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Some New Integral Relation of I- Function

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ABSTRACT

This paper deals with some new integral relation of I- function of one variable.
Keywords: I- function , Multivariable polynomial.

I. INTRODUCTION

The I- function of one variable is defined by Saxena [6] and we shall represent here in the following manner:

$$I[z] = \mathrm{I}^{m,n}_{p_i,q_i:r}\left[z\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]\end{matrix}\right.\right] = \frac{1}{2\pi w}\int_L \theta(s) z^s ds, \tag{1.1}$$

where $\omega = \sqrt{(-1)}, z(\neq 0)$ is a complex variable and

$$z^s = \exp[s\{\log|z| + w \arg z\}]. \tag{1..2}$$

In which log |z| represent the natural logarithm of |z| and arg |z| is not necessarily the principle value . An empty product is interpreted as unity, also,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right]}, \tag{1.3}$$

m,n, and pi $\forall$ i $\in$ (1,.....r) are non negative integers satisfying $0 \le n \le p_i$, $0 \le m \le q_i$, $\forall$ i $\in$ (1,.....r) , α_{ji}, (j=1,….. p_i; i = 1,……r) and β_{ji} (j = 1,….q_i ; i = 1,….r) are assumed to be positive quantities for standardization purpose . Also a_{ji} (j = 1,…., p_i; i = 1,……,r) and b_{ji} (j = 1,…….., q_i; i = 1,…..,r) are complex numbers such that none of the points.

$$S = \{(bn+v)|\beta_h|\}, h = 1,.....,m; v = 0,1,2,..... \quad , \tag{1.4}$$

which are the poles of Γ ($b_n - \beta_n S$), h = 1,……m and the points.

$$S = \{(a_l - \eta - 1)|\alpha_l| l = 1,.....,n; \eta = 0,1,2,....,\tag{1.5}$$

which are the poles of $\Gamma(1-a_l+\alpha_l s)$ coincide with one another, i.e. with

$$\alpha_l(b_n+v) \neq b_n(a_l-\eta-1) \tag{1.6}$$

for n, h = 0, 1,2,….; h = 1,…., m; l = 1,…..,n.

Further, the contour L runs from - $\omega\infty$ to + $\omega\infty$. Such that the poles of $\Gamma(b_n - \beta_n s)$, h = 1……, m; lie to the right of L and the poles $\Gamma(1 - a_l + \alpha_l s)$, l = 1,….,n lie to the left of L. The integral (1.1) converges, if |arg z| < ½ B π (B>0), A ≤ 0, where

$$A = \sum_{j=1}^{p_i} a_{ji} - \sum_{j=1}^{q_i} \beta_{ji}. \tag{1.7}$$

and

$$B = \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p_i} \alpha_{ji} + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q_i} \beta_{ji}, \tag{1.8}$$

$$\forall \ \mathrm{i} \ \in (1,.....,\mathrm{r}).$$

Gradshteyin I.S., Ryzhik, I.M. [4] given table of Integrals, series, Sharma [7] evaluated the integrals involving general class of polynomial with H-function, Srivastava, H.M. and Garg, M. [9]. established some integrals involving a general class of polynomials and the multivariable H- function . Recently ,Satyanarayana, B. and Pragathi Kumar, Y. [5] has evaluated Some finite integrals involving multivariable polynomials, Agarwal, P. [1] cstablished integral involving the product of Srivastava's polynomials and generalized Mellin-Barnes type of contour integral , Bhattar and Bhargava [3] established some integral formulas involving two $\bar{H}$ - function and multivariable's general class of polynomiyals. Satyanarayana, B. and Pragathi Kumar, Y. [5] has evaluated Some finite integrals involving multivariable polynomials. Following them, i will evaluated some new integrals involving multivariable polynomials, and I-function of one variable.

II. FORMULA REQUIRED

The following formulas will be required in our investigation

(i) The second class of multivariable polynomials given by Srivastava (9) is defined as follows:

$$S_{V_1,...,V_t}^{U_1,...,U_t}[x_1,...,x_t] = \sum_{K_1=0}^{[V_1/U_1]} ... \sum_{K_t=0}^{[V_t/U_t]} (-V_1)_{U_1k_1} ... (-V_t)_{U_tk_t} \ \mathrm{A}[V_1,k_1;....;V_t,k_t] \ \frac{x_1^{k_1}}{k_1!} ... \frac{x_t^{k_t}}{k_t!}. \tag{2.1}$$

(ii) The first class of multivariable polynomials introduced by Srivastava and Garg (9) is defined as follows :

$$S_V^{U_1,...,U_t}[x_1,...,x_t] = \sum_{K_1...k_t=o}^{U_1k_1+...U_tK_t \le V} (-V_1)_{U_1k_1} ... (-V_t)_{U_tk_t} \ \mathrm{A}[V_1,k_1;....;V_t,k_t] \ \frac{x_1^{k_1}}{k_1!} ... \frac{x_t^{k_t}}{k_t!} \tag{2.2}$$

III. SOME NEW FINITE INTEGRALS FORMULAE

In this section we prove two integral formulae, which involving multivariable polynomials, and I-function of one variable.

