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Bianchi Type-VI_0 Wet Dark Fluid Cosmological Model with Linearly Varying Deceleration Parameter in *f(R,T)* Gravity

A. Y. Shaikh[1], K. S. Wankhade[2], S. R. Bhoyar[3]
[1]Department of Mathematics , Indira Gandhi Mahavidyalaya, Ralegaon, Maharashtra, India
[2]Department of Mathematics , Y.C.Science College , Mangrulpir, Maharashtra, India
[3]Department of Mathematics , Phulsing Naik Mahavidyalaya Pusad, Maharashtra, India
Corresponding Author : Dr. S.R.Bhoyar, Phulsing Naik Mahavidyalaya,Pusad, Maharashtra, India

ABSTRACT

In this paper, Bianchi type- VIo cosmological model with Wet Dark Fluid (WDF) in ***f(R,T)*** gravity , where *R* is the Ricci scalar and *T* the trace of stress energy-momentum tensor, in the context of late time accelerating expansion of the universe has been studied. The exact solutions of the field equations are obtained by using a linearly varying deceleration parameter (LVDP). The universe ends with big rip. Our model initially shows acceleration for a certain period of time and then decelerates consequently. Several dynamical and physical behaviors of the model are also discussed in detail.

Keywords : Bianchi type VI_0 space-time, *f(R,T)* gravity; WDF, LVDP

I. INTRODUCTION

The discovery of modern cosmology is that the current universe is not only expanding butalso accelerating. This late time accelerated expansion of the universe has been confirmedby the high red-shift supernovae experiments (Riess et. al. (1998), Perlmutter et. al. (1999), Bennet et. al. (2003)).The universe consists of 76 % dark energy and 20 % dark matter. In view of the late time acceleration of the universe and the existence of the dark matter and dark energy, very recently, modified theories of gravity have been developed. There are various modified theories namely *f(R)*,*f(G)*,*f(R,G)* and *f(R,T)* .One of the interesting and prospective versions of modified gravity theories is the *f(R,T)*gravity proposed by Harko et al. (2011) wherein the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar *R* and the trace of the stress energy tensor *T*. In *f(R,T)*gravity, the field equations are obtained from the Hilbert-Einstein type variational principle. The *f(R,T)* gravity models can explain the late time cosmic accelerated expansion of the Universe. Adhav (2012) has obtained Bianchi type I cosmological model in *f (R,T)* gravity. Several Relativists (Katore and Shaikh(2012),Myrzakulov (2012), Mubasher et al. (2012), Sharif and Zubair (2012a), Sharif and Zubair (2012b), Sharif et al. (2013) , Ahmed and Pradhan (2014) , Reddy et al.(2014), Shamir (2015), Momeni et al. (2015),Santos and Ferst (2015),Shaikh and Wankhade (2015), Shaikh and Bhoyar (2015),Shaikh,A.Y.(2016), Katore and Shaikh(2016) A.Y.Shaikh(2016)) studied different cosmological models in *f(R,T)* theory of gravity. Singh and Bishi (2015) have investigated Bianchi I universe model with quadratic EoS in *f(R,T)* gravity with Λ .Sofuoglu (2016) has researched Bianchi type IX universe model in *f(R,T)* gravity. Shaikh and Wankhade (2017) investigated Hypersurface-Homogeneouscosmological model in *f(R,T)* theory of gravity with a term Λ.

The purpose of this paper is to study Bianchi type-VI_0 space time cosmological model in the frame of the newly established extension of the standard general relativity known as the *f(R,T)* theory of gravity

