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Unsteady-State Thermo Elastic Problem in a Thick Circular Plate Due to Heat Source with third Kind Boundary Conditions

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ABSTRACT

In this paper, by applying Marchi-Fasulo and Hankel integral transforms technique, to study the non-homogeneous unsteady-state boundary value problem of heat conduction in circular plate to determine the temperature, displacement and radial and angular thermal stresses.
Keywords: Heat generation, Thick circular plate, Marchi -Fasulo and Hankel Transforms.

I. INTRODUCTION

The high velocities of modern aircraft give rise to aerodynamics heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. Kulkarni V.S. and Deshmukh K.C. [1] have been determined the quasi-static thermal stresses in a thick circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. Nowacki [4] has determined steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Rajneesh Kumar et al. [5] have discussed thick annular disc in which sources are generated according to linear function of temperature due to partial heating and boundary condition of radiation type.

II. FORMULATION OF THE PROBLEM

Consider a thick circular plate under the unsteady-state temperature field of thickness 2h occupying the region D defined as $0 \le r \le a,\ -h \le z \le h$, due to heat generation. Let (r, φ, z) be cylindrical co-ordinate system and θ be temperature function of space and time the transient heat conduction equation is given as

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial z^2} + \frac{\xi(r,z,t,\theta)}{\lambda} = \frac{1}{K}\frac{\partial T}{\partial t} \qquad (2.1)$$

Where $\xi(r,z,t,\theta)$ is the internal heat source function, and the $K = \frac{\lambda}{\rho c}$, λ being the thermal conductivity of the material, ρ is density and c is the capacity.

Following [3] use the substitution

$$\xi(r,z,t,\theta) = \Phi(r,z,t) + \psi(t)\theta(r,z,t) \qquad (2.2)$$

$$T(r,z,t) = \theta(r,z,t)e^{-\int_0^t \psi(y)dy} \qquad (2.3)$$

$$\chi(r,z,t) = \Phi(r,z,t)e^{-\int_0^t \psi(y)dy} \qquad (2.4)$$

For the sake of brevity, we consider

$$\chi(r,z,t) = \frac{\delta(r-r_0)\delta(z-z_0)}{2\pi r_0}\exp(-\omega t),$$
$$0 \le r_0 \le a, -h \le z \le h \qquad (2.5)$$

Substitute the equation (2.2) to (2.5) in (2.1), we obtain

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\chi(r,z,t)}{\lambda} = \frac{1}{K}\frac{\partial T}{\partial t} \qquad (2.6)$$

where K is the thermal diffusivity of the material of the thick circular plate subjected to the initial condition and boundary conditions

$$T(r,z,t) = T_0 \ \text{at}\ t = 0 \qquad (2.7)$$

$$\left[T(r,z,t) + k_1\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = 0, -h \le z \le h, t > 0 \qquad (2.8)$$

$$\left[T(r,z,t) + k_2\frac{\partial T(r,z,t)}{\partial z}\right]_{z=-h} = f_1(r,t), 0 \le r \le a \qquad (2.9)$$

$$\left[T(x,y,z,t) + k_3\frac{\partial T(r,z,t)}{\partial z}\right]_{z=h} = f_2(r,t), 0 \le r \le a \qquad (2.10)$$

The displacement function in the cylindrical co-ordinate system are represented by the Goodier's thermoelastic displacement potential and Love's function as [5]

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial^2\phi}{\partial r\,\partial z} \qquad (2.11)$$

$$u_z = \frac{\partial\phi}{\partial z} + 2(1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}, \qquad (2.12)$$

In which Goodier's thermoelastic potential must satisfy the equation

$$\nabla^2\phi = \left(\frac{1+\nu}{1-\nu}\right)a_t\theta \qquad (2.13)$$

$$\nabla^2(\nabla^2 L) = 0 \qquad (2.14)$$

Where $\nabla^2 = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}$

The component of the stresses are represented by the use of the potential ϕ and Love's L function as

