© 2017 IJSRST | Volume 3 | Issue 7 | Print ISSN: 2395-6011 | Online ISSN: 2395-602X
Themed Section: Science and Technology

On Hot Electron Transport in Si(100) 2DEG at Low Lattice Temperature

A. K. Ghorai
Physics Department, Kalimpong College, Kalimpong,West Bengal, India

ABSTRACT

Electron temperature model is applied to calculate electron temperature and hot electron mobility at low lattice temperatures in semiconductor inversion layer where the electrons are quantized to form a two-dimensional electron gas (2DEG). The computation is carried out considering the interaction of electrons only with the deformation potential acoustic phonons and using the momentum relaxation time approximation. The derived expressions are used to find the electric field dependence of electron temperature and mobility characteristics in Si(100) 2DEG and the results thus obtained are compared with other theoretical results.
Keywords : Semiconductor, Hot electron, 2DEG, Phonon, Relaxation time

I. INTRODUCTION

A triangular quantum well is generally formed in semiconductor inversion and accumulation layers of modulation doped heterostructure and the main attraction of such systems is the very high mobility of the electrons because of spatial separation of electrons from their parent donors. In the triangular well, only the quantum mechanical ground state is populated by free electrons at low lattice temperatures ($T_L < 100\,\mathrm{K}$), making the system one of the best experimental realization of a 2DEG in nature [1-3].

The knowledge of mobility of free electrons in semiconducting materials is important for their application in solid state electronic devices. Different theoretical models were developed to study the mobility characteristics and the success of these models largely depends on the proper formulation of different type of scattering mechanisms. This apart, one must know which type of scattering is predominant in a particular material under the prevalent condition of lattice temperature T_L and free carrier concentration N_i. The free carriers are dominantly scattered by the optical and intervalley phonons above the lattice temperature of 100K and by the impurity ions at very lower lattice temperature range. In between these two extreme temperatures the free electrons interact strongly with intravalley acoustic phonons. The impurity scattering can be suppressed by dielectric engineering but the scattering by acoustic phonons is intrinsic to the semiconductor and cannot be eliminated. Thus the intrinsic mobility determined by acoustic phonon scattering alone provides an important upper limit for the achievable mobility of free electrons in a semiconductor. The acoustic mode lattice vibration induces changes in lattice spacing, which change the band gap from point to point. Since the crystal is "deformed" at these points, the potential associated is called the deformation potential. The free electrons in Si interact dominantly only with acoustic phonons through deformation potential at low lattice temperatures [4-9].

The energy distribution of the free electrons ensemble in semiconducting system is perturbed when their drift energy becomes comparable to their thermal energy because of the application of electric field E to the system. In principle, the high-field energy distribution can be derived from a direct solution of the Boltzmann transport equation taking the scattering mechanism into account. But the analytical solution of the transport equation under high-field condition is usually a formidable mathematical task and as such one has to take recourse of some simplifying assumptions which, more often than not, may compromise with the physical validity of the theoretical results. However, if the inter carrier collisions are frequent enough to randomize the energy and momentum of the electrons, the distribution function in the momentum space can be approximated by Maxwell-Boltzmann distribution at a field-dependent effective electron temperature T_e, which is always greater than the lattice temperature. The field-dependent electron temperature can be obtained from the solution of the energy balance equation of the electron-phonon system and the solution depends on the prevalent scattering rates of the free electrons [4,5,10]. Shinba et al [11] developed a comprehensive theory to study the hot electron effect in Si(100) inversion layer at low lattice temperatures. The hot electron mobility in Si inversion layer was studied by Hess et al [12] at higher temperatures. The purpose of the present article is to find the dependence of electron temperature and electron mobility upon electric field in 2DEG at low lattice temperatures using the effective electron temperature model and the result thus obtained is compared with that obtained by Shinba et al [11].