II. Wet Dark Fluid

Wet Dark Fluid (WDF) is a new candidate for Dark Energy (DE) in the script of generalized Chaplygin gas, where a physically motivated equation of state is offered with the properties relevant for a DE problem. The equation of state for a WDF is

$$\frac{p_{WDF}}{\omega}+\rho^{*}=\rho_{WDF}. \qquad (1)$$

Equation (1) is good approximation for many fluids, including water. The parameter ω and ρ^{*} are taken to be positive and restricted to $0\le\omega\le1$. Note that if c_s denotes the adiabetic sound speed in WDF, then c_s^2 (Babichev et. al. (2004)). To find the WDF energy density, the energy conservation equation is used

$$\rho^{*}_{WDF}+3H\left(p_{WDF}+\rho_{WDF}\right)=0. \qquad (2)$$

From equation of state (1) and using $3H=\frac{\dot{V}}{V}$ in equation (2), we obtain

$$p_{WDF}=\left(\frac{\omega}{1+\omega}\right)\rho^{*}+\frac{c}{V^{(1+\omega)}} \qquad , \qquad (3)$$

where c is the constant of integration and V is the volume expansion. WDF naturally includes these components, a piece that behave as a cosmological constant as well as a standard fluid with an equation of state $p=\omega\rho$. It is shown that if we take $c>0$, this fluid will not violate the strong energy condition $p+\rho\ge0$. Thus,

$$p_{WDF}+\rho_{WDF}=(1+\omega)\rho_{WDF}-\omega\rho^{*}=(1+\omega)\left(\frac{c}{V^{(1+\omega)}}\right)\ge0 \quad (4)$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW model by Holman and Naidu (2005).The Bianchi type-I universe filled with dark energy from a wet dark fluid has been investigated by Singh and Chaubey (2008). The Bianchi type-V universe filled with dark energy from a wet dark fluid has been studied by Chaubey (2009).Adhav et. al. (2010) has investigated Bianchi type III cosmological models with dark energy in the form of Wet Dark energy in the presence and absence of magnetic field. Plane Symmetric Universe filled with dark energy from a wet dark fluid has been considered by Katore et. al.(2011). Katore et. al.(2011) have discussed Bianchi type VIo universe with wet dark fluid in general relativity. The higher dimensional Bianchi type I Universe filled with dark energy from a wet dark fluid has been studied by Katore et. al. (2012). Mishra and Sahoo (2013) discussed the kink space-time with wet dark fluid (WDF) in the scale invariant theory of gravitation .Many Relativists (Jain et. al. (2012), Adhav et. al. (2013), Samanta (2013),Mishra and Sahoo (2014a,b), Deo et. al. (2016), Mete et. al. (2016), Shaikh,A.Y(2016)) studied cosmological models with WDF in General Relativity and theories of gravitations.

III. Gravitational field equations of $f(R,T)$ gravity

The $f(R,T)$ gravity is the generalization of General Relativity (GR). In this theory, the field equations are derived from a variation, Hilbert-Einstein type principle which is given as

$$S=\frac{1}{16\pi}\int\sqrt{-g}\;f(R,T)\,d^4x+\int\sqrt{-g}\;L_m d^4x, \quad (5)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T) of the matter T_{ij} ($T=g^{ij}T_{ij}$) and L_m is the matter Lagrangian density.

The stress energy tensor of matter is defined as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}. \qquad (6)$$

Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} and not on its derivatives, in this case

$$T_{ij}=g_{ij}L_m-\frac{\delta(L_m)}{\delta g^{ij}}. \qquad (7)$$

The $f(R,T)$ gravity field equations are obtained by varying the action S with respect to the metric tensor components g_{ij},

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+f_R(R,T)\left(g_{ij}\nabla^i\nabla_i-\nabla_i\nabla_j\right)= 8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\Theta_{ij}, \qquad (8)$$

where

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\alpha\beta}\frac{\partial^2 L_m}{\delta g^{ij}\partial g^{\alpha\beta}}. \qquad (9)$$

Here $f_R=\frac{\delta\, f(R,T)}{\delta R}$, $f_T=\frac{\delta\, f(R,T)}{\delta T}$

$$\Theta_{ij}=g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}} \qquad (10)$$

and ∇_i is the covariant derivative.

The contraction of equation (8) yields

$$f_R(R,T)R + 3\Pi f_R(R,T) - 2f(R,T) = (8\pi - f_T(R,T))T - f_T(R,T)\Theta$$

$$\text{with } \Theta = g^{ij}\Theta_{ij} . \quad (11)$$

Equation (10) gives a relation between Ricci scalar and the trace of energy momentum tensor.

