Numerical Investigations on Vortex Generated Oscillations through a Pair of Cylinders alongside Each Other

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ABSTRACT

The current investigation is concentrated on vortex generated oscillations and vortex shedding owing to elastically fixed pair of cylinders. Several non-dimensional key process parameters such as mass ratio, Reynolds number, transverse gap ratio, frequency ratio and reduced velocity, relating to the present problem are identified and their relative importance are also observed. The investigations are carried out at several frequency ratios and reduced velocities by considering the flow to be laminar and incompressible. The numerical model involves governing transport equations of continuity and momentum in two dimensions. The model uses finite volume method (FVM) with pressure based SIMPLE algorithm to carry out numerical simulation for getting the related flow fields. The oscillations created by the interacting vortices through a pair of cylinders are fully investigated in terms of cylinder trajectory and vibration frequency/amplitude.

Keywords: Cylinder, Reduced velocity, Frequency ratio, Trajectory, Deflection.

I. INTRODUCTION

Flow past cylinder is a very general research problem on account of not just minimalism but for voluminous technological usages and practices. The aerodynamic effects thus created, reasons for vibration in the cylinder (along both in-line and transverse directions), caused by vortex shedding and elastic excitation. The vortex shedding is related to the boundary separation phenomenon. When both vortex shedding frequency and the natural frequency of the vibrating cylinder are equal, resonance occurs.

Investigators are in exploration of novel materials and products having numerous, incomparable and unmatched properties such as light weighting, high strength-to-weight ratio, resistance to corrosion along with wear and tear, superb manufacturability and trouble-free recyclability, presently which can be used almost everywhere starting from industrial to household purposes. Definitely, the talked about materials/products will be highly susceptible to vibrations because of the light weighting. The flow induced vibration is one such cutting edge field upon which currently ever more focus is concentrated by the investigators.

The particulars about the flow over bluff bodies have been explained by Schlichting [1]. Tests on flow over cylinder at elevated Reynolds number is also demonstrated in literature by Roshko [2]. Numerical analysis of fluid flow problems is described in texts by Ralston and Rabinowitz [3]. Additionally, the numerical fluid flow is also exhibited by Patankar [4]. The particulars about the hydrodynamic stability is analyzed by Chandrasekhar [5]. The influences of surface roughness on flow past cylinder is also studied by Nakamura and Tomonari [6]. Fundamentals about boundary layer in fluid flow is presented by Schetz [7]. Physical and computational traits of fluid flow is exemplified by Cebeci and Bradshaw [8]. Basics of fluid mechanics is reported by Landau and Lifshitz [9]. In addition, the effect of boundary layer in fluid flow is specified by Young [10]. The intricacies about viscous flow is portrayed by White [11]. Simulation scheme for flow around circular cylinder is also stated by Kondo [12]. Iterative methods to solve linear systems of equations are depicted by Saad [13].
Stability analysis due to wake around cylinder is investigated by Barkley and Henderson [14]. Numerical solutions of linear systems of equations are also given away by Trefethen and Bau [15]. The rudiments of CFD has also been explained by Versteeg and Malalasekera [16]. The basics of fluid mechanics is also elucidated by Kundu and Cohen [17]. Fluid flow over porous cylinder with incessant suction/blowing is expounded by Fransson et al. [18]. Effects of traveling wave on von Karman vortex street about a cylinder is explicated by Wu et al. [19]. Influence of synthetic jet on wake around cylinder is detailed by Feng and Wang [20]. The interference of synthetic jet with wake vortex shedding near a cylinder besides its decomposition analysis is further expanded by Feng et al. [21, 22]. In addition, synthetic jet usage on separation control near cylinder is also extended by Feng and Wang [23]. The suction flow practice for diminishing vortex induced vibration of cylinder is also added by Chen et al. [24]. Also, the suction flow technique on unsteadiness reduction around the cylinder is further investigated by Chen et al. [25]. From the abovementioned researches, to the best of author’s facts and figures, it is apparent that there is no such comprehensive computational analysis relating to the influences of reduced velocity and frequency ratio on cylinder vibration in terms trajectory of cylinder centre together with the amplitude, frequency and deflection of cylinder. With this standpoint, the present research aims primarily at the computational investigations of the above stated intents (apart from the fluid flow behavior) around a pair of cylinders in abreast position. The current numerical model practices finite volume method (FVM) with pressure based SIMPLE algorithm to carry out simulation for getting the desired flow fields. Lastly, the computational results are analyzed and compared for realizing the expected physical feel and sagacity. The predictions of the model concerning various non-dimensional key process parameters are along expected lines. Furthermore, for comparison of results, a pilot-scale experimental arrangement is also planned for the future.

