Study of Pico-Second Time Domain Reflectometry Techniques using Microstripline Terminated in some Polar Liquids

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ABSTRACT

A microstripline having 50 ohms characteristic impedance has been delineated on FR-4 board material, by using standard chemical photo-etching technology. The dielectric constant and dissipation factor for FR-4 substrate material is 4.6 and 0.018 at 1MHz respectively. The fringing fields arising from terminal end of a microstripline have been employed to observe liquid responses. The microstripline has been vertically terminated into various polar liquids at different successive lengths. The response waveforms due to 40 ps rise time pulse from microstripline in a liquid are observed on the TDR unit in reflection mode at room temperature. The response waveform obtained is similar to the conventional co-axial transmission line connected to precision SMA cell containing liquid. The data analysis is carried out by standard Fourier transformation techniques. The microstripline terminated into a liquid can be used to estimate the dielectric properties of various liquids in broadband frequency region up to 10 GHz.

Keywords: TDR, Electric permittivity, Relaxation Time, Dielectric Dispersion.

I. INTRODUCTION

Time domain Reflectometry (TDR) has been extensively used to determine dielectric parameters of polar as well as non-polar liquids in the GHz. region [1-4]. In the standard TDR technique a step voltage pulse of rise time about 40 ps generated by a tunnel diode is driven along a precision transmission line connected to the SMA cell having 50 Ohms characteristic impedance. In the open load conditions, the transient reflected pulse response waveforms from air and loaded with the liquid under consideration, are digitized and stored in the memory of the broad band sampling oscilloscope. These waveforms are suitably used to obtain the complex reflection coefficient ρ* over the broad range of frequencies by using Fourier transforms. Also, the dispersion curves for complex permittivity data (real and imaginary parts) can be obtained.

TDR & Considerations of Transmission Line Theory:

A co-axial Transmission line in which the length section BC is filled with dielectric liquid and the end D is terminated in a such a way that no further reflections occur at the end of a line by connecting 50 ohm resistive load. When a step pulse is made incident on the line section BC (generally SMA cell with open air load), a partial reflection and transmission at the air-dielectric and dielectric-air interfaces take place. These multiple reflections and transmissions are schematically shown in Fig. 1.

Figure 1. Schematic representation of microwave Transmission line section filled by dielectric.

From the ideal transmission line and network theory the sample of the length d occupies the top section of co-
axial line. Input admittance for four terminals is given by,
\[ y_a = \frac{y_i + y_d}{1 + Z_j y_d} \] ---- (1)
where \( y_i \) is open circuit input admittance \( Z_s \) is a short circuit input impedance of a dielectric filled line and \( y_d \) is a terminal admittance. For a nonmagnetic sample \( y_o \) and \( Z_s \) are given by
\[ y_o = j \omega C_d \tan \left( \frac{z}{z} \right) \] ---- (2)
\[ z_s = j \omega L_d \tan \left( \frac{z}{z} \right) \] ---- (3)

with \( z = \frac{\omega d}{c} \sqrt{\varepsilon} \) ---- (4)

where \( C_d \) and \( L_d \) are the geometrical capacitance and inductance per unit line length respectively. The factor \( \tan \left( \frac{z}{z} \right) \) accounts for the propagation with finite speed in the sample. For the ideal coaxial line and for dielectric permittivity \( \varepsilon^* \) the admittance \( y_d \) relative to the characteristic conductance \( G_c = \sqrt{C_d / L_d} \). The input admittance of dielectric section and termination is given by
\[ y_{in} = \frac{i(\omega)}{v(\omega)} \] ---- (5)
where \( i \) and \( v \) denote the current and voltage respectively and \( \omega = 2\pi f \)

\[ v(\omega) = \int_0^z e^{-j\omega t} v(t) dt \] ---- (6)
\[ i(\omega) = \int_0^z e^{-j\omega t} i(t) dt \] ---- (7)

From the transmission line theory, the current \( i \) and voltage \( v \) in a co-axial line with characteristic admittance \( G_c \) are related to incident \( v_i(\omega) \) and reflected \( v_r(\omega) \) by,
\[ i(\omega) = G_c [v_i(\omega) - r_r(\omega)] \] ---- (8)
\[ v(\omega) = [v_r(\omega) + r_r(\omega)] \] ---- (9)

hence equation \( y_{in} = i(\omega)/v(\omega) \) becomes,
\[ y_{in} = G_c \frac{[v_i(\omega) - r_r(\omega)]}{[v_r(\omega) + r_r(\omega)]} \] ---- (10)

