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Asian Options & Monte Carlo Methods

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ABSTRACT

Price manipulation is reserved for commodity products with low trading volumes, Asian options play an important in pricing in such cases. Since there is no systematic solutions to arithmetic average options, iterative or numerical methods are used. Computer Simulation using Monte Carlo methods plays an important in this case. Various reduction techniques are also used to improve accuracy. This paper deals with Monte Carlo method's use in pricing options and also comparison with other options. Moreover paper also gives a thought to using Quasi Monte Carlo methods in pricing Asian options.

Keywords: Options, Pricing, Asian, Monte Carlo Methods.

I. INTRODUCTION

An option is a contract between a buyer and a seller to buy or to sell the underlying asset at an agreed price at a later date. The call option and the put option are the basic options. The agreed price is the *strike price*; the contract date is the *expiry date*.

European options, American options, Bermudan options, Asian options are some of the known options. European options are exercised only at the expiry date whereas American options can be exercised any time.
Black-Scholes option pricing formula gave a core framework for options pricing.

There are three option pricing methods used primarily. This are binomial methods, finite difference models and Monte Carlo models.

Binomial methods involves the option's theoretical value for discrete intervals over the option's duration. The model of this type starts with a binomial tree of discrete future possible underlying stock prices.

The finite difference model can be derived, once the equations used to value options can be expressed in terms of partial differential equations. The idea is to replace the partial derivatives occurring in partial differential equations by approximations based on series expansions (Taylors series) of functions near the points of interest.

II. ASIAN OPTIONS

Asian options are options in which the underlying variable is the average price over a period of time. Because of this, Asian options have a lower volatility and hence rendering them cheaper relative to their European counterparts.

They are commonly traded on currencies and commodity products which have low trading volumes.

III. MONTE CARLO METHODS

The Monte Carlo method is a statistical method of evaluation of mathematical functions using random samples. Monte Carlo methods need random numbers for computation. There is always some error possible, but larger the number of random samples taken, more accurate is the result.
In mathematical form, the Monte Carlo method is finding the definite integral of a function by choosing a large number of independent variable samples at random from within an interval or region, averaging the resulting dependent-variable values, and then dividing by the span of the interval or the size of the region over which the random samples were chosen.

Which differs from the classical method of approximating a definite integral, in which independent-variable samples are selected at equally-spaced points within an interval or region.

IV. MONTE CARLO METHODS FOR PRICING ASIAN OPTIONS

Steps in Applying Monte carlo methods for pricing Asian options

- ✓ Produce random numbers
- ✓ Transform to standard normnal distribution
- ✓ Sample from the distrinution og dstock prices
- ✓ Evaluate Asian option payoffs
- ✓ Obtain estimate of Asian payoff using averaging
- ✓ Obtain standard deviuation and use it for confidence interval

Theoretical Background:

Option prices are obtained using the following formula

$$C^{(i)} = \exp(-rT)\max(A^{(i)} - K, 0), \quad i = 1, 2, ..., n$$

Thus to estimator of monte carlo is

$$\hat{C} = \frac{1}{n}\sum_{i=1}^{n} C^{(i)}$$

Sample variance can be obtained by

$$s_C^2 = \frac{1}{n-1}\sum_{i=1}^{n}(C^{(i)} - \hat{C})^2$$

V. ERROR REDUCTION TECHNIQUES

The control variate method :

The control variates method is a variance reduction technique used in Monte Carlo methods. It exploits information about the errors in estimates of known quantities to reduce the error of an estimate of an unknown quantity.

The basic idea of control variate is to express expectation of F(x) as follows:

$$E(f(X)) = E(f(X) - h(X)) + E(h(X)),$$

where E(h(X)) can be computed and Var(f(X)−h(X)) is smaller than Var(f(X)).

Using Monte-Carlo method we can estimate E(f(X) − h(X)), and we add the value of E(h(X)).

