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GEM Equations and their Validity

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ABSTRACT

The characteristic equations similar to those satisfied by general EM waves have been deduced for the case of GEM waves. It has been observed that the expressions deduced lead to similar expressions valid for EM waves in absence of the interaction between E, H and G. From this study one may have a knowledge about the validity of the proposed GEM relations.

Keywords : Gravitoelectromagnetic, relativistic gravity, weak gravitational field, energy density, wave impedance.

I. INTRODUCTION

According to Einstein [1] gravitation exists in every space [2] and gets over any substance. It is also known that charge is an electric phenomenon while mass is gravitational. Again, implications of general theory of relativity are that gravity and electromagnetism may be related.

On the basis of this conjecture, several works have been done to interrelate electromagnetism with gravity [3,4]. In these works gravity and the force of electromagnetism have been considered such that the behaviour of gravity could be expressed in a way similar to that of magnetism. From these works [5,6] formal analogy between equations of electromagnetism and those of relativistic gravity could be shown [7]. It may be pointed out that interaction between light and matter have been discussed by Saldanha and Monken [8] whereas Maxwell's equations for a source-free region have been dealt with by Raymer and Smith [9].

Again, considering relation between the changes of electrical, magnetic and gravitational fields GEM (gravitoelectromagnetic) equations have, also, been proposed by several workers. It must be mentioned that any type proposed GEM relations would be justified theoretically if they lead to relations similar to those obeyed by EM (electromagnatic) waves in general.

In this dissertation we shall try to derive some relations with GEM equations which are similar to those satisfied by EM waves and examine whether these relations, deduced, transfer to those of EM waves if gravitational effects are absent. If, so, the GEM equations under consideration would be called as properly proposed.

At first, we shall consider the GEM equations proposed in [6] then those in [2] and try to find, theoretically, whether they are justified or not.

1. Proposed GEM equations :

In literatures we generally find different sets of GEM equations amongst which only two sets would be considered in this work.

Assuming weak gravitational field or for space-time reasonably flat following GEM equations have been proposed [6]

$$(i)\nabla.\boldsymbol{E}_g = -4\pi G\rho_g,$$

$$(ii)\nabla.\boldsymbol{B}_g = 0,\ (iii)\nabla\times\boldsymbol{E}_g = -\frac{\partial \boldsymbol{B}_g}{\partial t} \text{ and}$$

$$(iv)\nabla\times\boldsymbol{B}_g = 4(-\frac{4\pi G}{c^2}.\boldsymbol{J}_g + \frac{1}{c^2}\frac{\partial \boldsymbol{E}_g}{\partial t}) \quad \ldots(1)$$

where $\boldsymbol{E}_g$ = static gravitational field or gravitoelectric field (ms^{-2}), $\boldsymbol{B}_g$ = gravitational magnetic field (s)$^{-1}$, ρ_g = mass density (kgm^{-2}), $-\frac{1}{4\pi G}$ = permittivity of free space, G = gravitational constant (m^3.kg.s^{-2}), $\boldsymbol{J}_g$ = mass current density ($\boldsymbol{J}_g = \rho_g \boldsymbol{V}_\rho$, $\boldsymbol{V}_\rho$ = velocity of mass flow generating the gravitational magnetic field) (kg m^{-2} s^{-1}), c = speed of propagation of gravity (ms^{-1}), $\boldsymbol{B}_g = \mu_g \boldsymbol{H}_g$, and μ_g = permeability of the medium in presence of gravity.

Again, the second set of equations proposed in [2] are –

$$(i)\nabla.\boldsymbol{E} = 0,\ (ii)\nabla.\boldsymbol{G} = 0,\ (iii)\nabla.\boldsymbol{H} = 0,$$
$$(iv)\nabla\times\boldsymbol{E} = -\mu_{G0}\frac{\partial \boldsymbol{H}}{\partial t},\ (v)\nabla\times\boldsymbol{G} = -\mu_{E0}\frac{\partial \boldsymbol{H}}{\partial t} \text{ and}$$
$$(vi)\nabla\times\boldsymbol{H} = \varepsilon_{G0}\frac{\partial \boldsymbol{E}}{\partial t} - \varepsilon_{E0}\frac{\partial \boldsymbol{G}}{\partial t} \tag{2}$$

where ***E, H*** and ***G*** are respectively the strength of electrical, magnetic and gravitational fields, μ_{G0} , μ_{E0} and ε_{G0} , ε_{E0} are respectively the magnetic and electrical constants related to gravitational and electrical space-time. Without considering gravity we shall have $\mu_{G0} = \mu_0$, $\varepsilon_{G0} = \varepsilon_0$, μ_{E0} =0= ε_{E0} .