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left(\upsilon\nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \qquad (2.15)$$

$$\sigma_{\theta\theta}= 2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\upsilon\nabla^2 L-\frac{1}{r}\frac{\partial L}{\partial r}\right)\right\} \quad (2.16)$$

$$\sigma_{zz}= 2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\upsilon\nabla^2 L-\frac{1}{r}\frac{\partial L}{\partial r}\right)\right\} \quad (2.17)$$

$$\sigma_{rz}= 2G\left\{\frac{\partial^2\phi}{\partial r\,\partial z}+\frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 L-\frac{\partial^2 L}{\partial z^2}\right)\right\} \quad (2.18)$$

Where G and υ are the shear modulus and Poisson's ratio respectively, the boundary condition on the traction free surface functions are

$$\sigma_{rz} = \sigma_{zz} = 0 \ \text{ at } z = \pm h \quad (2.19)$$

Equation (2.1) to (2.19) constitutes the mathematical formation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite Hankel integral transform to (2.6), we obtain

$$\left[-\mu_m{}^2\bar{T}(m,z,t)+\frac{d^2\bar{T}(m,z,t)}{dz^2}\right]+\frac{\bar{\chi}(m,z,t)}{\lambda}=\frac{1}{K}\frac{d\bar{T}(m,z,t)}{dz} \quad (3.1)$$

$$\bar{T}=\bar{T}_0, \quad (3.2)$$

$$\left[\bar{T}(r,z,t)+k_2\frac{d\bar{T}(r,z,t)}{dz}\right]_{z=-h} = \bar{f_1}(r,t) \quad (3.3)$$

$$\left[\bar{T}(x,y,z,t)+k_3\frac{d\bar{T}(r,z,t)}{dz}\right]_{z=h} = \bar{f_2}(r,t) \quad (3.4)$$

$$\bar{\chi}(m,z,t)=\frac{\sqrt{2}}{a}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J_0(\mu_m r_0)}{J_0(\mu_m a)}\delta(z-z_0)\exp(-\omega t) \quad (3.5)$$

Where $\bar{T}$ denotes the Hankel integral transform of T and m is Hankel integral transform parameter.

Applying Marchi-Fasulo integral transform to (3.1) we obtain

$$\left[-\mu_m{}^2\bar{T}^*(m,n,t)-a_n{}^2\bar{T}^*(m,n,t)+\Phi\right]+\frac{\bar{\chi}^*(m,n,t)}{\lambda}=\frac{1}{K}\frac{d\bar{T}^*(m,n,t)}{dz} \quad (3.6)$$

Where $\Phi = P_n(h)\bar{f_2}-P_n(-h)\,\bar{f_1}$

$$\frac{d\bar{T}^*(m,n,t)}{dz}+K(\mu_m{}^2+a_n{}^2)\bar{T}^*(m,n,t)=H(\mu_m,a_n) \quad (3.7)$$

Where $H(\mu_m,a_n)=\frac{\sqrt{2}}{\lambda a}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J_0(\mu_m r_0)\exp(-\omega t)}{J_0(\mu_m a)}\{P_n(z_0)+\Phi\}$

The equation (3.7) has solution

$$\bar{T}^*(m,n,t)e^{K(\mu_m{}^2+a_n{}^2)t}=\frac{H(\mu_m,a_n)}{K(\mu_m{}^2+a_n{}^2-\omega)}e^{-K(\mu_m{}^2+a_n{}^2)t}+C \quad (3.8)$$

At initially t=0, $C=\bar{T}^*{}_0-\frac{H(\mu_m,a_n)}{K(\mu_m{}^2+a_n{}^2-\omega)}$ $\bar{T}^*(m,n,t) = \frac{H(\mu_m,a_n)}{K(\mu_m{}^2+a_n{}^2-\omega)}\exp(-\omega t)$

$$+\left[\bar{T}^*{}_0-\frac{H(\mu_m,a_n)}{K(\mu_m{}^2+a_n{}^2-\omega)}\right]e^{-K(\mu_m{}^2+a_n{}^2)t} \quad (3.9)$$

Where $\bar{T}^*$ denotes the Marchi Fasulo integral transform of $\bar{T}$ and n is Marchi Fasulo integral transform parameter.