$$\int_{-1}^{1} (1-x)^{\rho}(1+x)^{\sigma} \ \mathrm{S}_{V_1,...,V_t}^{U_1,...,U_t}[\mathrm{y}_1(1-x)^{m_1}(1+x)^{n_1},...,\mathrm{y}_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times I_{p_i,q_i:r}^{m,n}\left[z(1-x)^g(1+x)^h \left| \begin{matrix} [(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}] \\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,p_i}] \end{matrix} \right. \right] \mathrm{d}x$$

$$= 2^{\rho+\sigma+1} \sum_{K_1=0}^{[\mathrm{V}_1/\mathrm{U}_1]} ..... \sum_{K_t=0}^{[\mathrm{V}_t/\mathrm{U}_t]} (-V_1)_{U_1k_1} ... (-V_t)_{U_tK_t} \ \mathrm{A}[\mathrm{V}_1,k_1;.$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z2^{h+g} \left| \begin{matrix} (-\sigma-\sum_{i=1}^{t} m_i k_i, h;1),(-\rho-\sum_{i=1}^{t} n_i k_i\ ,g;1)[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}] \\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,q_i}](-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1) \end{matrix} \right. \right] \tag{3.1}$$

where $m_i>0$ (i=1,…,t), $n_i>0$ (i = 1,….,t) $h\geq 0$,$g\geq 0$ (not both are zero simultaneously).

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma} S_V^{U_1\ldots U_t}[y_1(1-x)^{m_1}(1+x)^{n_1},\ldots,y_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z(1-x)^{g}(1+x)^{h} \left| \begin{matrix} [(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}] \\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,p_i}] \end{matrix} \right. \right] \mathrm{d}x$$

$$=2^{\rho+\sigma+1}\sum_{k_1\ldots,k_t=0}^{U_1k_1+\ldots+U_tk_t}(-V_1)_{U_1k_1+\ldots+U_tk_t}A(V_1,k_1;\ldots;k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times I^{\mathrm{m,n+2}}_{p_i+2,\mathrm{q}_i+1:r}\left[z2^{h+g} \left| \begin{matrix} (-\sigma-\sum_{i=1}^{t} m_i k_i, h;1),(-\rho-\sum_{i=1}^{t} n_i k_i, g;1),[(a_j,\alpha_j)_{1,\mathrm{n}}],[(a_{ji},\alpha_{ji})_{n+1,\mathrm{p}_i}] \\ [(b_j,\beta_j)_{1,\mathrm{m}}],[(b_{ji},\beta_{ji})_{m+1,\mathrm{q}_i}],(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1) \end{matrix} \right. \right]. \tag{3.2}$$

Provided the conditions stated in results (3.1) are satisfied.

Proof : To establish integral in (3.1) , we express I-function occurring in its left –hand side interms of Mellin-Barnes contour integral given by (3.1), the second class of polynomial given by (2.1) . Then interchange the order of integration of summations and integration, we arrive the following :

$$\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1;\ldots;V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}$$

$$\text{x}\ \frac{1}{2\pi i}\int_L\phi(s)z^s\int_{-1}^{1}(1-x)^{\rho+gs+\sum_{i=1}^{t}m_ik_i}(1+x)^{\sigma+hx+\sum_{i=1}^{t}n_ik_i}dxds$$

$$=\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1;\ldots;V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}$$

$$\times\ \frac{1}{2\pi i}\int_L\phi(s)z^s ds\ 2^{\sigma+hs+\sum_{i=1}^{t}n_ik_i+\rho+gs+\sum_{i=1}^{t}m_ik_i+1}\frac{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+1)\Gamma(\rho+gs+\sum_{i=1}^{t}m_ik_i+1)}{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+\rho+gs+\sum_{i=1}^{t}m_ik_i+2)}$$

$$=\ 2^{\sigma+\rho+1}\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1;\ldots;V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times\ \frac{1}{2\pi i}\int_L\phi(s)z^s\frac{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+1)\Gamma(\rho+gs+\sum_{i=1}^{t}m_ik_i+1)}{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+\rho+gs+\sum_{i=1}^{t}m_ik_i+2)}$$

$$(z2^{h+g})^s ds\,.$$

Now we evaluate the above integral with help of integral (2.2). Interpreting the resulting contour integral of H-function we can easily arrive at desired result (3.1).

To establish integral in (3.2) can be easily established on the same lines similar to the proof of (3.1) .