2. Testing the first set of equations :

(A) Morphologically

From relations (1) and (2) we readily see that the mutual effects of electric, magnetic and gravitational fields are not explicit in (1) but this fact is explicitly shown in (2).

Again, (1) would be used for any medium having ρ_g and $\boldsymbol{J}_g$ whereas (2) would be valid for media having ρ_g = 0 = $\boldsymbol{J}_g$.

(B) Setting up of wave equations :

Taking curl of both sides of (iii) of (1) we get

$$-4\pi G\nabla\rho_g - \nabla^2\boldsymbol{E}_g = -\frac{16\pi G}{c^2}.\frac{\partial \boldsymbol{J}_g}{\partial t} - \frac{4}{c^2}\frac{\partial^2 \boldsymbol{E}_g}{\partial t^2} \tag{3}$$

Similarly, from (iv) of (1) we obtain

$$-\nabla^2\boldsymbol{B}_g = -\frac{4\pi G}{c^2}\nabla\times\boldsymbol{J}_g - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}_g}{\partial t^2} \tag{4}$$

Equations (3) and (4) resemble those of a cylindrical wave guide (or an optical fibre) having inhomogeneous medium which show that the energy of the propagating wave in the form of GEM field would be damped very quickly.

Now, in absence of mass in the medium through which waves are propagating the GEM equations (1) would be

$$(i)\nabla.\boldsymbol{E}_g = 0,\ (ii)\nabla.\boldsymbol{B}_g = 0,$$
$$(iii)\nabla\times\boldsymbol{E}_g = -\frac{\partial \boldsymbol{B}_g}{\partial t}, \text{ and } (iv)\nabla\times\boldsymbol{B}_g = \frac{1}{c^2}\frac{\partial \boldsymbol{E}_g}{\partial t}.$$

Here, $\rho_g = 0 = J_g$. So, these are the wave equations in homogeneous medium. It is seen that these equations are similar to the EM field equations widely used for vacuum. The only difference is that a suffix 'g' has been attached to ***E*** and ***B*** signifying that the fields are influenced by gravity.

Under the above mentioned conditions (3) and (4) respectively become

$$\nabla^2\boldsymbol{E}_g = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}_g}{\partial t^2} \tag{5}$$

$$\nabla^2\boldsymbol{B}_g = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}_g}{\partial t^2} \tag{6}$$

C) **Opinion about equations in (1) :**

It is observed that (5) and (6) are exactly similar to those satisfied by ordinary EM wave equations. Hence, the solutions would, also, be of the same form. Evidently, similarity would be there in different wave characteristics like mutual normalcy of $\boldsymbol{E}_g$, $\boldsymbol{H}_g$ and $\boldsymbol{k}$ (the wave vector); form of wave impedance; expression for electric and magnetic energy densities; power flow (Poynting vector); polarization of the wave; Brewster's law; degree of polarization etc. Also Fresnel equations describing the reflection and refraction of GEM waves would be of the same form as those of ordinary EM waves passing through vacuum.

It is seen that the proposed equations (1) would not give us expressions for different phenomena similar to those of EM waves in a straight forward way. Rather, they result into complicated relations. Of course, these equations give forth similar results for vacuum which are simple and straight forward. So, the GEM equations proposed in (1) would be easily valid for spaces where ρ_g and $\boldsymbol{J}_g$ are absent.