II. THEORETICAL MODEL

In the electron temperature model, the calculation of the electric field and electron temperature dependence of mobility of hot electrons with simplifying of a Maxwell-Boltzmann distribution with an electron temperature $T_e(E)$ is carried out by averaging momentum relaxation time over this distribution and calculating the mobility as a function of electron temperature, also calculating the relationship between the electric field E and electron temperature T_e, from momentum and energy balance equations. The power supplied to 2DEG can be used to obtain the electron temperature, drift velocity and mobility as a function of electric field. The dissipated power given to the electron system is transferred to the phonon system. When an electric field E is applied the electron gains energy at the rate of $e\mu E^2$, where e is the electronic charge and μ is the mobility of warm electrons. In steady state, if ϵ be the electron energy, the average rate of energy loss $\langle \frac{d\epsilon}{dt} \rangle$ of the non-equilibrium electron due to phonon scattering is equal to the rate of energy gain from the field and the equation is given as [4]

$$e\mu E^2 = \langle \frac{d\epsilon}{dt} \rangle. \quad (1)$$

At low temperature range, in Si(100) 2DEG, the main contributions to energy loss of electrons is assumed to be from acoustic phonon scattering due to deformation potential mechanism. We also assume that the distribution function of the central valley electrons to be a drifted Maxwell-Boltzmann distribution. Firstly, we further assume the expressions for mobility and electron temperature are of the same form as those for bulk semiconductor. In addition, the electron densities are assumed to be constant in the whole electric field range. We expect that the energy flow from the electronic system will be mainly through the acoustic phonon emission via deformation potential for $T_e < 40$ K temperature range. The power loss and mobility equations for deformation potential acoustic phonon scattering are considered here from earlier works [4,13,14]. In steady state, when all the acoustic modes are fully excited, the average energy loss per unit time of an electron to the crystal lattice due to acoustic phonon emission via deformation potential at electron temperature T_e for degenerate ensemble is given by,

$$\langle \frac{d\epsilon}{dt} \rangle_{dp} = \frac{2^{5/2}\mathcal{E}_a^2 m^{*5/2} k_B^{3/2}}{\pi^{3/2}\hbar^4 \rho_v} T_e^{3/2}\left(\frac{T_e}{T_L} - 1\right)\frac{F_{1/2}(\eta)}{F_1(\eta)} \quad (2)$$

Here $\mathcal{E}_a$ is the deformation potential constant, k_B the Boltzmann constant, $\hbar$ the Dirac constant, m^* the effective mass of the electron and ρ_v is the mass density. The reduced Fermi energy $\eta = \epsilon_F/k_B T_e$ and in 2DEG Fermi energy $\epsilon_F = \pi\hbar^2 N_i/n_v m_\parallel^*$, n_v is the number of equivalent valleys at the surface and $m_\parallel^*$ is the effective mass of the electron parallel to the interface. $F_y(\eta)$ is the Fermi integral given by

$$F_j(\eta) = \int_0^{\propto} \frac{x^j dx}{1 + \exp[x - \eta]}.$$

The hot electron mobility due to deformation potential acoustic phonon scattering in degenerate ensemble is

$$\mu_{dp}(T_e) = \frac{\pi e \hbar^4 \rho_v u_l^2}{\sqrt{2}\mathcal{E}_a^2 m^{*5/2} k_B^{3/2} T_L} T_e^{-1/2} \frac{F_1(\eta)}{F_{3/2}(\eta)}. \quad (3)$$

Here u_l is the acoustic velocity.

In a different approach, at low lattice temperature, the present author has calculated the average rate of electron energy loss per unit time in non-degenerate 2DEG due to interaction of electrons with deformation potential acoustic phonons. In the calculation the true phonon distribution given by the truncated Laurent's expression is used. The expression is given as [6]

$$N_q(x) = \sum_{m=0}^{\infty} \frac{B_m}{m!} x^{m-1} \quad ; \quad x \leq \bar{x},$$
$$\approx 0 \quad ; \quad x > \bar{x}, \qquad (4)$$

