \alpha(1 - (-\log F)^k)/k$	$z = (2/(3 + t_3)) - (\ln 2/\ln 3)$; $k = 7.8590\,z + 2.9554\,z^2$; $\alpha = l_2 k/(1 - 2^{-k})\Gamma(1 + k)$; $\xi = l_1 + (\alpha(\Gamma(1 + k) - 1)/k)$
5	GPA	$R_T = \xi + \alpha(1 - (1 - F)^k)/k$	$\xi = l_1 + l_2(k + 2)$; $k = (4/(t_3 + 1)) - 3$ $\alpha = (1 + k)(2 + k)l_2$
6	NOR	$R_T = \mu + \sigma\varphi^{-1}(F)$	$\mu = l_1$; $\sigma = l_2\sqrt{\pi}$

In Table 1, F(R) (or F) is the cumulative distribution function (CDF) of R; P is the probability of exceedance; ϕ^{-1} is the inverse of the standard normal distribution function and $\varphi^{-1} = Z_P = (P^{0.135} - (1 - P)^{0.135})/0.1975$; ξ, α, k are the location, scale and shape parameters respectively; μ (or $\overline{R}$), σ (or S_R) and C_S (or Ψ) are the average, standard deviation and coefficient of skewness of the recorded rainfall data; sign(k) is plus or minus 1 depending on the sign of k ; R_T is the estimated rainfall by probability distributions corresponding to return period T (in year).

Goodness-of-Fit Tests

GoF tests are essential for checking the adequacy of probability distributions to the series of recorded rainfall data. Out of a number GoF tests available, the widely accepted GoF tests are χ^2 and KS, which are used in the study. The theoretical descriptions of GoF tests statistic (Charles Annis, 2009) are as follows:
χ^2 statistic:

$$\chi^2 = \sum_{j=1}^{NC} \frac{(O_j(R) - E_j(R))^2}{E_j(R)} \qquad \ldots (4)$$

where, $O_j(R)$ is the observed frequency value of jth class, $E_j(R)$ is the expected frequency value of jth class and NC is the number of frequency classes. The rejection region of χ^2 statistic at the desired significance level (η) is given by $\chi_C^2 \geq \chi_{1-\eta, NC-m-1}^2$. Here, m denotes the number of parameters of the distribution and χ_C^2 is the computed value of χ^2 statistic by PDF.

KS statistic:

$$KS = \underset{i=1}{\overset{N}{Max}}(F_e(R_i) - F_D(R_i)) \qquad \ldots (5)$$

where, $F_e(R_i) = M/(N + 1)$ is the empirical CDF of R_i and $F_D(R_i)$ is the computed CDF of R_i (Zhang, 2002). Here, M is the rank assigned to each R_i. If the computed values of GoF tests statistic given by the distribution are less than that of the theoretical values at the desired significance level (η) then the distribution is found to be acceptable for modelling the series of rainfall data.

Diagnostic Index

The selection of most suitable distribution for estimation of rainfall is performed through diagnostic index, say RMSE, which is defined by:

$$RMSE = \sqrt{(1/N)\sum_{i=1}^{N}(R_i - R_i^*)^2} \qquad \ldots (6)$$

Here, R_i is the recorded rainfall of ith sample and R_i^* is the estimated rainfall i^{th} sample by probability distribution. The distribution having the least RMSE is considered as better suited distribution for estimation of rainfall.

III. APPLICATION

In this paper, a study on estimation of PFD for different return periods for 12 catchments of BRB upstream of river Swan is carried out. The estimated 1-day maximum rainfall obtained from the selected probability distribution through GoF tests and diagnostic index is considered as an input to estimate the PFD using CWC guidelines. The Annual 1-day Maximum Rainfall (AMR) recorded at Una district for the period of 20 years from 1995 to 2014, as presented in Figure 1, is used. The descriptive statistics such as average rainfall ($\overline{R}$), standard deviation, coefficient of variation, Coefficient of Skewness (C_S) and coefficient of kurtosis of the recorded AMR are computed as 173.0 mm, 70.3 mm, 40.6 %, 0.885 and 0.378 respectively.

IV. RESULTS AND DISCUSSIONS

By applying the procedures of LMO of six probability distributions, parameters were determined and used for estimation of 1-day maximum rainfall for different return periods. Table 2 gives the 1-day maximum rainfall estimates for different return periods adopting six probability distributions. These estimates were used to develop the rainfall frequency curves and presented in Figure 2.
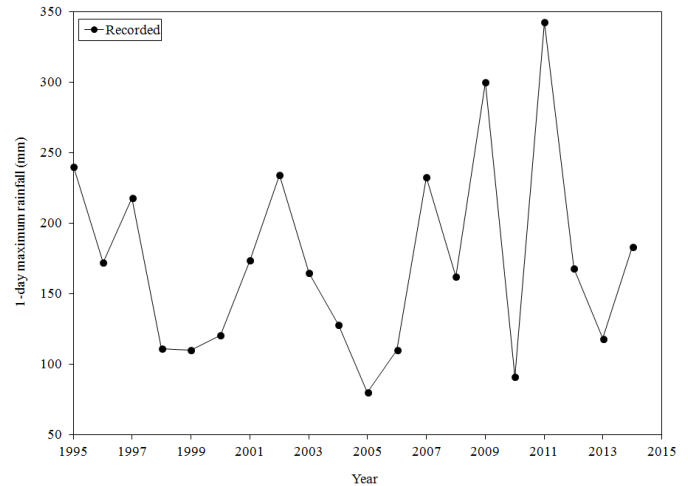