3. Testing the second set of equations :

Let us try to test (2)

a) Setting up of wave equation :

Taking curl of both sides of (iv), (v) and (vi) of (2) we respectively get

$$\nabla^2 \boldsymbol{E} = \mu_{G0}\varepsilon_{G0}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \mu_{G0}\varepsilon_{E0}\frac{\partial^2 \boldsymbol{G}}{\partial t^2} \tag{7}$$

$$\nabla^2 \boldsymbol{G} = \mu_{E0}\varepsilon_{G0}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \mu_{E0}\varepsilon_{E0}\frac{\partial^2 \boldsymbol{G}}{\partial t^2} \tag{8}$$

and
$$\nabla^2 \boldsymbol{H} = (\mu_{G0}\varepsilon_{G0} - \mu_{E0}\varepsilon_{E0})\frac{\partial^2 \boldsymbol{H}}{\partial t^2} \tag{9}$$

Here, the second term of (7) and first term of (8) contain interaction between electrical and gravitational characteristics of the medium. Also (7) and (8) show that $\boldsymbol{E}$ and $\boldsymbol{G}$ depend upon the result of the time variations of the two. In absence of gravity (7), (8) and (9) would lead to similar equations as those for EM waves.

b) Solutions.

The solutions of the equations (7), (8) and (9) would be respectively

$$\boldsymbol{E} = \boldsymbol{E}_0 \exp(i\boldsymbol{k}.\boldsymbol{r} - i\omega t) \tag{10}$$

$$\boldsymbol{G} = \boldsymbol{G}_0 \exp(i\boldsymbol{k}'.\boldsymbol{r} - i\omega' t) \tag{11}$$

$$\boldsymbol{H} = \boldsymbol{H}_0 \exp(i\boldsymbol{k}.\boldsymbol{r} - i\omega t) \tag{12}$$

These are similar to those of EM waves.

c) Relation between field vectors and wave vectors.

Using (10), (11) and (12) it could be shown that

$$(a)\boldsymbol{k}\times\boldsymbol{E} = \mu_{G0}\omega\boldsymbol{H}\ ,$$
$$(b)\boldsymbol{k}'\times\boldsymbol{G} = \mu_{E0}\omega\boldsymbol{H} \text{ and}$$
$$(c)\boldsymbol{k}\times\boldsymbol{H} = -\varepsilon_{G0}\omega\boldsymbol{E} + \varepsilon_{E0}\omega'\boldsymbol{G} \tag{13}$$

It is seen from (13) that $\boldsymbol{E}$ and $\boldsymbol{G}$ are on the same axis but opposite to each other and their resultant is normal to $\boldsymbol{k}$ and $\boldsymbol{H}$ as shown in figure 1. This fact is similar to that of a general EM wave. In absence of the effect of gravity (13) will show the relation between $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{k}$ in case of propagation of EM waves.

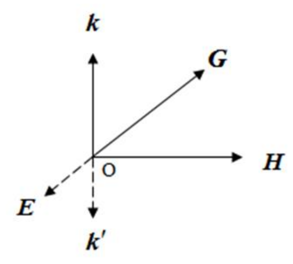

Figure 1. Orientation of $\boldsymbol{E}$, $\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{k}$ and $\boldsymbol{k}'$ according to (13 c)

d) Energy flow :

It has been found that expressions of energy density and momentum density of EM waves in a linear medium have been given in [8]. The expression for EM energy flowing out in unit time from a surface enclosing a volume [10] is

$$\oiint_s (\boldsymbol{E} \times \boldsymbol{H}).d\boldsymbol{s} = -\int_V \frac{1}{2}[\mu_0 \frac{\partial(\boldsymbol{H})^2}{\partial t} + \varepsilon_0 \frac{\partial(\boldsymbol{E})^2}{\partial t}]dV - \int_V \boldsymbol{J}.\boldsymbol{E}dV \tag{14}$$

A similar expression for GEM field would be found out from (2iv) and (2vi) as given below

$$\oiint_s (\boldsymbol{E} \times \boldsymbol{H}).d\boldsymbol{s} = -\int_V \frac{1}{2}[\mu_{G0} \frac{\partial(\boldsymbol{H})^2}{\partial t} + \varepsilon_{G0} \frac{\partial(\boldsymbol{E})^2}{\partial t}]dV + \int_V \varepsilon_{E0}\boldsymbol{E}.\frac{\partial \boldsymbol{G}}{\partial t}dV \tag{15}$$

The left hand sides of (14) and (15) represent the amount of EM energy crossing a closed surface in unit time.