II. DESCRIPTION OF PHYSICAL PROBLEM

The physical problem includes the fluid flow with velocity \( U \) over a pair of elastically fixed cylinders of diameter \( D \) each. The fluid flow is assumed to be incompressible and the problem concerns with two degrees of freedom (relating to oscillations along both \( x \) and \( y \) directions). A two dimensional rectangular computational domain is chosen in order to save simulation time. Transverse gap ratio (\( T/D \)) chosen to be 3. The particulars about the physical problem with computational domain are demonstrated in Figure 1 as well.

![Figure 1. Physical problem with computational domain](image)

III. MATHEMATICAL FORMULATION

A. Governing equations

The dimensionless form of the governing transport equations of continuity and momentum relating to the current physical problem are as described underneath.

\[
\nabla' \cdot \mathbf{u}' = 0 \tag{1}
\]

\[
\frac{\partial \mathbf{u}'}{\partial t'} + \nabla' (\mathbf{u}' \cdot \mathbf{u}') = -\nabla' p' + \frac{1}{Re} \nabla'^2 \mathbf{u}' \tag{2}
\]

Where, \( Re \) (i.e. \( \rho UD/\mu \)) is the Reynolds number, \( \mu \) is the dynamic viscosity, \( D \) is the reference length, \( U \) is the reference speed, \( u \) is the velocity field, \( t \) is the time, \( p \) is the modified static pressure and \( \rho \) is the density.

The current studies deliberate elastically fixed rigid cylinders which oscillate along both \( x \) and \( y \) directions. The oscillation is represented by an equivalent spring mass with damper system under fluid force. Non-dimensional form of equation relating to the specified equivalent system is as mentioned below.

\[
M' \ddot{x}_c' + C' \dot{x}_c' + K' x_c' = F_x' (\ddot{x}_c', \dot{x}_c', x_c') \tag{3}
\]

\[
M' \ddot{y}_c' + C' \dot{y}_c' + K' y_c' = F_y' (\ddot{y}_c', \dot{y}_c', y_c') \tag{4}
\]

Where, \( M' = \text{dimensionless mass} \), \( C' = \text{dimensionless damping coefficient (zero for the present instance)} \), \( K' = \text{ }}
dimensionless stiffness coefficient, \( F^* = \text{dimensionless fluid force} \).

Furthermore,
\[
M^* = \frac{M}{\rho D^2 L}, K_x^* = \frac{K_x}{\rho U^2 L}, F_x^* = \frac{F_x}{\rho U^2 D L},
K_y^* = \frac{K_y}{\rho U^2 L}, F_y^* = \frac{F_y}{\rho U^2 D L}
\]

Where, \( L = \text{axial dimension of cylinder, } C_L = \text{lift coefficient, the variables } x_c^*, y_c^*, \dot{x}_c^*, \dot{y}_c^*, \ddot{x}_c^* \text{ and } \ddot{y}_c^* \text{ are dimensionless displacement, velocity and acceleration along x and y directions respectively, } t^* = \text{dimensionless time (} = tU/D) \).

\[
\begin{align*}
\dot{x}_c^* &= \frac{x_c D}{U^2}, \quad \dot{y}_c^* = \frac{y_c D}{U^2}, \quad x_c^* = \frac{x_c D}{U^2}, \\
\dot{y}_c^* &= \frac{y_c D}{U^2}, \quad \ddot{x}_c^* = \frac{\ddot{y}_c D}{U^2}, \quad \ddot{y}_c^* = \frac{\ddot{y}_c D}{U^2}
\end{align*}
\]

The dimensionless form is considered for enabling flow consistency and easy coupling. The results of the current study are also expressed in terms of mass ratio \( m^* \) and reduced velocity \( U^* \).

\[
m^* = 4M/\rho \pi D^2 L, U^* = U/f_{ny} D
\]

Where, \( f_{ny} = \text{natural frequency of the system in vacuum and } f_{ny} = \sqrt{R/M} / 2\pi \).

**B. Boundary conditions**

The specified velocity boundary condition is used at the inlet. However, the specified pressure boundary condition is considered at the exit. The lateral boundaries are introduced with symmetry (i.e. far field) boundary conditions (i.e. both x-velocity gradient and y-velocity are taken as zero at lateral boundaries). And also, the no-slip boundary condition is considered at the surface of the cylinders.

**C. Parameters**

In the current studies, the numerical simulations are carried out for fluid flow over a pair of freely oscillating cylinder. The fluid considered is air with invariable density and viscosity of 1 kg.m\(^{-3}\) and 0.001 kg.m\(^{-1}\).s\(^{-1}\), respectively. The diameter of each cylinder is taken as 0.01 m. The flow over the pair of cylinders is set at inlet velocity of 1.6 m/s corresponding to Reynolds number of 160. In other words, the flow is considered to be laminar. The cylinders are unrestricted to vibrate along transverse and in-line directions. The mass ratio is taken to be three. The damping coefficient is set as zero for attaining high amplitudes due to vibrations. The reduced velocity ranges between 4 and 10. Three different frequency ratios of 1.0, 1.5 and 2.0 are taken into account.

