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Existence of Neutrino Mass : A New Mathematical Formulation

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ABSTRACT

It is assumed that there exists a transformation law, even in the standard model framework, that transforms mass eigen states of neutrino to the flavour eigen states. The transformation matrix from mass eigen states to flavor states is assumed to be singular and therefore becomes unable to generate any mass of the neutrino. It is considered that a small deviation from that singular matrix leads to unitary PMNS matrix, which is able to diagonalize the mass matrix. A formulation is done to indicate the possible spontaneous symmetry breaking which can generate physical neutrino mass in eV scale. A mapping between symmetry transformations and a set of real numbers (0,k) is defined on the backdrop of the neutrino mass generation.

Keywords: Neutrino mass, Beyond Standard Model, CKM Matrix, Mass and flavor eigen states of neutrinos.

I. INTRODUCTION

The existence of neutrino mass draws attention to the scientists and researchers as a direct consequence of the Solar as well as atmospheric neutrino problem evolving neutrino oscillation phenomenon. In the framework of standard model the neutrino is considered as the massless particle. First time Pontecorvo proposed the idea of neutrino oscillations that leads to the nonzero neutrino mass [1]. After that the solar neutrino puzzle and atmospheric neutrino anomaly became the strong evidences in favour of the existence of non-zero neutrino mass. There was a revolution in the field of neutrino physics when at the Neutrino'98 (June, 1998) conference in Takayama, Japan, the Super-Kamiokande collaboration announced the discovery of oscillations of the cosmic ray neutrinos, which would clearly indicate the existence of neutrino mass. After it becomes clear that the neutrino is not massless at all the theoreticians come forward to explain the mass generation mechanism of the neutrino beyond the standard model. There are two classes of models where introduction of lepton number violating interactions leads to radiative generation of small neutrino mass. The first one is the Zee model [2] where a charged $SU(2)_L$ singlet field η^+ is added to the standard model along with a second Higgs doublet. In the second model, called Babumodel [3], one charged field η^+ along with a doubly charged field h^{++} are added to the standard model, but no second Higgs doublet is introduced. There is another model [4] where the neutrino mass is generated without including the right handed neutrino in the theory. In this model an additional $SU(2)_L$ triplet Higgs field (Δ^{++}; Δ^{+}; Δ^{0}) is added and it breaks the lepton number by two units and lead to the Majorana mass [5] of the neutrinos. A natural and elegant way to generate the neutrino mass is to include the right handed neutrinos in the model which leads to the left right symmetric model [6, 7, 8]. In the standard model the gauge bosons as well as fermionic particles get their respective masses when the $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken down, although neutrino remains massless even after such kind of spontaneous symmetry breaking. Therefore, the neutrino is not supposed to get mass not through electro-weak symmetry breaking, but through other kind of spontaneous symmetry breaking. In 1979 Weinberg [9] introduced the dimension five operator that could generate the neutrino mass. In the framework of standard model the calculations related to the various electro-weak processes may be carried out just by putting the neutrino mass by hand and it cannot hamper the previous result since the neutrino mass is very small. But one may ask how such neutrino mass can be generated beyond the standard model theory and what may be the reason for such smallness of the neutrino mass. In the perspective of generating small Majorana neutrino mass a simple and economical minimal standard electro-weak model by addition of two heavy

Higgs scalar triplets was proposed by Ma and Sarkar [10]. There are few more, for example, mass generation in SO(4) model [11], which could explain the generation of neutrino mass.

II. EXTENSION OF THE STANDARD MODEL

To generate the neutrino mass there is no way other than the standard model of electro-weak theory to be extended, because it is not at all possible to generate the neutrino mass in the framework of it. In the neutrino mass model the mass matrix is diagonalized in such a way that the flavour eigen states of the neutrino are supposed to be the mixture of mass eigen states by the following transformation law.

$$U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}=\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \qquad (1)$$

where, U is a unitary matrix that diagonalizes the mass matrix M_ν as $U^t M_\nu^2 U = \mathrm{diag}(m^2_1, m^2_2, m^2_3)$.