It is also seen that the first two terms on the right hand sides of (14) and (15) are similar over and above the similarity of the left hand sides. But, the third terms on the right hand sides of these equations are not similar due to the interaction between $\boldsymbol{E}$, $\boldsymbol{G}$ and $\boldsymbol{H}$. Comparing (14) and (15) we may say that $\varepsilon_{E0}\boldsymbol{E}.\frac{\partial \boldsymbol{G}}{\partial t}$, represents the rate of GE (gravitoelectric) energy transferred from the GEM field due to the effect of *G*.
Again, from (iv) and (v) of (2) we shall have

$$\oiint_s (\boldsymbol{E} \times \boldsymbol{G}).d\boldsymbol{s} = -\int_V [\mu_{G0}\boldsymbol{G}.\frac{\partial \boldsymbol{H}}{\partial t} - \mu_{E0}\boldsymbol{E}.\frac{\partial \boldsymbol{H}}{\partial t}]dV \tag{16}$$

This relation shows that the time rate of GE energy flowing out through the boundary surface is related to the time rate of change of ***H*** over and above the magnitudes of ***G*** and ***E***. Similarly, (v) and (vi) of (2) lead to

$$\oiint_s (\boldsymbol{H} \times \boldsymbol{G}).d\boldsymbol{s} = -\int_V \frac{1}{2}[\varepsilon_{E0} \frac{\partial(\boldsymbol{G})^2}{\partial t} - \mu_{E0} \frac{\partial(\boldsymbol{H})^2}{\partial t}]dV + \int_V \varepsilon_{G0}\boldsymbol{G}.\frac{\partial \boldsymbol{E}}{\partial t}dV \tag{17}$$

This shows that the time rate of GM (gravitomagnetic) energy flowing out through the boundary surface is related to the time rate of change of gravitational and magnetic energies stored in the volume enclosed by the surface.

Adding (15), (16) and (17) we can write

$$\oiint_s (\boldsymbol{E} \times \boldsymbol{H} + \boldsymbol{E} \times \boldsymbol{G} + \boldsymbol{H} \times \boldsymbol{G}).d\boldsymbol{s} = -\int_V \frac{1}{2}[(\mu_{G0} - \mu_{E0}) \frac{\partial(\boldsymbol{H})^2}{\partial t} + \varepsilon_{G0} \frac{\partial(\boldsymbol{E})^2}{\partial t} + \varepsilon_{E0} \frac{\partial(\boldsymbol{G})^2}{\partial t}]dV$$
$$+ \int_V [\mu_{E0}\boldsymbol{E}.\frac{\partial \boldsymbol{H}}{\partial t} - \mu_{G0}\boldsymbol{G}.\frac{\partial \boldsymbol{H}}{\partial t} + \varepsilon_{E0}\boldsymbol{E}.\frac{\partial \boldsymbol{G}}{\partial t} + \varepsilon_{G0}\boldsymbol{G}.\frac{\partial \boldsymbol{E}}{\partial t}]dV \tag{18}$$

Thus, the GEM energy flowing out in unit time from a surface enclosing a volume plus the energy stored in the volume by the GEM field is equal to the energy transferred from the GEM field.

In absence of the interaction with gravity μ_{E0}, ε_{E0}, and ***G*** are zero. Hence, (18) leads to

$$\oiint_s (\boldsymbol{E} \times \boldsymbol{H}).d\boldsymbol{s} = -\int_V \frac{1}{2}[\mu_{G0} \frac{\partial(\boldsymbol{H})^2}{\partial t} + \varepsilon_{G0} \frac{\partial(\boldsymbol{E})^2}{\partial t}]dV \tag{19}$$

which resembles (14) for a charge free region.

e) Wave impedance :