where $x = \hbar q u_l / k_B T_L$, q is the magnitude of phonon wave vector. $B_m{}'$s are Bernoulli numbers and $\bar{x} < 2\pi$. For the practical purpose $\bar{x}$ may be taken to be 3.5. Thus the average rate of electron energy loss per unit time in non-degenerate 2DEG due to interaction of electron with deformation potential acoustic phonon is obtained as [15]

$$\langle \tfrac{d\epsilon}{dt} \rangle_{dp} = \mathcal{B}_{dp} \left[\frac{\sqrt{\pi} T_n}{2p^{3/2}} + \frac{\sqrt{T_n}}{p^2} + \sum_{m=0}^{\infty} \frac{B_m}{m!} \frac{T_n^{m/2}}{p^{(m+3)/2}} \times \left\{ \Gamma\left(\frac{m+3}{2}\right) - \Gamma\left(\frac{m+3}{2}, \frac{p\bar{x}^2}{T_n}\right) \right\} \right] \qquad (5)$$

where $\mathcal{B}_{dp} = \left(\mathcal{E}_a^2 \varepsilon_s^{1/2} (k_B T_L)^{5/2} \right) / \left(8\sqrt{\pi} \hbar^3 d \rho_v u_l^4 \right)$, $p = k_B T_L / 16\varepsilon_s$, $T_n = T_e / T_L$ $\varepsilon_s = \frac{1}{2} m_\parallel^* u_l^2$. Here $\Gamma(a)$ and $\Gamma(a,b)$ are respectively Gamma function and incomplete Gamma function. The parameter d is the width of the layer of lattice atoms with which the electrons can interact.

The deformation potential acoustic phonon controlled hot electron mobility μ_{dp} in a 2DEG can be estimated as a function of electron temperature T_e using a simple model valid for Maxwell-Boltzmann distribution as [4]

$$\mu_{dp} = \frac{e}{m_\mu^*} \langle\langle \tau_{dp} \rangle\rangle = \frac{e}{m_\mu^*} \frac{\langle \epsilon \tau_{dp} \rangle}{\langle \epsilon \rangle}, \qquad (6)$$

where m_μ^* is the mobility effective mass of the electron and τ_{dp} is the momentum relaxation time due to free carrier scattering with the deformation acoustic phonons. The average value of a parameter A for a two-dimensional non-degenerate ensemble is given by

$$\langle A \rangle = \int_0^{\infty} A f_0(\epsilon) d\epsilon, \qquad (7)$$

where the hot electron distribution function $f_0(\epsilon)$ is given by the Maxwell-Boltzmann distribution at an effective electron temperature T_e as [4]

$$f_0(\epsilon) = \frac{N_i}{N_C^{2D}} \exp(-\epsilon / k_B T_e).$$

Here the effective density of state N_C^{2D} in 2DEG can be given as[16]

$$N_C^{2D} = \frac{m_\parallel^*}{\pi \hbar^2} k_B T_e .$$

Under the condition of quasi-elastic collisions and the phonon distribution given by Eq.(4), the present author reported the momentum relaxation time of non-degenerate free electrons due to electron-phonon scattering in a 2DEG as [17]

$$\frac{1}{\tau_{dp}(\epsilon)} = \mathcal{A}_{dp} \, \lambda \, \mathcal{F}(\epsilon). \qquad (8)$$

Where

$$\mathcal{A}_{dp} = \left(\frac{\mathcal{E}_a^2 m_\parallel^{*3/2}}{4\sqrt{2}\hbar^3} \right) \left(\frac{1}{\pi d \rho_v u_l} \right) \left(\frac{k_B T_L}{\sqrt{\varepsilon_s}} \right)^2, \quad \lambda = \frac{4\sqrt{\varepsilon_s}}{k_B T_L}.$$