Applying inverse of Hankel and Marchi-Fasulo integral transform to (3.9), we obtain

$$T(r,z,t)=\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J_0(\mu_m r)}{J_0(\mu_m a)}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}$$
$$\times\left\{\Lambda_{m,n}\exp(-\omega t)+\left[\bar{T}^*{}_0-\Lambda_{m,n}\right]e^{-K(\mu_m{}^2+a_n{}^2)t}\right\} \quad (3.10)$$

Where $\Lambda_{m,n}=\frac{H(\mu_m,a_n)}{K(\mu_m{}^2+a_n{}^2-\omega)}$

Substitute the value of $T(r,z,t)$ in (2.3), the temperature distribution is obtain

$$\theta(r,z,t)=\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J_0(\mu_m r)}{J_0(\mu_m a)}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left\{\begin{matrix}\Lambda_{m,n}\exp(-\omega t)\\+\left[\bar{T}^*{}_0-\Lambda_{m,n}\right]e^{-K(\mu_m{}^2+a_n{}^2)t}\end{matrix}\right\}e^{\int_0^t\psi(y)dy} \quad (3.11)$$

DETERMINATION OF DISPLACEMENT

Substitute the value of $\theta(r,z,t)$ from (3.11) in (2.13), we obtain

$$\phi(r,z,t)=\left(\frac{1+v}{1-v}\right)a_t\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J_0(\mu_m r)}{J_0(\mu_m a)}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\times\left\{\begin{matrix}\Lambda_{m,n}\exp(-\omega t)+\\\left[\bar{T}^*{}_0-\Lambda_{m,n}\right]e^{-K(\mu_m{}^2+a_n{}^2)t}\end{matrix}\right\}e^{\int_0^t\psi(y)dy} \quad (3.12)$$

Similarly, the solution for Love's function L are assumed so as to satisfy the governed condition of equation (2.14) as

$$L(r,z,t)=\left(\frac{1+v}{1-v}\right)a_t\frac{\sqrt{2}}{a}\times\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J_0(\mu_m r)}{J_0(\mu_m a)}\sum_{n=1}^{\infty}\left[\begin{matrix}\sinh(a_n z)+\\ z\mathrm{cozh}(a_n z)\end{matrix}\right]\times\left\{\begin{matrix}\Lambda_{m,n}\exp(-\omega t)+\\\left[\bar{T}^*{}_0-\Lambda_{m,n}\right]e^{-K(\mu_m{}^2+a_n{}^2)t}\end{matrix}\right\}e^{\int_0^t\psi(y)dy} \quad (3.13)$$

Substitute the value of ϕ and L in (2.11), (2.12) respectively.

$$u_r=\left(\frac{1+v}{1-v}\right)a_t\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J'_0(\mu_m r)}{J_0(\mu_m a)}\times\sum_{n=1}^{\infty}\left[\begin{matrix}P_n(z)-(a_n+1)\mathrm{cozh}(a_n z)-\\ a_n z\sinh(a_n z)\end{matrix}\right]\times\left\{\begin{matrix}\Lambda_{m,n}\exp(-\omega t)+\\\left[\bar{T}^*{}_0-\Lambda_{m,n}\right]e^{-K(\mu_m{}^2+a_n{}^2)t}\end{matrix}\right\}e^{\int_0^t\psi(y)dy} \quad (3.14)$$

$$u_z=\left(\frac{1+v}{1-v}\right)a_t\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{J_0(\mu_m r)}{J_0(\mu_m a)}\times\sum_{n=1}^{\infty}[-a_n(Q_n\sin(a_n z)+W_n\mathrm{Cos}(a_n z))-4va_n\sin(a_n z)-a_n{}^2(\sinh(a_n z)+z\mathrm{cozh}(a_n z))]$$

$$\times\left\{\begin{matrix}\Lambda_{m,n}\exp(-\omega t)+\\ [\overline{T}^{*}{}_{0}-\Lambda_{m,n}]e^{-K(\mu_m{}^2+a_n{}^2)t}\end{matrix}\right\}\ e^{\int_0^t\psi(y)dy} \tag{3.15}$$