At this place, we may find out the EM, GM and GE impedances as follows.
We have (13) showing the relation between $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{G}$, $\boldsymbol{k}$ and $\boldsymbol{k}'$.
From (13a) we get

$$\left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right| = \mu_{G0}c_1 \tag{20}$$

where $c_1 = \frac{\omega}{k} = \frac{1}{\sqrt{(\mu_{G0}\varepsilon_{G0})}}$ (21)

Also, from (13b) we obtain

$$\left|\frac{\boldsymbol{G}}{\boldsymbol{H}}\right| = \mu_{E0}c_2 \tag{22}$$

where $c_2 = \frac{\omega'}{k'} = \frac{1}{\sqrt{(\mu_{E0}\varepsilon_{E0})}}$ (23)

Again, from (20) and (22) we can write

$$\left|\frac{\boldsymbol{G}}{\boldsymbol{E}}\right| = \left|\frac{\boldsymbol{G}}{\boldsymbol{H}}\right| \Big/ \left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right| = \frac{\mu_{E0} c_2}{\mu_{G0} c_1} \tag{24}$$

Total impedance of the GEM wave would be

$$Z = \left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right| + \left|\frac{\boldsymbol{G}}{\boldsymbol{H}}\right| + \left|\frac{\boldsymbol{G}}{\boldsymbol{E}}\right| = \mu_{G0} c_1 + \mu_{E0} c_2 + \frac{\mu_{E0} c_2}{\mu_{G0} c_1} \tag{25}$$

It could be shown that in absence of the effect of gravity total impedance (Z) of the wave would be

$$Z = \mu_{G0} c_1 = \left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right| = \sqrt{\frac{\mu_{G0}}{\varepsilon_{G0}}} \tag{26}$$

which is equal to the impedance of the EM waves in vacuum.

The ratio $\left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right|$ for the GEM wave could also be found out by substituting the value of G from (24) in (13c) when we obtain.

$$\left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right| = \frac{\mu_{G0}\omega}{-\varepsilon_{G0}\mu_{G0}\omega c_1 + \varepsilon_{E0}\mu_{E0}\omega' c_2} \tag{27}$$

If the gravitational effect be absent (i.e. for $\mu_{E0} = 0 = \varepsilon_{E0}$) then

$$\left|\frac{\boldsymbol{E}}{\boldsymbol{H}}\right| = \sqrt{\frac{\mu_{G0}}{\varepsilon_{G0}}} \tag{28}$$

Since, we are taking the modulus, the negative sign in the right hand side has been dropped. Relation (28) is the usual one for EM field as it is the same as those given in (20) or (26).

f) Energy densities and their ratios :

Let us find out different ratios of electrostatic, magnetic and gravitational energy densities.

From (18) it is seen that in case of GEM field due to the mutual interaction of ***E***, ***H*** and ***G***

Electrostatic energy density

$$u_e = -\frac{1}{2}\int \varepsilon_{G0} \frac{\partial (\boldsymbol{E})^2}{\partial t} = -\frac{1}{2}\varepsilon_{G0} (\boldsymbol{E})^2 \tag{29}$$

Magnetic energy density

$$u_m = -\frac{1}{2}\int (\mu_{G0} - \varepsilon_{E0}) \frac{\partial (\boldsymbol{H})^2}{\partial t} = -\frac{1}{2}(\mu_{G0} - \varepsilon_{E0})(\boldsymbol{H})^2 \tag{30}$$

and Gravitational energy density

$$u_G = -\frac{1}{2}\int \varepsilon_{E0} \frac{\partial (\boldsymbol{G})^2}{\partial t} = -\frac{1}{2}\varepsilon_{E0} (\boldsymbol{G})^2 \tag{31}$$

Hence, total energy density of the GEM wave is

$$u = u_e + u_m + u_G = -\frac{1}{2}[\varepsilon_{G0}(\boldsymbol{E})^2 + (\mu_{G0} - \varepsilon_{E0})(\boldsymbol{H})^2 + \varepsilon_{E0}(\boldsymbol{G})^2]$$

which leads to $u = -\frac{1}{2}[\varepsilon_{G0}(\boldsymbol{E})^2 + \mu_{G0}(\boldsymbol{H})^2]$ (32)

in absence of **G.** It is similar to the energy density of an EM wave.