The function $\mathcal{F}(\epsilon)$ is given as

$$\mathcal{F}(\epsilon) = \pi + \sum_{m=1}^{\infty} \frac{\pi B_{2m} (2m-1)!!}{(2m)! \, (2m)!!} \lambda^{2m} \epsilon^m = \mathcal{F}_1(\epsilon); \qquad \text{for} \quad \lambda\sqrt{\epsilon} \leq \bar{x},$$

and $$\mathcal{F}(\epsilon) = \sum_{m=0}^{\infty} \frac{2B_{2m}}{(2m)!} \mathcal{C}_{2m}(\bar{\theta}) \lambda^{2m} \epsilon^m + \lambda\sqrt{\epsilon - \epsilon_1} = \mathcal{F}_2(\epsilon) ; \qquad \text{for} \quad \lambda\sqrt{\epsilon} > \bar{x} . \qquad (9)$$

Here

$$\mathcal{C}_i(\theta) = -\frac{\sin^{i-1}\theta \cos\theta}{i} + \frac{i-1}{i} \mathcal{C}_{i-2}(\theta),$$

$$\mathcal{C}_0(\theta) = \theta, \quad \bar{\theta} = \sin^{-1}(\bar{x}/\lambda\sqrt{\epsilon}), \quad \epsilon_1 = (\bar{x}/\lambda)^2.$$

Using Eqs.(6), (7), (8) and (9) we may get the mobility of warm electrons in a 2DEG due to deformation potential acoustic phonon scattering in non-degenerate ensemble as

$$\mu_{dp}(T_e) = \frac{e}{m_\mu^* \mathcal{A}_{dp} \lambda (k_B T_e)^2} \left[\int_0^{\epsilon_1} \frac{\epsilon}{\mathcal{F}_1(\epsilon)} e^{-\epsilon/k_B T_e} d\epsilon + \int_{\epsilon_1}^{\infty} \frac{\epsilon}{\mathcal{F}_2(\epsilon)} e^{-\epsilon/k_B T_e} d\epsilon \right]. \qquad (10)$$

It is obvious that the integrations in Eq.(10) are not amenable to analytical evaluation and as such one has to take recourse to some numerical technique [18] for their evaluation.

When the equipartition law of phonon distribution is assumed, the average rate of electron energy loss in non-degenerate 2DEG due to interaction of electron with deformation potential acoustic phonon is obtained as [15]

$$\langle\frac{d\epsilon}{dt}\rangle_{dp} = \mathcal{B}_{dp}\left[\frac{\sqrt{\pi}}{2p^{3/2}}(1+T_n) + \frac{\sqrt{T_n}}{p^2}\right]. \qquad (11)$$

Under the condition of equipartition law the momentum relaxation time of non-degenerate free electrons due to electron-phonon scattering in a 2DEG can be given as [17]

$$\frac{1}{\tau_{dp}(\epsilon)} = \mathcal{A}_{dp}\,\lambda\,\pi. \qquad (12)$$

III. RESULTS AND DISCUSSION

To observe the field dependence of the hot electron temperature as well as the mobility characteristics as obtained by the above formulations, an n-channel (100) oriented Si inversion layer is considered with the material parameter values [11] : $\mathcal{E}_a = 12$ eV, $u_l = 9.04 \times 10^5$ cm s^{-1}, $\rho_v = 2.33$ gm cm^{-3}, $\epsilon_{sc} = 11.7$, effective mass of electron $m^* = 0.32m_0$, longitudinal effective mass $m_l^* = 0.916m_0$, transverse effective mass $m_t^* = 0.190m_0$, m_0 being the free electron mass. At low lattice temperatures one may consider presumably the electrons occupy only the lowest subband when the layer thickness d is given by $(\hbar^2\epsilon_{sc}/2m_\perp^* e^2 N_i)^{1/3}\gamma_0$. Here ϵ_{sc} is the permittivity of the semiconductor, $m_\perp^*$ is the effective mass perpendicular to the surface, and γ_0 is the zero-th root of the equation $A_i(-\gamma_n) = 0$; $A_i(-z)$ being Airy function. For the (100) surface of Si the six valleys are not equivalent. The two equivalent valleys for which $m_\parallel^* = m_t^*$, $m_\perp^* = m_l^*$ and $m_\mu^* = m_l^*$ occupy the lower subband than the other four equivalent valleys [3,8].