Figure 1: Times series plot of recorded AMR

TABLE 2
ESTIMATED 1-DAY MAXIMUM RAINFALL USING SIX PROBABILITY DISTRIBUTIONS

Probability Distribution	Estimated rainfall for different return periods					
	2-year	5-year	10-year	20-year	50-year	100-year
EXP	148.7	221.3	276.1	331.0	403.5	458.4
EV1	161.0	225.7	268.6	309.7	362.9	402.7
EV2	149.1	208.9	261.1	323.3	426.5	524.8
GEV	158.8	223.4	268.4	313.1	373.7	421.0
GPA	156.6	233.7	278.7	314.6	351.3	372.7
NOR	173.0	232.1	262.9	288.4	317.1	336.2

Rainfall Frequency Curves (RFCs)

The 1-day maximum rainfall estimates obtained from six probability distributions (using LMO) are used to develop the RFCs and presented in Figure 2.

From the trend lines of the fitted curves, as presented in Figure 2, it can be seen that there is a perfect line of agreement in the upper and lower tail regions while estimating the rainfall using EV1 distribution for the data under study.
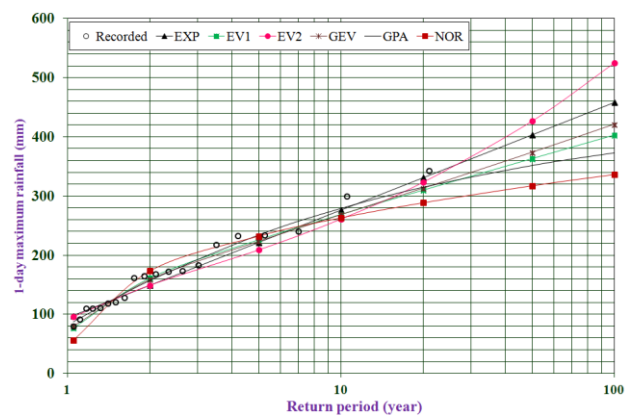

Figure 2: Plots of recorded 1-day maximum rainfall and RFCs using six probability distributions

Analysis Based on GoF Tests

For the present study, the degree of freedom (NC-m-1) is considered as one for 3-parameter distributions (GEV and GPA) and two for 2-parameter distributions (EXP, EV1, EV2 and NOR) while computing the χ^2 statistic values for the data under study. The GoF tests results were computed from Eqs. (4) and (5), and given in Table 3.

TABLE 3
COMPUTED AND THEORETICAL VALUES OF GOF TESTS

Probability distribution	χ^2		KS	
	Computed value	Theoretical value at 5% level	Computed value	Theoretical value at 5% level
EXP	2.000	5.990	0.149	0.294
EV1	2.000	5.990	0.090	0.294
EV2	4.000	5.990	0.163	0.294
GEV	2.000	3.841	0.091	0.294
GPA	2.500	3.841	0.099	0.294
NOR	4.000	5.990	0.120	0.294

From Table 3, it may be noted that the computed values of χ^2 and KS tests results obtained from six probability distributions are not greater than the theoretical values at 5% level of significance, and at this level, all six distributions are found to be acceptable for modelling the series of AMR recorded at Una district.

Analysis Based on Diagnostic Index

For the selection of most suitable probability distribution for estimation of rainfall, the diagnostic index, say RMSE values of six probability distributions are computed from Eq. (6) and given in Table 4.

TABLE 4
COMPUTED VALUES OF RMSE OF SIX
PROBABILITY DISTRIBUTIONS

Data series	RMSE (mm) values using					
	EXP	EV1	EV2	GEV	GPA	NOR
Una	13.6	13.0	17.9	12.8	11.4	19.4

From Table 4, it may be noted that the RMSE values computed from GPA, GEV and EV1 distributions are the first, second and third minimum when compared to the corresponding values of other probability distributions. But, from Figure 2, it can be seen that the rainfall estimates obtained from GPA and GEV distributions are not convincible in upper tail region when compared to the rainfall estimates of EV1 distribution.

By considering the trend lines of the fitted curves by the probability distributions in the tail regions, it is identified that the EV1 is the most appropriate distribution for estimation of rainfall at Una district, which was also confirmed through GoF tests results.