Using matter Lagrangian L_m the stress energy tensor of the matter is given by

$$T_{ij} = (p_{WDF} + \rho_{WDF})u_i u_j + p_{WDF} g_{ij} , \quad (12)$$

where $u^i = (0,0,0,1)$ denotes the four velocity vector in co-moving coordinates which satisfies the condition $u^i u_i = 1$. ρ_{WDF} and p_{WDF} is energy density and pressure of the fluid respectively.

The variation of stress energy of perfect fluid has the following expression

$$\Theta_{ij} = -2T_{ij} - pg_{ij} . \quad (13)$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij}. Several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al.(2011) gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} . \quad (14)$$

Here we consider a particular form of the function $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$ where λ_1 and λ_2 are any parameters such that

$$f(R,T) = \lambda_1 R + \lambda_2 T. \quad (15)$$

Using equation (12) , the gravitational field equations (13) reduces to

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)T_{ij} + \frac{\lambda_2}{\lambda_1}\left(p + \frac{1}{2}T\right)g_{ij} \quad (16)$$

IV. Metric and Solution of Field Equations

We take a spatially homogeneous and anisotropic Bianchi type VI_0 space-time in the form

$$ds^2 = -dt^2 + a_1^2 dx^2 + a_2^2 e^{2x} dy^2 + a_3^2 e^{-2x} dz^2 , \quad (17)$$

where the metric functions a_1, a_2, a_3 are the functions of t alone.

For the space-time given by Eq.(17), the following are the formulae for physical and kinematical parameters which are useful to solve the field equations of this theory.

The spatial volume and the average scale factor is defined as

$$V = a^3 = a_1 a_2 a_3 . \quad (18)$$

The average Hubble parameter is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\frac{V_4}{V} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}.\right). \quad (19)$$

The expressions for scalar expansion θ and shear scalar σ are given by

$$\theta = 3H , \quad (20)$$

$$\sigma^2 = \frac{3}{2}H^2\Delta . \quad (21)$$

The average anisotropy parameter is

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad (22)$$

where $H_i (i = 1,2,3)$ represents the directional Hubble parameter in the directions of x, y, z respectively.

Another important dimensionless kinematical quantity is the deceleration parameter q , which tells whether the universe exhibits accelerating volumetric expansion or not

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \quad (23)$$

Using co-moving coordinates and equations (5) and (12), the $f(R,T)$ gravity field equations, (16), for metric (17) can be written as

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{1}{a_1^2} = -\left(\frac{16\pi + 3\lambda_2}{\lambda_1}\right)\frac{p_{WDF}}{2} + \frac{\lambda_2}{\lambda_1}\frac{\rho_{WDF}}{2} , \quad (24)$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} - \frac{1}{a_1^2} = -\left(\frac{16\pi + 3\lambda_2}{\lambda_1}\right)\frac{p_{WDF}}{2} + \frac{\lambda_2}{\lambda_1}\frac{\rho_{WDF}}{2} \quad (25)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{1}{a_1^2} = -\left(\frac{16\pi + 3\lambda_2}{\lambda_1}\right)\frac{p_{WDF}}{2} + \frac{\lambda_2}{\lambda_1}\frac{\rho_{WDF}}{2} \quad (26)$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{1}{a_1^2} = \left(\frac{16\pi + 3\lambda_2}{\lambda_1}\right)\frac{\rho_{WDF}}{2} - \frac{\lambda_2}{\lambda_1}\frac{p_{WDF}}{2} \quad (27)$$

$$\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2} = 0 \quad (28)$$

where a dot hereinafter denotes ordinary differentiation with respect to cosmic time.

On integrating equation (28) we get

$$a_3 = c_1 a_2, \quad (29)$$

where c_1 is constant of integration and assume it as unity.