It can be said that in the framework of standard model no such transformation law exists so that the mass eigen states can be transformed into the flavor states, otherwise the neutrino could get mass as a result of electro-weak symmetry breaking. In other wards within the standard model consideration no such U matrix diagonalizing the mass matrix M can exist. Therefore, one can think the transformation rule given by (1) is valid only beyond the standard model. Now the question may arise that whether there exists any other transformation between mass eigen states and flavor eigen states even in the framework of standard model in some kind of futile form. For the sake of consistency it can be assumed that there exists a transformation law between these two sets of eigen states of neutrino even in the standard model, although such transformation fails to generate the neutrino mass. Without hampering the standard model postulates transformation rule may be introduced as

$$A\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}=\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \qquad (2)$$

where, the matrix A is singular. It signifies that the transformation A fails to generate the flavor eigen states from the mass eigen states. Therefore, the transformation matrix A remains 'sterile' in the framework of standard model. Now one may assume that a small deviation from the matrix A generates the matrix U i.e.,

$$U = A + kI \qquad (3)$$

where k is taken to be small scalar. Now once the matrix U is generated the transformation between mass eigen states and flavour eigen states may be possible. Thus a small change in the transformation matrix A brings it beyond the standard model and mass generation of neutrino occurs. Let us try to construct A from the knowledge about the CKM Matrix U [12]. Using the equation (3) one can find

$$A=\begin{pmatrix} c_{12}c_{13}-k & c_{13}s_{12} & s_{13}e^{-i\delta}\\ -c_{23}s_{12}-s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12}-s_{12}s_{23}s_{13}e^{i\delta}-k & c_{13}s_{23}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-c_{13}s_{12}s_{13}e^{i\delta} & c_{23}c_{13}-k \end{pmatrix} \qquad (4)$$

where, $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$ (i, j = 1, 2, 3)

It has been found experimentally that $\theta_{12}=34.1°$, $\theta_{23}=41.6°$ and $\theta_{13}<10.3°$ [13].

Since A is a singular matrix, so equating its determinant as zero we can obtain two equations.

$$k^3-[c_{12}c_{13}+c_{12}c_{23}+c_{23}c_{13}-s_{12}s_{23}s_{13}\cos\delta]k^2+ [c_{12}c_{13}(1+c_{12}c_{23}+c_{23}c_{13})+ c_{23}c_{13}s_{12}^2-s_{12}s_{23}s_{13}+s_{13}^2c_{12}c_{23}\cos\delta]k-1=0 \qquad (5a)$$

and

$$[c_{12}c_{23}s_{13}-s_{12}s_{23}k]ks_{13}\sin\delta=0 \qquad (5b)$$

From the equation (5b) the upper bound of the k is obtained (k < .081) as the experimental value of θ_{13} is not known to us precisely. If we could know the value of θ_{13} with accuracy then not only the exact value of k could be obtained (5b), but such value could also be verified with the help of equation (5a). Let us now establish a general relation between the matrices

A and U both in and beyond the standard model by defining a well defined mapping. One can express such relation as

$$U = A + \varepsilon I \quad (6)$$

Clearly ε takes the value k beyond the standard model whereas it is equal to 0 in the framework of standard model. Therefore, there must be a relationship between the value of ε and some kind of symmetries to be broken down spontaneously. To establish such relation let us consider a set **S** of all kind of continuous local and global symmetries (spontaneously breakable). One may now define a mapping ε : **S** → $\mathbf{R}_v$ where, $\mathbf{R}_v$ is a set of real numbers containing two elements 0 and k. Now, ε is defined in such a way that for every S belonging in **S**

$$\varepsilon(S) = k \quad (7)$$

if the spontaneous breaking of S symmetry generates the neutrino mass. On the other hand if the spontaneous breaking of S symmetry fails to generate the neutrino mass it is mapped into 0.

For example,

$$\varepsilon(SU(2)_L \times U(1)_Y) = 0$$

Brahmachari et al. [14] showed that the neutrino mass generation could be possible by the spontaneous breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry and therefore such symmetry, as per our definition of ε, must be mapped into k. It is worth noting that $U(1)_{B-L}$ is a global symmetry. In the nutshell the ε, defined here, is nothing but the transformation of the symmetry transformations broken down spontaneously. Clearly such mapping is onto, but not one-to-one. Therefore, the inverse mapping does not exist.

III. CONCLUSION

When one talks about the neutrino mass that clearly implies the original small physical mass of neutrino, which is very much consistent with the experimental value. It is to be noted that if the right handed neutrino is introduced in the standard model theory then the Dirac mass of the neutrino can be generated like any other fermion, but such mass will be very large. Therefore, as per our consideration electro-weak symmetry cannot be mapped into k as it fails to produce the physical neutrino mass in the eV scale. The model developed in this article does not contradict the existing theory of neutrino mass generation mechanism as well as the electro-weak theory that fails to generate neutrino mass; rather it assures that the neutrino mass cannot be produced in the framework standard model because of the relation (2). In the true sense the physical states of neutrino remain hidden in the standard model framework. It also reveals that the extension of the standard model (in terms of small neutrino mass production) occurs through a small deviation of the matrix A that produces the matrix U, compatible to generate neutrino mass through seesaw mechanism. We can thus view the neutrino mass generation in the framework as well as beyond the standard model. In this formalism we may also get the set of symmetries which, if broken down spontaneously, are able to generate neutrino mass.

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