From (29), (30) and (31) we may find out different ratios of these energy densities. Thus,

$$\frac{u_e}{u_m} = \frac{\mu_{G0}}{\mu_{G0} - \varepsilon_{E0}} \tag{33}$$

(This ratio is not unity unlike the case of an EM wave). In a similar manner we find out

$$\frac{u_G}{u_e} = \frac{\mu_{E0}}{\mu_{G0}} \tag{34}$$

and $$\frac{u_G}{u_m} = \frac{\mu_{E0}}{\mu_{G0} - \varepsilon_{E0}} \tag{35}$$

It is seen that none of these ratios are unity due to the influence of gravity. It could be easily shown that in absence of gravitational field the ratio $\frac{u_e}{u_m} = 1$ while other two are zero.

This is the case of EM wave. Adding (33), (34) and (35) we may write

$$S = \frac{u_e}{u_m} + \frac{u_G}{u_e} + \frac{u_G}{u_m} = \frac{\mu_{G0}^2 + \mu_{E0}(\mu_{G0} - \varepsilon_{E0}) + \mu_{G0}\mu_{E0}}{(\mu_{G0} - \varepsilon_{E0})\mu_{G0}} \tag{36}$$

In absence of the effect of gravity the sum of the ratios becomes

$$S = 1 \tag{37}$$

which is nothing but the ratio of electrostatic and magnetic energy densities as mentioned earlier. It should be mentioned that the ratios must be taken in the way as in (36) otherwise, this sum would become infinity in absence of gravitational effect which is impossible.

g) Polarization :

Let us discuss about the polarizations of the GEM wave. We have three fields ***E*** , ***H*** and ***G***. It is well known that change of both fields ***E*** and ***G*** produces the magnetic field ***H***. The directions of ***E*** and ***G*** would be opposite to each other. Thus, ***H*** may assumed to be due to the change in the difference between ***E*** and ***G*** after they have been multiplied by $\varepsilon_{G0}\omega$ and $\varepsilon_{E0}\omega'$ respectively so that we may put $\varepsilon_{G0}\omega''\boldsymbol{E}' = \varepsilon_{G0}\omega\boldsymbol{E} - \varepsilon_{E0}\omega'\boldsymbol{G}$ in equation (vi) of (2) where ω'' may be taken as the frequency of $\boldsymbol{E}'$, the resultant of ***E*** and ***G***. The waves are shown in figure 2.

Now, the GEM wave is linearly polarized with the polarization vector $\boldsymbol{E}'$, the resultant of E and ***G*** as mentioned earlier. The orientation of $\boldsymbol{E}'$, ***H*** and ***k*** are shown in figure 3.

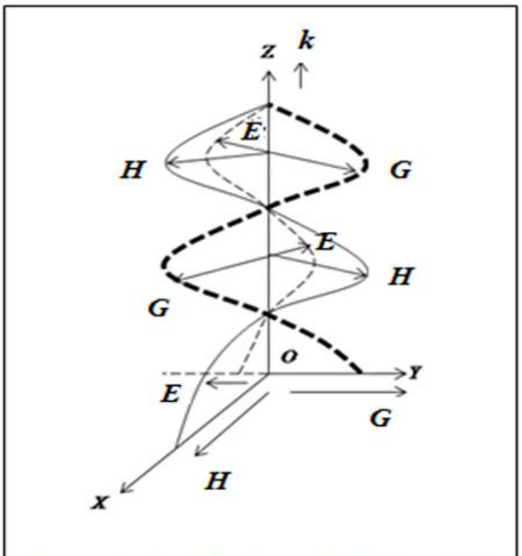

Figure 2. Oscillation of ***E***, ***H*** and ***G***

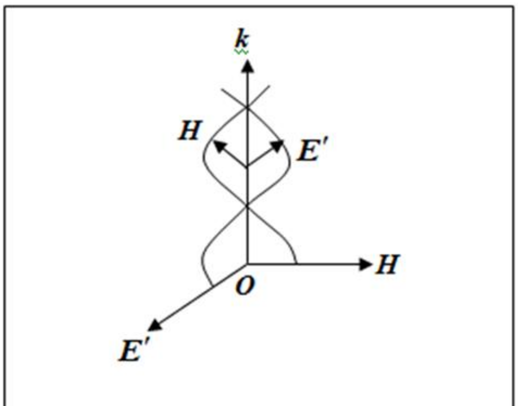

Figure 3. Orientation of $\boldsymbol{E}'$ and ***H***

In figure 4 the plane ZOP is the plane of vibration while the plane ZOH is the plane of polarization which are mutually normal. Now, according to the theory of polarization any plane polarized wave could be divided into two plane polarized components perpendicular to each other as in figure 4.