DETERMINATION OF STRESS FUNCTIONS

The stress functions are obtained as

$$\sigma_{rr}=2G\left(\frac{1+\nu}{1-\nu}\right)a_t\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{1}{J_0(\mu_m a)}\times\left\{\Lambda_{m,n}\exp(-\omega t)+[\overline{T}^{*}{}_{0}-\Lambda_{m,n}]e^{-K(\mu_m{}^2+a_n{}^2)t}\right\}$$

$$\sum_{n=1}^{\infty}\left[-\frac{P_n(z)}{\lambda_n}\left[\frac{J''_0(\mu_m r)}{\mu_m{}^2}+J_0(\mu_m r)\right]-\frac{1}{\mu_m{}^2}[2\upsilon a_n{}^2\operatorname{cozh}(a_n z)J_0(\mu_m r)-[(a_n+1)\operatorname{cozh}(a_n z)+a_n z\sinh(a_n z)]]J''_0(\mu_m r)]\right]\ e^{\int_0^t\psi(y)dy} \tag{3.16}$$

$$\sigma_{\theta\theta}=2G\left(\frac{1+\nu}{1-\nu}\right)a_t\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{1}{J_0(\mu_m a)}\times\left\{\Lambda_{m,n}\exp(-\omega t)+[\overline{T}^{*}{}_{0}-\Lambda_{m,n}]e^{-K(\mu_m{}^2+a_n{}^2)t}\right\}\times$$

$$\sum_{n=1}^{\infty}\left[-\frac{P_n(z)}{\lambda_n}\left[r\frac{J'_0(\mu_m r)}{\mu_m{}^2}+J_0(\mu_m r)\right]-\frac{1}{\mu_m{}^2}[2\upsilon a_n{}^2\operatorname{cozh}(a_n z)J_0(\mu_m r)-[(a_n+1)\operatorname{cozh}(a_n z)+a_n z\sinh(a_n z)]r^{-1}J'_0(\mu_m r)]\right]\ e^{\int_0^t\psi(y)dy} \tag{3.17}$$

$$\sigma_{zz}=2G\left(\frac{1+\nu}{1-\nu}\right)a_t\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{1}{J_0(\mu_m a)}\times\left\{\Lambda_{m,n}\exp(-\omega t)+[\overline{T}^{*}{}_{0}-\Lambda_{m,n}]e^{-K(\mu_m{}^2+a_n{}^2)t}\right\}\times$$

$$\sum_{n=1}^{\infty}\left[-\frac{P_n(z)}{\lambda_n}\left[\frac{J''_0(\mu_m r)}{\mu_m{}^2}+J_0(\mu_m r)\right]+\frac{1}{\mu_m{}^2}[(2\upsilon+a_n)a_n{}^2\operatorname{cozh}(a_n z)a_n{}^3z\sinh(a_n z)]]J_0(\mu_m r)\right]\ e^{\int_0^t\psi(y)dy} \tag{3.18}$$

$$\sigma_{rz}=2G\left(\frac{1+\nu}{1-\nu}\right)a_t\frac{\sqrt{2}}{a}\sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{1}{J_0(\mu_m a)}\times\left\{\Lambda_{m,n}\exp(-\omega t)+[\overline{T}^{*}{}_{0}-\Lambda_{m,n}]e^{-K(\mu_m{}^2+a_n{}^2)t}\right\}\times$$

$$\sum_{N=1}^{\infty}[a_n(Q_n\sin(a_n z)+W_n\operatorname{Cos}(a_n z))J_0(\mu_m r)+[2\upsilon a_n\sinh(a_n z)+a_n{}^2[\sinh(a_n z)+z\operatorname{cozh}(a_n z)]]J'_0(\mu_m r)]]\ e^{\int_0^t\psi(y)dy} \tag{3.19}$$

SPECIAL CASE

Setting,

$$\psi(y)=-y,\ T_{0=0},\ \Phi=0,\ \chi(r,z,t)=\frac{\delta(r-r_0)\delta(z-z_0)}{2\pi r_0}\exp(-\omega t)$$

$$\text{And }\int_0^t\psi(y)dy=\frac{-t^2}{2},\ \overline{T}^{*}{}_{0}=0 \tag{3.20}$$