Using equation (29) , the set of equations (24)- (28) reduces to

$$2\frac{\ddot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 + \frac{1}{a_1^2} = -\left(\frac{16\pi + 3\lambda_2}{\lambda_1}\right)\frac{p_{WDF}}{2} + \frac{\lambda_2}{\lambda_1}\frac{\rho_{WDF}}{2}, \quad (30)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{1}{a_1^2} = -\left(\frac{16\pi + 3\lambda_2}{\lambda_1}\right)\frac{p_{WDF}}{2} + \frac{\lambda_2}{\lambda_1}\frac{\rho_{WDF}}{2}, \quad (31)$$

$$2\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 - \frac{1}{a_1^2} = \left(\frac{16\pi + 3\lambda_2}{\lambda_1}\right)\frac{\rho_{WDF}}{2} - \frac{\lambda_2}{\lambda_1}\frac{p_{WDF}}{2}. \quad (32)$$

Since field equations (30)-(32) are three equations which are highly nonlinear having four unknowns a_2, a_3, p_{WDF}, and ρ_{WDF}, an extra condition is needed to solve the system completely. A law of variation for Hubble parameter proposed by Berman (1983) and Berman and Gomide (1988) within the context of FRW space times in general relativity that yields constant deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2} = m - 1$ (where a is the scale factor and $m \geq 0$ is a constant). In Berman's law the deceleration parameter can get value $q \geq -1$, and since $-1 \leq q < 0$ corresponds to the accelerating expansion, many authors have studied cosmological models using this law in the context of dark energy following the discovery of current acceleration of the universe.

In this paper, we propose a generalized, linearly varying deceleration parameter.

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -kt + m - 1 \quad (33)$$

where $k \geq 0$ and $m \geq 0$ are constants and $k = 0$ reduces to the law of Berman (1983) which yields models with constant deceleration parameter. Reddy et. al. (2016) have considered a minimally interacting holographic dark energy Bianchi type-IX cosmological model in Saez-Ballester (1986) scalar-tensor theory of gravitation and to obtain the cosmological model they have used linearly varying deceleration parameter proposed by Akarsu and Dereli (2012) specified by equation (33). Bianchi type-III holographic dark energy model in Brans-Dicke theory with constant deceleration parameter have been studied by Umadevi and Ramesh(2015) while Reddy et al(2016) obtained Kantowski –Sachs holographic dark energy model the deceleration parameter given by Eq.(33). Here we are concerned with linearly varying deceleration parameter given by Eq. (33) when $k > 0\, and\, m \geq 0.$

Solving (33) one obtains three different form of solutions for the scale factor:

$$a = k_1 e^{\frac{2}{\sqrt{m^2 - 2c_1 k}} \operatorname{arctanh}\left(\frac{kt - m}{\sqrt{m^2 - 2c_1 k}}\right)} \qquad for\ k > 0\ and\ m \geq 0 \quad (34)$$

$$a = k_2 (mt + c_2)^{\frac{1}{m}} \qquad for\ k = 0\ and\ m > 0 \quad (35)$$

$$a = k_3 e^{c_{3t}} \qquad for\ k = 0\ and\ m = 0 \quad (36)$$

where $k_1, k_2, k_3, c_1, c_2, c_3$ are constants of integration.

The last two of these solutions are for constant q and hence corresponds to the solutions under Constant Deceleration Parameter (CDP) ansatz. For convenience, Reddy et. al. (2016) considered the solution for $k > 0, l > 0$ and choose the integration constant $c_1 = 0$. The reason for considering the solution only for $k > 0, l > 0$ is not only for simplicity but also for compatibility with the observed universe. The condition $k > 0$ means that we are dealing with increasing acceleration $\dot{q} = -k < 0$. $k > 0$ is the only way to shift the deceleration parameter to values higher than (−1).

With above assumptions and following Reddy et. al. (2016) , the equation (34) reduces to

$$a(t) = k_1 e^{\frac{2}{m} \operatorname{arctanh}\left(\frac{kt}{m} - 1\right)} \quad (37)$$

In order to get exact solutions for the set of equations (30)-(32), we assume that $a_1 = V^{\gamma}$ where γ is any constant.