**IV. NUMERICAL METHOD**

The present numerical model uses finite volume method (FVM) with pressure based SIMPLE algorithm to carry out simulation for getting the desired flow fields. Second order upwind with first order implicit scheme is applied for the present physical problem formulation. The grid independence test reveals the optimum grid size that corresponds to grids with 160 nodes on the circumference of each cylinder. Likewise, the time independence test also ensures solution stability with a sound simulation time. The meshing inside the computational domain along with the enlarged view of meshing near cylinder surface is illustrated in Fig. 2.

**Figure 2. (a) Computational domain with meshing (b) Enlarged view of meshing near cylinder surface**
V. RESULTS AND DISCUSSION

Numerical simulations are performed (for flow around a pair of cylinders alongside each other), to study the influences of reduced velocity and frequency ratio on cylinder vibration in terms trajectory of cylinder centre in conjunction with the amplitude, frequency and deflection of cylinder.

The trajectory of (i.e. the path followed by) the cylinder centre at different reduced velocities is as depicted in Figure 3. It is observed that the trajectory of the cylinder centre follows the illustrious Lissajous figure at low reduced velocity. And also, the amplitude of vibration is high at low reduced velocity. Furthermore, the amplitude of vibration decreases with the increase in reduced velocity. In addition, with the increase in reduced velocity, the regular Lissajous figure gradually becomes irregular.

Figure 3. Trajectory of cylinder centre at different reduced velocity

Figure 4 unveils the frequency ratio (i.e. the ratio of the vibrating frequency to the natural frequency of the oscillating cylinder) vs. the reduced velocity curves. It is observed that the vibrating frequency is always lower than that of natural one. Furthermore, the frequency ratio increases with the reduced velocity.

Figure 4. Frequency ratio vs. reduced velocity

Figure 5 demonstrates the ratio of vibrating frequency of oscillating cylinder to the shedding frequency of stationary cylinder vs. the reduced velocity curves at different frequency ratios. It is observed that the shedding frequency of the rigidly fixed cylinder is always more than the vibration frequency of the oscillating cylinder for nearly every case with an exception at the reduced velocity of 4 for frequency ratios of 1 and 1.5. In addition, both the curves intended for frequency ratios of 1 and 1.5 overlap for the reduced velocity between 4 and 7. On the other hand, at the higher reduced velocity values (≥ 8), the curves for frequency ratios 1 and 2 coincide to a certain extent.

Figure 5. Ratio of vibrating frequency of oscillating cylinder to shedding frequency of stationary cylinder vs. reduced velocity

Figure 5 illustrates the ratio of cylinder deflection in stream direction to cylinder diameter vs. the reduced velocity at different frequency ratios. It is observed that the deflection (in the stream direction) of each cylinder from the mean position increases with the reduced velocity. However, at a particular velocity ratio, the said deflection decreases with increase in frequency ratio. In addition, the talked about deflection gets increased by an order for increase in reduced velocity from 4 to 10.

Figure 6 illustrates the ratio of cylinder deflection in stream direction to cylinder diameter vs. the reduced velocity at different frequency ratios. It is observed that the deflection (in the stream direction) of each cylinder from the mean position increases with the reduced velocity. However, at a particular velocity ratio, the said deflection decreases with increase in frequency ratio. In addition, the talked about deflection gets increased by an order for increase in reduced velocity from 4 to 10.

Figure 6. Ratio of cylinder deflection in stream direction to cylinder diameter vs. reduced velocity

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Numerical simulations are performed for flow past a pair of cylinders in alongside each other, to study the influences of reduced velocity and frequency ratio on cylinder vibration in terms trajectory of cylinder centre together with the amplitude, frequency and deflection of cylinder. The present numerical model uses finite volume method (FVM) with pressure based SIMPLE algorithm to carry out simulation for getting the desired flow fields. Second order upwind with first order implicit scheme is applied for the present physical flow fields. The grid independence test reveals second order upwind scheme is used for the present physical flow fields. It also ensures solution stability with a sound simulation time. The predictions of the model concerning several non-dimensional key process parameters are in line with the expectations. The comparison with an in-house experimental setup is planned for the future. With the specified model conditions, it is observed that the reduced velocity of 7 along with the frequency ratio of 1.5 renders pragmatically complete results and is the reasonable one.

VI. CONCLUSION

VII. REFERENCES