Accordingly, $\boldsymbol{E}'$ could be represented by

$$\boldsymbol{E}_1' = \hat{n}_1 E'_{10} \exp(i\boldsymbol{k}.\boldsymbol{r} - i\omega t) \tag{38}$$

and $$\boldsymbol{E}_2' = \hat{n}_2 E'_{20} \exp(i\boldsymbol{k}.\boldsymbol{r} - i\omega t) \tag{39}$$

Three cases may arise ---

1) When $\boldsymbol{E}'_1$ and $\boldsymbol{E}'_2$ are in phase, the resultant wave would be plane polarized having the plane of vibration of $\boldsymbol{E}'$ inclined at an angle $\theta = tan^{-1}(\frac{E'_{20}}{E'_{10}})$ as in figure 5 where, E'_{10} and E'_{20} are the amplitudes of $\boldsymbol{E}'_1$ and $\boldsymbol{E}'_2$ respectively.

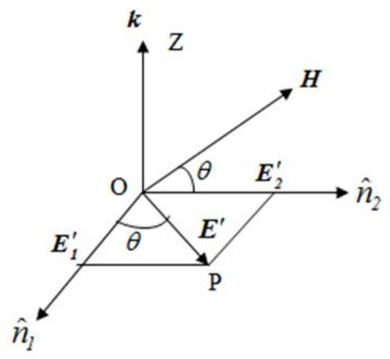

Figure 4. Electric field of a linearly polarized wave

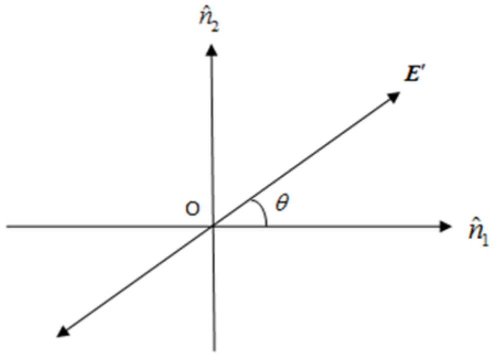

Figure 5. Plane polarized vibration

2) If there be a phase difference between $\boldsymbol{E}'_1$ and $\boldsymbol{E}'_2$ then the wave would be elliptically polarized provided $E'_{10} \neq E'_{20}$ Here, the axes of the ellipse may not coincide with the co-ordinate axes .

Again, if the phase difference between $\boldsymbol{E}'_1$ and $\boldsymbol{E}'_2$ be $\pi/2$ and $E'_{10} \neq E'_{20}$ then the resultant wave would be represented by figure 6.

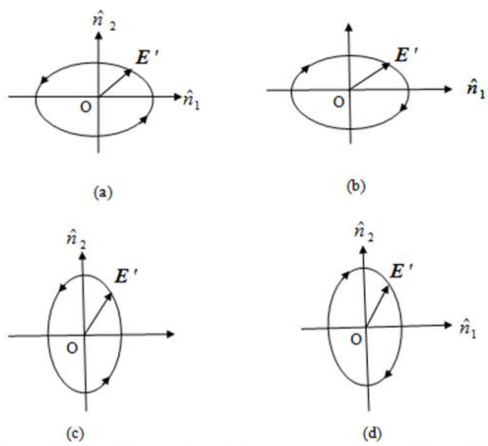

Figure 6. Elliptically polarized GEM wave. (a) and (c) are left handed while (b) and (d) are right handed vibrations. For (a), (b) $E'_{10} > E'_{20}$ and for (c), (d) $E'_{10} < E'_{20}$

When the phase difference between $\boldsymbol{E}'_1$ and $\boldsymbol{E}'_2$ is $\pi/2$ and $\boldsymbol{E}'_{10} = \boldsymbol{E}'_{20}$ then the resultant wave would be circularly polarized as shown in figure 7.