Using equations (29) and (37), we obtain the exact values of the scale factor as

$$a_1 = k^{3\gamma} e^{\frac{6\gamma}{m} \operatorname{arctanh}\left(\frac{kt}{m} - 1\right)} \quad (38)$$

$$a_2 = a_3 = k^{\frac{3(1-\gamma)}{2}} e^{\frac{6(1-\gamma)}{2m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)} \tag{39}$$

Metric (17) with the help of equations (38) and (39) can be written as

$$ds^2 = -dt^2 + \left[k^{6\gamma} e^{\frac{12\gamma}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}\right]dx^2 + \left[k^{3(1-\gamma)} e^{\frac{6(1-\gamma)}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}\right]\left(e^{2x}dy^2 + e^{-2x}dz^2\right) \tag{40}$$

Using equations (38),(39) , we get the energy density and pressure as

$$p_{WDF} = \frac{\lambda_1}{\left[\lambda_2^2 - (16\pi + 3\lambda_2)^2\right]}\left[\frac{(16\pi + 3\lambda_2)\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - \lambda_2\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]}{\left(2mt - kt^2\right)^2} + \frac{8(4\pi + \lambda_2)}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}}\right] \tag{41}$$

$$\rho_{WDF} = \frac{\lambda_1}{\left[\lambda_2^2 - (16\pi + 3\lambda_2)^2\right]}\left[\frac{\lambda_2\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - (16\pi + 3\lambda_2)\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]}{\left(2mt - kt^2\right)^2} + \frac{8(4\pi + \lambda_2)}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}}\right] \tag{42}$$

Using equations (41),(42) , we obtain the parameter ω and ρ^* as follows

$$\omega = \left\{\frac{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}\left\{(16\pi + 3\lambda_2)\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - \lambda_2\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]\right\} + 8(4\pi + \lambda_2)\left(2mt - kt^2\right)^2}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}\left\{\lambda_2\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - (16\pi + 3\lambda_2)\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]\right\} + 8(4\pi + \lambda_2)\left(2mt - kt^2\right)^2}\right\} \tag{43}$$

$$\rho^* = \frac{24\lambda_1(\gamma - 2)(4\pi + \lambda_2)}{\left[\lambda_2^2 - (16\pi + 3\lambda_2)^2\right]\left(2mt - kt^2\right)}\left[\frac{48(1-\gamma)\left[3 + (kt-m)\right]}{\left(2mt - kt^2\right)^2} + \frac{4}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}}\right] \tag{44}$$

4.1 A particular case of $f(R,T) = R + 2\lambda T$.

For values of $\lambda_1 = 1$ and $\lambda_2 = 2\lambda$ in (15) we get $f(R,T) = R + 2\lambda T$.

In this case the pressure , energy density, parameter ω and ρ^* are as follows

$$p_{WDF} = \frac{1}{4\left[\lambda^2 - (8\pi + 3\lambda)^2\right]}\left[\frac{2(8\pi + 3\lambda)\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - 2\lambda\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]}{\left(2mt - kt^2\right)^2} + \frac{16(2\pi + \lambda)}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}}\right] \tag{45}$$

$$\rho_{WDF} = \frac{1}{4\left[\lambda^2 - (8\pi + 3\lambda)^2\right]}\left[\frac{2\lambda\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - 2(8\pi + 3\lambda)\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]}{\left(2mt - kt^2\right)^2} + \frac{16(2\pi + \lambda)}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}}\right] \tag{46}$$

$$\omega = \left\{\frac{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}\left\{(8\pi + 3\lambda)\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - \lambda\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]\right\} + 8(2\pi + \lambda)\left(2mt - kt^2\right)^2}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}\left\{\lambda\left[216(1-\gamma)^2 + 48(1-\gamma)(kt-m)\right] - (8\pi + 3\lambda)\left[72(1-\gamma)^2 + 144(1-\gamma)\gamma\right]\right\} + 8(2\pi + \lambda)\left(2mt - kt^2\right)^2}\right\} \tag{47}$$

$$\rho^* = \frac{48(\gamma - 2)(2\pi + \lambda)}{4\left[\lambda^2 - (8\pi + 3\lambda)^2\right]\left(2mt - kt^2\right)}\left[\frac{48(1-\gamma)\left[3 + (kt-m)\right]}{\left(2mt - kt^2\right)^2} + \frac{4}{k_1^2 e^{\frac{4}{m}\operatorname{arctan}h\left(\frac{kt}{m}-1\right)}}\right] \tag{48}$$