Substitute the values we obtain, the expression for the temperature and stresses,

$$\theta(r,z,t)=\sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z)P_n(z_0)}{\lambda_n}\times\sum_{m=1}^{\infty}\frac{k_1{}^2}{K(\mu_m{}^2+a_n{}^2-\omega)(k_1{}^2+\mu_m{}^2)}\frac{J_0(\mu_m r)J_0(\mu_m r_0)}{J_1{}^2(\mu_m a)}\times\left(e^{-\omega t}-e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \tag{3.21}$$

$$\phi=\left(\frac{1+\nu}{1-\nu}\right)a_t\sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z)P_n(z_0)}{\lambda_n}\times\sum_{m=1}^{\infty}\frac{k_1{}^2}{-\mu_m{}^2K(\mu_m{}^2+a_n{}^2-\omega)(k_1{}^2+\mu_m{}^2)}\frac{J_0(\mu_m r)J_0(\mu_m r_0)}{J_1{}^2(\mu_m a)}\times\left(e^{-\omega t}-e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \tag{3.22}$$

$$L=\left(\frac{1+\nu}{1-\nu}\right)a_t\sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z_0)}{\lambda_n}\sum_{m=1}^{\infty}\frac{k_1{}^2}{-\mu_m{}^2K(\mu_m{}^2+a_n{}^2-\omega)(k_1{}^2+\mu_m{}^2)}\frac{J_0(\mu_m r)J_0(\mu_m r_0)}{J_1{}^2(\mu_m a)}[\sinh(a_n z)+z\operatorname{cozh}(a_n z)]\times\left(e^{-\omega t}-e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \tag{3.23}$$

$$u_r=\left(\frac{1+\nu}{1-\nu}\right)a_t\sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z_0)}{\lambda_n}\times\sum_{m=1}^{\infty}\frac{k_1{}^2}{-\mu_m K(\mu_m{}^2+a_n{}^2-\omega)(k_1{}^2+\mu_m{}^2)}\frac{J'_0(\mu_m r)J_0(\mu_m r_0)}{J_1{}^2(\mu_m a)}\times[P_n(z)-2\operatorname{cozh}(a_n z)+z\sinh(a_n z)]\times\left(e^{-\omega t}-e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \tag{3.24}$$

$$u_z=\left(\frac{1+\nu}{1-\nu}\right)a_t\sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z_0)}{\lambda_n}\times\sum_{m=1}^{\infty}\frac{k_1{}^2}{-\mu_m{}^2K(\mu_m{}^2+a_n{}^2-\omega)(k_1{}^2+\mu_m{}^2)}\frac{J_0(\mu_m r)J_0(\mu_m r_0)}{J_1{}^2(\mu_m a)}\times\left(e^{-\omega t}-e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}}\times\left[\begin{matrix}P'_n(z)+\frac{2(1-\nu)}{\mu_m{}^2}[\sinh(a_n z)+z\operatorname{cozh}(a_n z)]\\ -a_n{}^2(3\sinh(a_n z)+z\operatorname{cozh}(a_n z))\end{matrix}\right] \tag{3.25}$$

$$\sigma_{rr} = 2G\left(\frac{1+v}{1-v}\right) a_t \sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z_0)}{\lambda_n} \times \sum_{m=1}^{\infty}\frac{k_1{}^2}{K(\mu_m{}^2+a_n{}^2-\omega)(k_1{}^2+\mu_m{}^2)}\frac{J_0(\mu_m r_0)}{J_1{}^2(\mu_m a)} \times$$

$$\sum_{n=1}^{\infty}\left[-P_n(z)\left[\frac{J''_0(\mu_m r)}{\mu_m{}^2}+J_0(\mu_m r)\right] - \frac{1}{\mu_m{}^2}[2\upsilon a_n{}^2 \mathrm{cozh}(a_n z)J_0(\mu_m r) - [(a_n+1)\mathrm{cozh}(a_n z) + a_n z\sinh(a_n z)]J''_0(\mu_m r)]\right]$$