Computation of Peak Flood Discharge

It was required to estimate PFD for 12 catchments of BRB upstream of river Swan. The area of the catchments is presented in Table 5. From an observation of catchment size and at the Google Earth of the region of these catchments it was estimated that these are small catchments that respond quickly to rainfall, t_c (time of concentration) $\cong$ 1-hour.

TABLE 5
CATCHMENT AREA OF DIFFERENT STREAMS

S.No.	Name of catchment	Area (km^2)	S.No.	Name of catchment	Area (km^2)
1	Babchar Khad	3.750	7	Joh Khad	14.500
2	Raipur Khad	3.125	8	Bangi Khad	5.500
3	Mundwara Khad	4.625	9	Kothi Khad	7.250
4	Bhatoli Khad	4.375	10	Jhakhar Khad	12.250
5	Dangah Khad	7.250	11	Kohwali Khad	5.250
6	Pithripur Khad	12.000	12	Phakruwali Khad	3.750

In the absence of the short duration rainfall, say,1-hour, 2-hour, 3-hour, etc., the same was computed from estimated 1-day maximum rainfall by using conversion factors, as given in Central Water Commission (CWC) report entitled 'Flood estimation report for Western Himalayas-Zone 7' (CWC, 1994). For the present study, the estimated 1-day maximum rainfall is multiplied with the factor of 0.425 to compute the 1-hour value of distributed rainfall and presented in Table 6.

TABLE 6
1-HOUR DISTRIBUTED RAINFALL FOR DIFFERENT RETURN PERIODS

1-hour distributed rainfall (I: mm) for					
2-year	5-year	10-year	20-year	50-year	100-year
68.4	95.9	114.2	131.6	154.2	171.1

The distributed 1-hour rainfall was used as input for computation of PFD as the catchment areas of different tributaries of BRB are in the range of 3.125 km^2 to 14.5 km^2.

These streams are ungauged and hence the PFD for ungauged catchments is computed by using rational formula, which is given below:

$$q = 0.278 * C \, I \, A \quad\quad\quad \ldots (7)$$

where, q is peak discharge (m^3/s), C is runoff coefficient, I is rainfall intensity (mm/hour) and A is catchment area (km^2).

By considering topography of the river basin, the value of the C is considered as 0.55 while computing the PFD. The computed PFD for 12 catchments of BRB are presented in Table 7, which could be taken as design flood for the streams.

TABLE 7
PEAK FLOOD DISCHARGE (m^3/s) FOR 12 CATCHMENTS OF BRB

S. No.	Name of the catchment	PFD (m^3/s) for					
		2-year	5-year	10-year	20-year	50-year	100-year
1	Babchar Khad	39.2	55.0	65.5	75.5	88.4	98.1
2	Raipur Khad	32.7	45.8	54.5	62.9	73.7	81.8
3	Mundwara Khad	48.4	67.8	80.7	93.1	109.1	121.0
4	Bhatoli Khad	45.8	64.2	76.4	88.0	103.2	114.5
5	Dangah Khad	75.9	106.3	126.5	145.9	171.0	189.7
6	Pithripur Khad	125.5	176.0	209.5	241.5	283.0	314.0
7	Joh Khad	151.7	212.7	253.1	291.8	341.9	379.4
8	Bangi Khad	57.5	80.7	96.0	110.7	129.7	143.9
9	Kothi Khad	75.9	106.3	126.5	145.9	171.0	189.7
10	Jhakhar Khad	128.2	179.7	213.8	246.5	288.9	320.6
11	Kohwali Khad	54.9	77.0	91.6	105.7	123.8	137.4
12	Phakruwali Khad	39.2	55.0	65.5	75.5	88.4	98.1

V. CONCLUSIONS

The paper describes briefly the study carried out for EVA of rainfall using a computer aided procedure for determination of parameters of six probability distributions (using LMO). The selection of most suitable distribution was evaluated by GoF tests (using χ^2 and KS) and diagnostic index (using RMSE). By using the maximum value of 1-hour distributed rainfall, runoff coefficient and catchment area of different streams, the PFD for 12 ungauged catchments of BRB were computed through rational formula. The following conclusions are drawn from the study:

i) The GoF tests results supported the use of all six probability distributions (using LMO) for modelling the series of AMR.

ii) Based on GoF tests results, diagnostic index and rainfall frequency curves, EV1 distribution was found to be most suitable distribution for estimation of 1-day maximum rainfall.

iii) The estimated 1-day maximum rainfall was used to compute 1-hour maximum value of distributed rainfall adopting CWC guidelines described in Flood estimation report for Western Himalayas-Zone 7.

iv) By using the 1-hour distributed rainfall, the PFD for 12 ungauged catchments of BRB upstream of river Swan was computed from rational formula.

v) The study suggested that the PFD, as given in Table 7, could be considered for design of flood protection measures for river Swan and its tributaries joining the BRB, Himachal Pradesh.

VI. ACKNOWLEDGEMENTS

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