The electric field dependent electron temperature in Si(100) 2DEG can be evaluated by different formulations mentioned in the theoretical model. The results are plotted in Fig.1 and for a comparison the corresponding dependence as obtained by Shinba et al [11] which confirms fairly the experimental data is also plotted in the same figure. It is seen from the figure that the result obtained from the formulation of 3DEG (curve-1) considering the degeneracy of the electron ensemble agrees quite well both quantitative as well as qualitative aspects with the results of Shinba et al. But there is enough quantitative as well as qualitative discrepancy between the result of Shinba et al and the result obtained from the expressions of 2DEG theory considering the equipartition law of phonon distribution (curve-2). But when the non-equipartition of phonon distribution is introduced in the 2DEG theory, both quantitative and qualitative agreement (curve-3) is better than equipartition theory. It is obvious that the results plotted in Fig.1 for the carrier concentration and lattice temperature of interest, the carrier ensemble would definitely be degenerate in a 2DEG system and the theory developed by considering degenerate statistics must give the better agreement. The slight disagreement of the curve-1 in Fig.1 from the result of Shinba et al may be attributed to the consideration of equipartition law of phonon distribution which is introduced to derive the expressions for the result of curve-1.

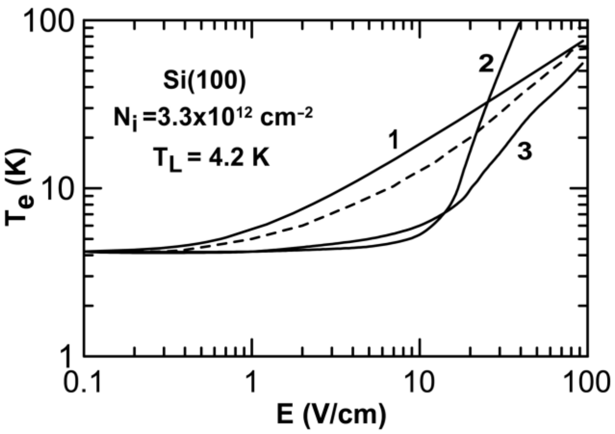

Figure 1 : Dependence of electron temperature upon electric field in Si(100) 2DEG system. Dashed curve is the result from Shinba et al [11]. The curve marked 1 is the result obtained from the theory of 3DEG. Curve marked 2 and 3 are respectively, the results of the theory of 2DEG due to equipartition law of phonon distribution and true phonon distribution.

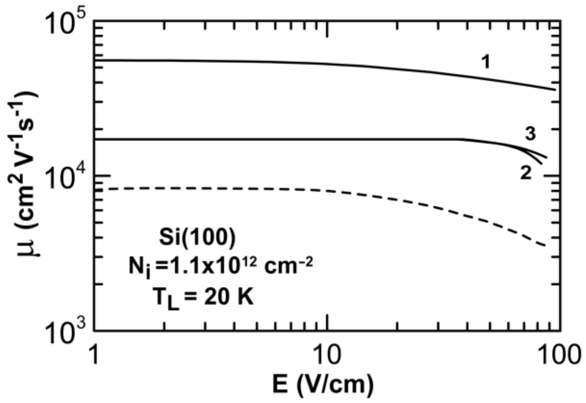

Figure 2: Dependence of electron mobility upon electric field in Si(100) 2DEG system. Dashed curve is the result of Shinba et al [11]. The curve marked 1 is the result obtained from the theory of 3DEG. Curve marked 2 and 3 are respectively, the results of the theory of 2DEG due to equipartition law of phonon distribution and true phonon distribution.

In Fig.2 the electric field dependence of the mobility characteristics of the electrons in Si(100) 2DEG are shown. In the figure it is observed that the qualitative agreement between the results of Shinba et al and the results from different theories mentioned in the theoretical model is very good. The results obtained from 2DEG theory show better quantitative agreement with the result of Shinba et al than the results obtained in the light of 3DEG theory.