$$\times\left(e^{-\omega t} - e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \qquad (3.26)$$

$$\sigma_{\theta\theta} = 2G\left(\frac{1+v}{1-v}\right) a_t \sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z_0)}{\lambda_n} \times \sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{1}{J_0(\mu_m a)} \times$$

$$\sum_{n=1}^{\infty}\left[-\frac{P_n(z)}{\lambda_n}\left[r\frac{J'_0(\mu_m r)}{\mu_m{}^2}+J_0(\mu_m r)\right] - \frac{1}{\mu_m{}^2}[2\upsilon a_n{}^2 \mathrm{cozh}(a_n z)J_0(\mu_m r) - [(a_n+1)\mathrm{cozh}(a_n z) + a_n z\sinh(a_n z)]r^{-1}J'_0(\mu_m r)]\right]$$

$$\times\left(e^{-\omega t} - e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \qquad (3.27)$$

$$\sigma_{zz} = 2G\left(\frac{1+v}{1-v}\right) a_t \sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z_0)}{\lambda_n} \times \sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{1}{J_0(\mu_m a)} \times$$

$$\sum_{n=1}^{\infty}\left[-\frac{P_n(z)}{\lambda_n}\left[\frac{J''_0(\mu_m r)}{\mu_m{}^2}+J_0(\mu_m r)\right] + \frac{1}{\mu_m{}^2}[(2\upsilon + a_n)a_n{}^2 \mathrm{cozh}(a_n z) + a_n{}^3 z\sinh(a_n z)]J_0(\mu_m r)\right] \times$$

$$\left(e^{-\omega t} - e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \qquad (3.28)$$

$$\sigma_{rz} = 2G\left(\frac{1+v}{1-v}\right) a_t \sum_{n=1}^{\infty}\left(\frac{(a_n h)\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right)\frac{4(k_2+k_3)}{\pi a^2}\frac{P_n(z_0)}{\lambda_n} \times \sum_{m=1}^{\infty}\frac{-1}{\mu_m{}^2}\frac{\mu_m}{(k_1{}^2+\mu_m{}^2)^{\frac{1}{2}}}\frac{1}{J_0(\mu_m a)} \times$$

$$\sum_{N=1}^{\infty}[a_n(Q_n\sin(a_n z) + W_n\mathrm{Cos}(a_n z))J_0(\mu_m r) + [2\upsilon a_n\sinh(a_n z) + a_n{}^2[\sinh(a_n z) + z\mathrm{cozh}(a_n z)]]J'_0(\mu_m r)]]$$

$$\times\left(e^{-\omega t} - e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \qquad (3.29)$$

NUMERICAL CALCULATION

Numerical calculation have been carried out for a copper thick circular plate with radius a =1 m, $k_1 = k_2 = k_3 = 1$ h=1m, thermal diffusivity K=4.42 ft^2/hr,thermal conductivity $\lambda = 224$Btu/hr ft F^0 , coefficient of linear thermal expansion $a_t = 23 \times 10^{-6}$, poisson ratio $v = 0.48$, modulus of elasticity $E = 6.9 \times 10^6$, shear modulus $G = 2.7 \times 10^6$, $\beta_1 = \beta_2 = 1$, $\alpha_1 = k_2$, $\alpha_2 = k_3$ and we obtain, the temperature function

$$\theta(r,z,t) = 0.011\sum_{n=1}^{\infty}\left(\frac{(a_n)\cos^2(a_n)-\cos(a_n)\sin(a_n h)}{a_n{}^2}\right)\frac{P_n(z)P_n(0.5)}{\lambda_n} \times \sum_{m=1}^{\infty}\frac{1}{(\mu_m{}^2+a_n{}^2-1)(1+\mu_m{}^2)}\frac{J_0(\mu_m r)J_0(\mu_m 0.5)}{J_1{}^2(\mu_m)} \times\left(e^{-t} - e^{-K(\mu_m{}^2+a_n{}^2)t}\right)e^{-\frac{t^2}{2}} \qquad (3.30)$$

By substitute the values we find displacement and radial and angular thermal stresses.