For the non-conducting sample \( y_{in} = j \omega C_g \varepsilon^* \)
where \( C_g \) is the geometrical capacitance of the sample and \( \varepsilon^* \) is its frequency dependent complex dielectric constant. The complex permittivity can be defined as,
\[ \varepsilon^* = \frac{G_c}{j \omega C_g} \frac{[v_i(\omega) - r_r(\omega)]}{[v_r(\omega) + r_r(\omega)]} \cot \frac{\pi}{2} \]

or
\[ \varepsilon^* = \frac{c}{j \omega d} \frac{[v_i(\omega) - r_r(\omega)]}{[v_r(\omega) + r_r(\omega)]} \cot \frac{\pi}{2} \] ---- (11)

where \( c \) is the speed of electromagnetic radiation in air and \( d \) is the electrical length of line section filled with dielectric. Here function \( z = \frac{\omega d}{c} \sqrt{\varepsilon^*} \) accounts for propagation and multiple reflections in the sample and the co-axial line geometry given by,
\[ z \cot z = \frac{1}{\frac{1}{3} \left( \sqrt{\varepsilon^*} \right)^2} - \frac{1}{45} \left( \frac{1}{\sqrt{\varepsilon^*}} \right)^2 \varepsilon^* + \cdots \] ---- (12)

The series expansion is being valid for \( |Z| < \pi \).

The condition \( |Z| < 1 \) corresponding to a maximum frequency \( \omega \) is given by,
\[ \left| \frac{\omega d}{c} \sqrt{\varepsilon^*} \right| < 1 \Rightarrow \omega < \frac{c}{d} \varepsilon^{*1/2} \]
is not a necessary upper limit but is one above which the solution rapidly becomes sensitive to small errors, especially near \( z = \pi / 2 \) for which \( z \cot z = 0 \).

Therefore one has to take above equation as a restriction on cell design and possible frequency range of interest. Assuming \( z \cot (z) = 1 \) we can write,
\[ v_r = r_1 \frac{[1 + (j \omega d / c \varepsilon^*)]}{[1 - (j \omega d / c \varepsilon^*)]} \] ---- (13)

If dielectric sample is replaced by air \( \varepsilon^* = 1 \) then,
\[ v_r = r_1 \frac{[1 + (j \omega d / c)]}{[1 - (j \omega d / c)]} \] ---- (14)

where \( r_1 \) is the reflected pulse for \( \varepsilon^* = 1 \) and \( r_x \) is the reflected pulse when the dielectric sample is present inside the cell. Eliminating \( v_0 \) in above equations,
\[ \frac{r_1}{r_x} = \frac{[1 + (j \omega d / c \varepsilon^*)]}{[1 - (j \omega d / c \varepsilon^*)]} \] ---- (15)

now
\[ \rho^* = \frac{c}{j \omega d} \frac{[r_1 - r_x]}{[r_1 + r_x]} = \frac{(\varepsilon^* - 1)}{1 + \left( \frac{\omega d^2}{c^2} \right) \varepsilon^*} \] ---- (16)

The quantities \( [r_1 - r_x] \) and \( [r_1 + r_x] \) will be called \( p(\omega) \) and \( q(\omega) \) respectively. We also denote reflection coefficient as in the low frequency limits, \( (\omega \rightarrow 0) \),
\[
\rho^* = \frac{c}{j\omega d} \frac{p(\omega)}{j\omega d} q(\omega) = (\varepsilon^* - 1) \quad ---- (17)
\]

The basic equation for determining relative complex permittivity \(\varepsilon^*\) of the sample derived from transmission line theory is conveniently written in simple form as

\[
\varepsilon^*(\omega) = \frac{c}{j\omega d} \frac{[\tau - r_x]}{[\tau + r_x]} z \cot z \quad ---- (18)
\]

where \(z = \frac{\omega d}{c} \sqrt{\varepsilon^*}\) and \(r_0\) and \(r_x\) are the Fourier transform of the pulses from the cell without sample \([R_i(t)]\) and with sample \([R_q(t)]\) respectively.