The introduction of an appropriate control variate provides a very efficient variance reduction technique, however, in some problems it may be difficult to find a suitable control variate.

$$S_{t_j}^{(i)} = S_{t_{j-1}}^{(i)} \exp\left(\left(r - q - \frac{1}{2}\sigma^2\right)(t_j - t_{j-1}) + \sigma\sqrt{t_j - t_{j-1}}Z^{i,j}\right),$$
$$j = 1, 2, ..., m, \quad i = 1, 2, ..., n$$

The sample averages are

$$A^{(i)} = \frac{1}{m}\sum_{j=1}^{m} S_{t_j}^{(i)}, \quad i = 1, 2, ..., n$$

$$G^{(i)} = \left(\prod_{j=1}^{m} S_{t_j}^{(i)}\right)^{1/m}, \quad i = 1, 2, ..., n$$

and sample geometric option prices are calculated as

$$C_G^{(i)} = \exp(-rT)\max(G^{(i)} - K, 0), \quad i = 1, 2, ..., n$$

The Antithetic Variate Method:

For each C(i) we generate $\widetilde{C^{(i)}}$, The Monte Carlo Estimator is

$$\hat{C} = \frac{1}{n}\sum_{i=1}^{n}\frac{C^{(i)} + \tilde{C}^{(i)}}{2}$$

and a 95% confidence interval is

$$[\hat{C} - 1.96\frac{s_C}{\sqrt{n}}, \ \hat{C} + 1.96\frac{s_C}{\sqrt{n}}]$$

with

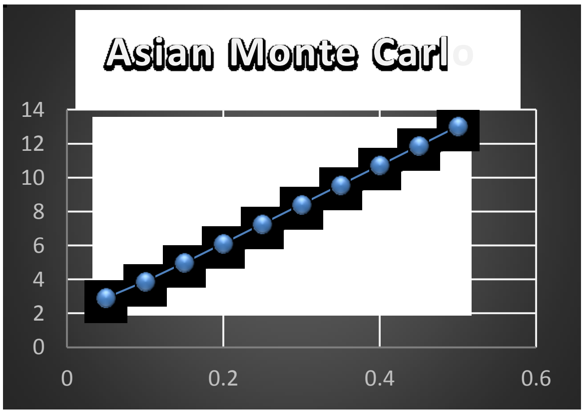
$$s_C^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{C^{(i)} + \tilde{C}^{(i)}}{2} - \hat{C}\right)^2$$

VI. SIMULATION RESULTS

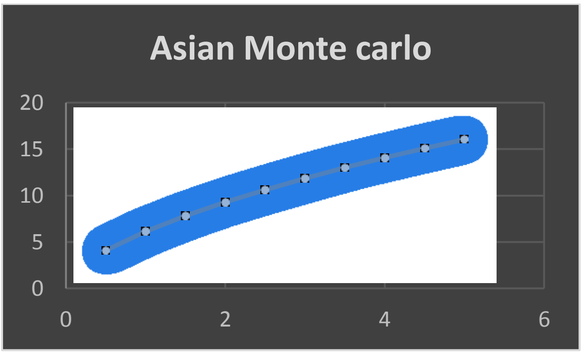
fixing K=100, m=12, n=50000 q=0 r=0.05 & S0=100 , table shows option prices & standard deviations for different values of volatility & maturity

σ	T	$\hat{C}$	$s_C/\sqrt{n}$
0.05	1	2.92	0.0114
0.10	1	3.89	0.0200
0.15	1	4.99	0.0287
0.20	1	6.13	0.0377
0.25	1	7.28	0.0471
0.30	1	8.43	0.0568
0.35	1	9.58	0.0670
0.40	1	10.73	0.0776
0.45	1	11.88	0.0886
0.50	1	13.03	0.1001

σ	T	$\hat{C}$	$s_C/\sqrt{n}$
0.20	**0.5**	4.09	0.0256
0.20	**1.0**	6.13	0.0377
0.20	**1.5**	7.80	0.0476
0.20	**2.0**	9.28	0.0561
0.20	**2.5**	10.62	0.0638
0.20	**3.0**	11.85	0.0708
0.20	**3.5**	13.00	0.0773
0.20	**4.0**	14.08	0.0835
0.20	**4.5**	15.09	0.0892
0.20	**5.0**	16.05	0.0947