For the above particular values of $\lambda_1 = 1$ and $\lambda_2 = 2\lambda$,the metric in Eqn. (17) together with eqns. (45)- (49) represents Bianchi type VI_0 cosmological model with WDF in $f(R,T)$ gravity with $f(R,T) = R + 2\mu T$, which is more general.

The spatial volume V , average scale factor a, scalar expansion θ , the mean Hubble directional Hubble parameter H shear scalar σ and anisotropy parameter Δ for the metric (17) are obtained as

The mean Hubble parameter

$$H = \frac{2(1-\gamma)}{(2mt - kt^2)}. \tag{49}$$

Expansion Scalar

$$\theta = 3H = \frac{6(1-\gamma)}{(2mt - kt^2)}. \tag{50}$$

Shear Scalar

$$\sigma^2 = \frac{6(8\gamma^2 - 10\gamma + 5)}{t^2(2m - kt)^2}. \tag{51}$$

Mean Anisotropic Parameter

$$\Delta = \frac{(8\gamma^2 - 10\gamma + 5)}{(1 - \gamma^2)}. \tag{52}$$

The universe will exhibit decelerating expansion if $q > 0$, an expansion with constant rate if $q = 0$, accelerating power law expansion if $-1 < q < 0$, exponential expansion(also known as de Sitter expansion) if $q = -1$ and super-exponential expansion if $q < -1$ (Carroll et al. 2003, Caldwell et al. 2003). In our model, the universe has finite lifetime. It starts with a big bang at $t_0 = 0$ and ends at $t_{end} = \frac{2m}{k}$. The energy density of the fluid and the scale factor diverge in finite time as $t \to t_{end} = \frac{2m}{k}$. This is the big rip behavior first suggested by Caldwell et al. (2003). We also observed that the universe begins with $q_0 = m - 1$, enters into the accelerating phase $q < 0$ at $t_a = \frac{m-1}{k}$, enters into super-exponential phase($q < -1$) at $t_{se} = \frac{m}{k}$ and ends with $q_{end} = -m - 1$. Spatial volume and average scale factor diverge in finite time , whereas expansion scalar and Hubble parameter tends to zero as $t \to \infty$. If we consider the ratio $\lim_{t \to \infty} \frac{\sigma}{\theta} = constant$ then our model does not approaches isotopically. It may also be observed that when $\gamma = 1$, $\Delta = 0$ so that the universe becomes isotropic which resembles with Bishi and Mahanta (2015). It is also observed that the EoS parameter is function of cosmic time *t* only. It is observed that energy density is decreasing function of time.

V. CONCLUSION

Bianchi type- VIo cosmological models with WDF in $f(R,T)$ gravity , where *R* is the Ricci scalar and *T* the trace of stress energy-momentum tensor in the context of late time accelerating expansion of the universe has been studied. To obtain the cosmological model we have used linearly varying deceleration parameter proposed by Akarsu and Dereli (2012). The model also shows initial singularity and finite lifetime with $t_{end} = \frac{2m}{k}$ which resembles with the investigations of S. Singh (2014) and Akarsu and Dereli (2012). It starts with a big bang and ends at a finite time. The energy density, pressure and scale factors diverge in finite time as $t \to t_{end} = \frac{2m}{k}$. This is called as Big Rip (Caldwell et. al.(2003)). We also observed that the universe begins with $q_0 = m-1$, enters into the accelerating phase $q < 0$ at $t_a = \frac{m-1}{k}$, enters into super-exponential phase($q < -1$) at $t_{se} = \frac{m}{k}$ and ends with $q_{end} = -m-1$ which resembles with Adhav et. al. (2012).

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