If we consider only single reflection then \(z \cot z = 1\) and equation (18) can be written as

\[
\varepsilon^*(\omega) = \frac{c}{j\omega d} \frac{[\tau - r_x]}{[\tau + r_x]} \quad ---- (19)
\]

Equation indicates that the dielectric constant of unknown sample can be found if the time profile of the incident \(R_i\) and reflected \(R_p\) pulses are recorded within a frequency range determined by the time limits of \(R_i\) and \(R_x\).

II. METHODS AND MATERIAL

HP-54750A digitizing oscilloscope along with HP-54754A differential TDR module has been used in the present experimental setup. The matching 50 Ohms Microstrip has been properly connected to the matching microwave transmission line. An incident step pulse of 200 mV magnitude and rise time about 40 ps is coupled to the microstripe. The reflected step pulse is observed from the open end termination from the microstripe. The reflected pulses(step) without sample \(R_q(t)\) in air and with microstripe terminated into the liquid sample \(R_q(t)\), were digitized in to 1024 points in the memory of the oscilloscope and transferred to a PC through standard 1.44 MB floppy diskette drive. Several polar liquids have been tested without disturbing the electrical connection of the microstripe.

DATA ANALYSIS:

The Fourier Transformation

The time domain data is converted in to frequency domain data using Fourier transform. While performing Fourier transform, one should be careful about the nature of the curve of which transform is to be obtained. Since nature of curves of \(p(t)\) and \(q(t)\) is different, different methods of Fourier transforms are to be used.

The Fourier transform of \(p(t)\) (closed waveform) is obtained by the summation method [5], defined by the equation,

\[
p(\omega) = \frac{T}{\pi} \sum_{n=0}^{N} \exp(-j\omega T) p(nT) \quad ---- (20)
\]

The Fourier transformation using summation method has some limitations that for all the sampling intervals the nature of pulse form must be known. Furthermore the transform \(p(t)\) is simply the area under the curve \(p(t)\) which has an initial peak followed by a decay to zero or a finite limiting value strictly to infinite time.

The pulse form of \(q(t)\) is not known exactly. The \(q(t)\) rises monotonically to a long time limit. Therefore summation method of Fourier transforms cannot be used for \(q(t)\) curve. The Fourier transform of such type of open curves can be obtained by the Samulan method [6] defined by the expression as follows.

\[
q(\omega) = \frac{T}{\pi} \left[ \sum_{n=0}^{N} (q(nT) - q(n-1)T) \exp(-j\omega T) \right] \quad ---- (21)
\]

In equation (1) and (2) \(\omega\) is angular frequency, \(T\) is the sampling interval or time difference between two adjacent points and \(N\) is number of points. In our experiment \(N\) is 1024 points and \(T\) depends on time window and number of points per waveform (\(N\)). For example if time window is 5 ns then \(T\) is 4.88 ps for 1024 points per waveform. Thus the time domain data is converted into frequency domain data in the frequency range of 10 MHz to 20 GHz.

The frequency domain data obtained from Fourier transform is further used to calculate frequency dependent complex reflection coefficient \(\rho^*(\omega)\) given by equation (17), the single reflection method has the advantage of giving a reflection coefficient of magnitude 0.3 < \(|\rho| < 1\) over the whole frequency spectrum present in the incident pulse. It can thus be considered as a true wide band frequency method even reaching >20 GHz if sufficient accuracy in \(\rho(\omega)\) can be achieved. The demands on the accuracy in \(\rho(\omega)\) are quite severe at high permittivity and high frequencies and the present method can be used for unknown liquids of medium permittivity.

Determination of effective pin length
An effective central pin length for SMA cell having length ‘d’ inside the liquid can be determined by considering equation (16) in the low frequency limit \( \omega \to 0 \), the equation

\[
\rho^* = \left( c / j \omega d \right) \left[ p(\omega) / q(\omega) \right]
\]

reduces to,

\[
\epsilon_0 = \left( \frac{c}{d} \right) \left( \frac{P_{\text{area}}}{q_o - q_x} \right)
\]

Where \( P_{\text{area}} \) is the area under \( p(t) \) curve. The \( q_x \) and \( q_o \) are the values of \( q(t) \) at \( t = \infty \) and \( t = 0 \) respectively. The values of \( P_{\text{area}}, q_x \) and \( q_o \) can be determined experimentally by using liquids of known static permittivity \( \epsilon_0 \). For different known liquids we can plot graphs of \( P_{\text{area}}/(q_x - q_o) \) versus \( (\epsilon_0 - 1) \). The slope of line gives right hand side of equation (22).