Graph of Asian MC for different volatility



Graph of Asian MC for different Maturity Values

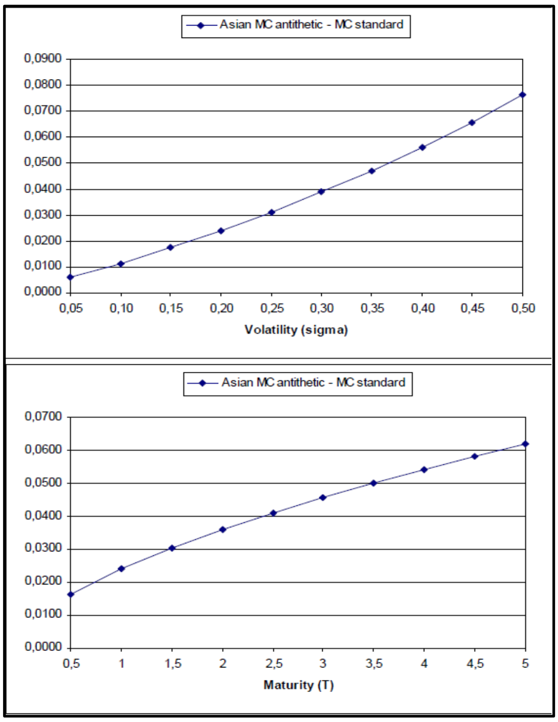
MC option prices for different volatilities using Antithetic method

σ	T	$\hat{C}$	$s_C/\sqrt{n}$
0.05	1	2.93	0.0028
0.10	1	3.90	0.0079
0.15	1	5.01	0.0131
0.20	1	6.15	0.0186
0.25	1	7.31	0.0243
0.30	1	8.47	0.0303
0.35	1	9.63	0.0367
0.40	1	10.79	0.0434
0.45	1	11.95	0.0506
0.50	1	13.10	0.0581

MC option prices for different maturity values using Antithetic method

σ	T	$\hat{C}$	$s_C/\sqrt{n}$
0.20	**0.5**	4.11	0.0127
0.20	**1.0**	6.15	0.0186
0.20	**1.5**	7.83	0.0232
0.20	**2.0**	9.31	0.0272
0.20	**2.5**	10.66	0.0307
0.20	**3.0**	11.89	0.0340
0.20	**3.5**	13.05	0.0370
0.20	**4.0**	14.13	0.0399
0.20	**4.5**	15.15	0.0426
0.20	**5.0**	16.12	0.0452

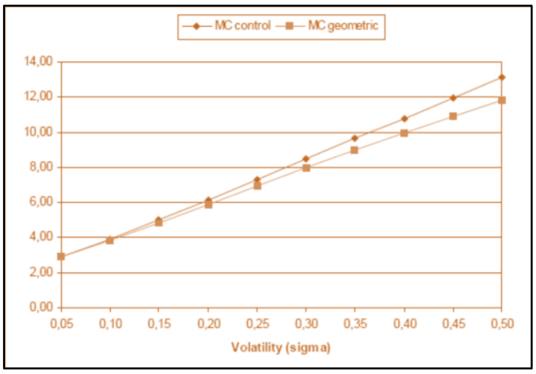
Graph of the above observations



MC option prices for different volatilities
& different maturity values using Control Variate method

σ	T	$\hat{C}$	$s_C/\sqrt{n}$
0.05	1	2.93	0.0001
0.10	1	3.90	0.0004
0.15	1	5.01	0.0009
0.20	1	6.16	0.0016
0.25	1	7.31	0.0024
0.30	1	8.47	0.0035
0.35	1	9.64	0.0049
0.40	1	10.80	0.0065
0.45	1	11.96	0.0085
0.50	1	13.11	0.0107

σ	T	C	$s_C/\sqrt{n}$
0.20	**0.5**	4.11	0.0007
0.20	**1.0**	6.16	0.0016
0.20	**1.5**	7.84	0.0025
0.20	**2.0**	9.32	0.0034
0.20	**2.5**	10.66	0.0045
0.20	**3.0**	11.90	0.0055
0.20	**3.5**	13.05	0.0066
0.20	**4.0**	14.14	0.0078
0.20	**4.5**	15.16	0.0090
0.20	**5.0**	16.12	0.0102



VII. Conclusion

Monte carlo methods/Simulation has been used to price Asian options . From numerical results it is concluded that the standard method is not accurate enough. The random numbers used in computation may show sign of correalation thus different techniques of random number generation for generating sophisticated random numbers should be used. Also Quasi random sequences can be analysed and used here. Antithetic method did not reduce deviation to that extent compared to control variate method.

Also Asian options based on geometric average as a control variate show good results.

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