IV. CONCLUSION

The figures reveal that there is disagreement particularly in quantitative aspect between the results obtained from the present formulations and the result obtained by the comprehensive theory of Shinba et al. Much of the disagreement may be ascribed to a lack of our knowledge of the sensitive material parameters like acoustic constant about which there has been a long standing controversy in literature [10]. This apart, at low lattice temperature and carrier concentration of interest here, the effect of screening of the electron ensemble and degeneracy of electron system must be considered in developing the theory of electron transport in 2DEG. Moreover the effect of finite phonon energy in the energy balance equation has to be taken into account in the process of electron-phonon collision. Besides these, the perturbation of the phonon system should also be given due consideration under the prevalent condition of low lattice temperatures [4]. Thus the remaining disagreement is likely to be reduced if we consider these factors.

V. REFERENCES

[1] H. L. Störmer, "Electron mobilities in modulation-doped GaAs-(AlGa)As heterostructure, Surf. Sci., 132(1-3), pp.519-526, 1983.

[2] F. Stern and W. E. Howard, "Properties of semiconductor surface inversion layers in the electric quantum limit", Phys. Rev., 163(3), pp. 816-835, 1967.

[3] T. Ando, A. B. Fowler and F. Stern, "Electronic properties of two-dimensional systems", Rev. Mod. Phys., 54(2), pp. 437-672, 1982.

[4] E. M. Conwell, High Field Transport in Semiconductors, Academic Press, New York, 1967.

[5] L. Reggiani, Hot-Electron Transport in Semiconductors, Springer-Verlag, Berlin, 1985.

[6] C. Jacoboni and L. Reggiani, "The Monte Carlo method for the solution of charge transport in semiconductors with applications to covalent materials", Rev. Mod. Phys., 55(3), pp. 645-705, 1983.

[7] A. K. Ghorai and D. P. Bhattacharya, "Effect of finite-energy acoustic phonons on the zero-field mobility characteristics of high-purity semiconductors at low lattice temperatures", Phys. Rev. B, 47(20), pp.13858-13860, 1993.

[8] A. K. Ghorai and D. P. Bhattacharya, "Lattice-controlled electron transport characteristics in quantized surface layers at low temperature", Surf. Sci., 380(2-3), pp. 293-301, 1997.

[9] A. K. Ghorai, "Phonon Controlled Temperature Dependence of Electron Mobility in 2DEG of GaAs Surface Layer", IJSRST, 3(6), pp. 281-287, 2017.

[10] B. R. Nag, Theory of Electrical Transport in Semiconductors, Pergamon Press, Oxford, 1972.

[11] Y. Shinba, K. Nakamura, M. Kukuchi and M. Sakata, "Hot Electron in Si(100) Inversion Layer at Low Lattice Temperatures", J. Phys. Soc. Jpn., 51(1), pp.157-163, 1982.

[12] K. Hess and C. T. Sah, "Hot Carriers in Silicon Surface Inversion Layers", J. Appl. Phys., 45(3), pp.1254-1257, 1974.

[13] K. Segeer, Semiconductor Physics: an introduction, 7.ed. Springer-Verlag, Berlin, 1999.

[14] H. P. Lee, D. Vakhshoori, Y. H. Lo, S. Wang, "Models for electron mobility and temperature of two-dimensional electron gas at low and moderate fields", J. Appl. Phys. 57(10), pp. 4814, 1985.

[15] A. K. Ghorai. "Non-Equilibrium Two Dimensional Electron-Lattice Interaction at Low Temperature", Int. J. Sci. Res. (IJSR), 5(11), pp.80-83, 1916.

[16] E. F. Schubert, Doping in III-V Semiconductors, Cambridge University Press, 2005.

[17] A. K. Ghorai, "Effect of Screening on Acoustic Phonon Limited Electron Mobility in Quantized Surface Layer in Semiconductor", Int. J. Sci., Eng. Tech. Res. (IJSETR), 2(7), pp. 1453-1457, 2013.

[18] S. Wolfram, Mathematica, Addison-Wesley Press, Redwood City, 1988.