or

\[
\left( \frac{d}{c} \right) = \left( \frac{P_{\text{area}}/(q_x - q_o)}{(\epsilon_0 - 1)} \right) = \text{Slope} \quad ---- (22)
\]

or the electrical Pin length \( d = c \times \text{slope} \). In above analysis [8-11] the velocity of light \( c \) to be used as effective velocity \( c_{\text{eff}} \) as microstrip is slow wave structure. Where \( c_{\text{eff}} = \frac{c}{\sqrt{\epsilon_{\text{eff}}}} \) and pin length \( d \) as \( d_{\text{eff}} \) microstrip-liquid terminal length [12].

### III. RESULTS AND DISCUSSION

**Microstrip-Liquid Quarter-wave Resonance:**

The complex reflection coefficient spectra i.e. complex permittivity spectra (Figs.2 - 6) for Propanol, Ethanol, Methanol, Water and Formamide, are obtained by transforming time domain data into frequency domain by suitable Fourier Transform and Samulan methods [6,7], shows the systematic oscillatory variations in \( \epsilon^*(\omega) \) instead of smooth variations, in the frequency range 0.01-20 GHz. The oscillatory behavior in \( \epsilon^*(\omega) \) resonance/anti-resonance may be attributed to microstrip half-wave resonance.

The difference in successive resonant peak values in the frequency range 0.01-20 GHz is observed to be of the order of \( \sim 1 \) GHz. By considering the microstrip as half wave (\( \lambda/2 \)) resonator [5],

Length of microstrip (L) = \( n \lambda/2 \) for order of resonance (n) = 1, 2, 3, …

\[
L = \frac{n\lambda}{2} = \frac{c}{2\sqrt{\epsilon_{\text{eff}}}} f_n \quad ----- (23)
\]

for microstrip made from FR-4 glass epoxy material \( \epsilon_{\text{eff}} = 3.6, \) at \( \sim 1 \) GHz and \( L = 74 \) mm, \( n = 1, \ c = 3.10^{11} \) mm-sec-1. So, fundamental resonant frequency obtained \( f_n = \frac{c}{\sqrt{\epsilon_{\text{eff}} \cdot 2L}} = 1.08 \) GHz

which is nearly same as adjacent peak difference in successive resonant peaks as expected.

The noisy appearance observed in all the raw spectra may be due to the superposing of the reflections generated at discontinuities from cable-connector-microstrip system. The electric permittivity (\( \epsilon_0 \)) and relaxation time (\( \tau \)) are evaluated by applying non-linear least square fit method in low frequency limit [9]. From the evaluated values of static permittivity (\( \epsilon_0 \)) and relaxation time (\( \tau \)) from raw spectra, it is noticed that (\( \epsilon_0 \)) and (\( \tau \)) values varies slowly with increase in physical terminal-lengths in liquids. This terminal-length dependence in their estimated values is not expected. This variation may be attributed to the inaccurate values for effective pin-lengths from earlier work used.

![Figure 2. Typical raw spectra for propanol at terminal length 10 mm](image1)

![Figure 3. Typical raw spectra for ethanol at terminal length 10 mm](image2)
The estimated values static permittivity ($\varepsilon_0$) and relaxation time ($\tau$) from raw spectra are given in Table 1. The evaluated values are fairly consistent with reported literature values. Further the raw spectra can be corrected by suitable bilinear calibration, Newton-Raphson iterative methods or similar methods using root function.

Table 1: Estimated values of permittivity ($\varepsilon$) and relaxation time ($\tau$) in ps from raw spectra at 25°C. The values indicated in parenthesis are standard literature values at 25°C

IV. CONCLUSION

From the above discussions, it is possible to estimate static electric permittivity ($\varepsilon$) and relaxation time ($\tau$) from the dielectric responses of polar liquids with the variation in terminal-lengths using a microstrip-liquid embedded system without disconnecting the co-axial cable, which is otherwise difficult to manipulate the central conductor pin length in the SMA cell. The unwanted microstrip resonant signals/peaks need corrections.

V. REFERENCES