Graphical Analysis

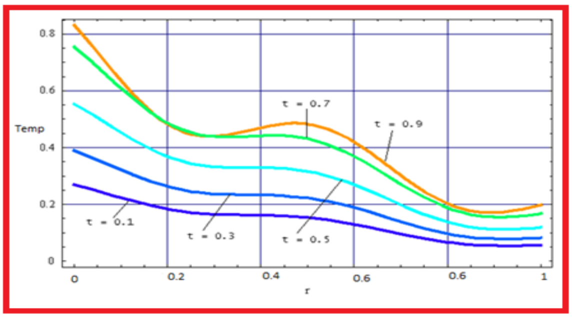

Fig.1: shows that variation of temperature $T(r,z,t)$ Vs r. It is clear that temperature slightly decreases at time t=0.1 sec., t=0.3 sec., t=0.5 sec., t=0.7 sec., t=0.9 sec.

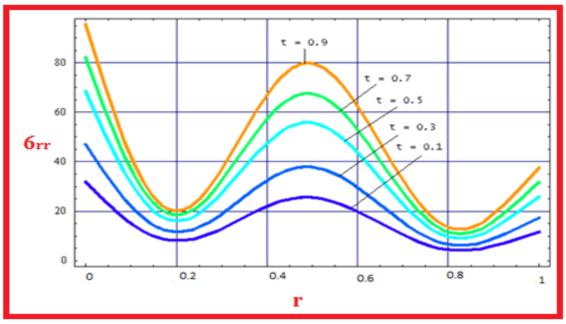

Fig.2: shows that variation of radial stress σ_{rr}Vs r. It is clear that radial stress slightly decreases at time t=0.1 sec., t=0.3 sec., t=0.5 sec., t=0.7 sec., t=0.9 sec. up to zero at z=0.4

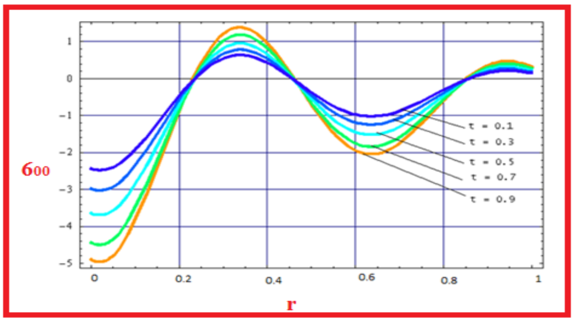

Fig.3: shows that variation of tangential stress $\sigma_{\theta\theta}$Vs z. It is clear that tangential stress suddenly decreases at time t=0.1 sec., t=0.3 sec., t=0.5 sec., t=0.7 sec., t=0.9 sec. and then slightly increases.

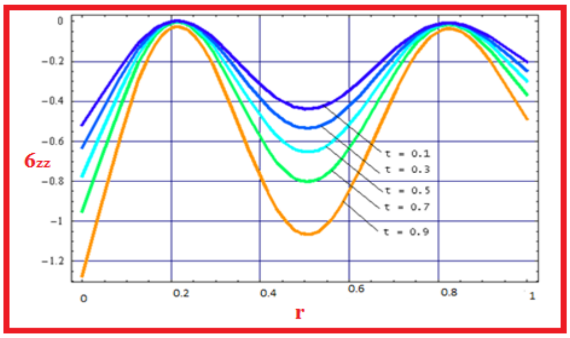

Fig.4: shows that variation of axial stress σ_{zz}Vs r. It is clear that axial stress slightly decreases at time t=0.1 sec., t=0.3 sec., t=0.5 sec., t=0.7 sec., t=0.9 sec. due to internal heat. Axial stress distribution verses r with different value of t.

IV. CONCLUSION

In this paper, we have discussed thermoelastic problem of thick circular plate in which heat source is inside the thick circular plate and obtained the temperature distribution, displacements and radial and angular thermal stresses. We analysed particular case with mathematical model for $\psi(y) = -y$ and numerical calculations are carried out by using Marchi-Fasulo and Hankel integral transforms technique. The result presented here will be useful in engineering problem particularly in the determination of stress in a thick circular plate.

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