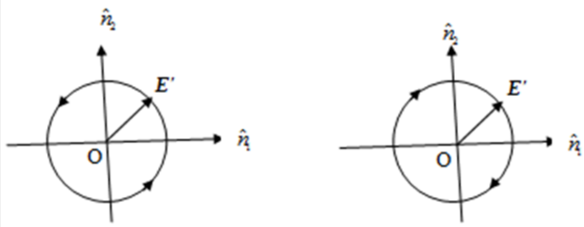

Figure 7 Circular polarization of GEM wave

Thus at a fixed point in space the resultant electric field $\boldsymbol{E}'$ is constant in magnitude but sweeps around a circle with frequency ω.

It must be mentioned that the polarizations discussed above are similar to those of EM wave as we have taken the resultant electric field $\boldsymbol{E}'$ in the GEM field which acts as $\boldsymbol{E}$ of an EM wave and $\boldsymbol{E}'$ will transform to $\boldsymbol{E}$ in absence of the effect of gravitation.
It is to be mentioned that Maxwell's equations were derived including polarization of the medium [11].

h) Fresnel Equations :

If a GEM wave be penetrating a boundary of two media, there would be reflection and refraction of the wave at the boundary. To study the phenomenon we proceed as follows. Let the field vector $\boldsymbol{E}'$ be normal to the plane of incidence. The situation of transmission from medium 1 to medium 2 and reflection of the wave in medium 1 are shown in figure 8

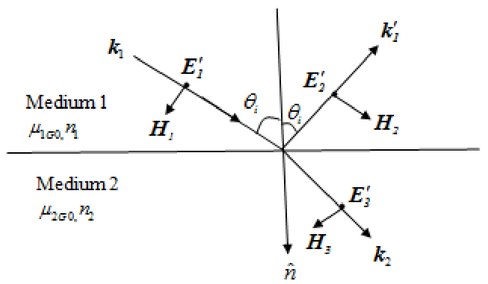

Figure 8. Reflection and refraction with E' perpendicular to plane of incidence

Considering continuity of tangential components of $\boldsymbol{E}'$ and $\boldsymbol{H}$ at the boundary we shall obtain, after some mathematical steps, the relative field amplitudes in non-magnetic media for the cases of reflected and transmitted wave respectively as

$$\left(\frac{E'_{20}}{E'_{10}}\right)_\perp = \frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)} \tag{40}$$

$$\left(\frac{E'_{30}}{E'_{10}}\right)_\perp = \frac{2\cos\theta_i \sin\theta_r)}{\sin(\theta_i + \theta_r)} \tag{41}$$

It is to be noted that Snell's law has been used in deducing (40) and (41). These relations show that Fresnel equations for GEM waves are of the same nature as general EM waves as found in literatures [12]. It could, also, be easily shown that Fresnel equations for GEM and EM wave will also be similar for $\boldsymbol{E}'$ parallel to the plane of incidence. This may be assumed due to the fact that we have taken $\boldsymbol{E}'$ in place of $\boldsymbol{E}$ used for EM wave and also $\boldsymbol{E}'$ transforms to $\boldsymbol{E}$ in absence of gravitational effect.

Now, from Fresnel equations we can find out other characteristic equations valid in case of GEM waves which are similar to those valid, generally, for EM waves e.g. the co-efficients of reflection and refraction, Brewster's law, degree of polarization etc.

It could be mentioned that $\boldsymbol{H}$ and $\boldsymbol{E}'$ are invariant under gauge transformations but $\boldsymbol{E}$ and $\boldsymbol{G}$ are not so individually which could be easily shown.

II. CONCLUSION

The study made here supports equations (2) as they lead to the expressions for different characteristics of general EM waves in absence of gravitational field. So, among different GEM equations proposed by different workers, those equations considered in (2) are more justified.

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