IV. SPECIAL CASES OF (3.1) AND (3.2)

Take A $(V_1,k_1;\ldots;V_t,k_t) = A_1(V_1,k_1)\ldots A_t(V_t,k_t)$ in (3.1) the multivariable polynomial $S_{V_1\ldots V_t}^{U_1\ldots U_t}(x_1,\ldots,x_t)$ reduced to the product of well known general class of polynomials $S_v^U(x)$ and the result (3.1) reduced to following form

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}\prod_{i=1}^{t}S_{V_i}^{U_i}[y_i(1-x)^m(1+x)^n]$$

$$\times I_{p_i,q_i:r}^{m,n}\left[z(1-x)^g(1+x)^h\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,q_i}]\end{matrix}\right.\right]\mathrm{dx}$$

$$2^{\sigma+\rho+1}\sum_{k_1=0}^{[V_1/U_1]}\cdots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1\ldots,V_tk_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times I_{p_i+2,q_i+1:r}^{m,n+2}\left[z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t}m_ik_i,h,1),(-\rho-\sum_{i=1}^{t}n_ik_i,g;1)[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right] \quad (4.1)$$

(a) Substituting r=1 in (3.1), we obtain :

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}\,S_{V_1,\ldots,V_t}^{U_1,\ldots,U_t}[y_1(1-x)^{m_1}(1+x)^{n_1},\ldots,y_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times H_{p,q}^{m,n}\left[z(1-x)^g(1+x)^h\left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\ (b_j,\beta_j)_{1,q}\end{matrix}\right.\right]\mathrm{d}x$$

$$=2^{\rho+\sigma+1}\sum_{K_1=0}^{[V_1/U_1]}\cdots\cdots\sum_{K_t=0}^{[V_t/U_t]}\frac{(-V_1)_{U_1k_1}}{k_1!}\ldots\frac{(-V_t)_{U_tK_t}}{k_t!}\,A[V_1,k_1;\ldots;V_t,k_t]\,\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}\,\sum_{i=1}^{t}(m_i+n_i)\,k_i$$

$$\times H_{p,q}^{m,n}\left[z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t}m_ik_i,h;1),(-\rho-\sum_{i=1}^{t}n_ik_i\ ,g;1)(a_j,\alpha_j)_{1,p}\\ (b_j,\beta_j)_{1,q}(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right] \quad (4.2)$$

(b) Substituting $\alpha_j=\beta_j=1$ in (4.2) we obtain

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}S_{V_1\ldots V_t}^{U_1\ldots U_t}[y_1(1-x)^{m_1}(1+x)^{n_1}\ldots y_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times G_{p,q}^{m,n}\left[z(1-x)^g(1+x)^h\left|\begin{matrix}(a_j)_{1,p}\\ (b_j)_{1,q}\end{matrix}\right.\right]dx$$

$$2^{\sigma+\rho+1}\sum_{k_1=0}^{[V_1/U_1]}\cdots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1\ldots,V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times G^{m,n+2}_{p+2,q+1}\left[z2^{h+g} \middle| \begin{matrix} (-\sigma-\sum_{i=1}^{t} m_i k_i, h, 1), (-\rho-\sum_{i=1}^{t} n_i k_i, g; 1)(a_j)_{1,p} \\ (b_j)_{1,q}, (-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1, h+g; 1) \end{matrix} \right].$$

(4.3)

(a) Substituting r=1 in (3.2), we obtain :

$$\int_{-1}^{1} (1-x)^p (1+x)^q S_V^{U_1 \ldots U_t}[y_1(1-x)^m(1+x)^n, \ldots, y_t(1-x)^m(1+x)^n]$$

$$\times \mathrm{H}^{m,n}_{p,q}\left[z(1-x)^g (1+x)^h \middle| \begin{matrix} (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{matrix} \right] \mathrm{d}x$$

$$= 2^{\rho+\sigma+1} \sum_{k_1 \ldots, k_t=0}^{U_1 k_1 + \ldots + U_t k_t} (-V_1)_{U_1 k_1 + \ldots + Utkt} A(V_1, k_1; \ldots, k_t) \frac{y_1^{k_1}}{k_1!} \ldots \frac{y_t^{k_t}}{k_t!} 2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times H^{m,n+2}_{p+2,q+1}\left[z2^{h+g} \middle| \begin{matrix} (-\sigma-\sum_{i=1}^{t} m_i k_i, h, 1), (-\rho-\sum_{i=1}^{t} n_i k_i, g; 1), (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q}, (-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1, h+g; 1) \end{matrix} \right] . \quad (4.4)$$

(b) Substituting $\alpha_j=\beta_j=1$ in (4.4), we obtain

$$\int_{-1}^{1} (1-x)^\rho (1+x)^\sigma S_V^{U_1 \ldots U_t}[y_1(1-x)^m(1+x)^n, \ldots, y_t(1-x)^m(1+x)^n]$$

$$\times \mathrm{G}^{m,n}_{p,q}\left[z(1-x)^g (1+x)^h \middle| \begin{matrix} (a_j)_{1,p} \\ (b_j)_{1,q} \end{matrix} \right] \mathrm{d}x$$

$$= 2^{\rho+\sigma+1} \sum_{k_1 \ldots, k_t=0}^{U_1 k_1 + \ldots + U_t k_t} (-V_1)_{U_1 k_1 + \ldots + U_t k_t} A(V_1, k_1; \ldots, k_t) \frac{y_1^{k_1}}{k_1!} \ldots \frac{y_t^{k_t}}{k_t!} 2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times G^{m,n+2}_{p+2,q+1}\left[z2^{h+g} \middle| \begin{matrix} (-\sigma-\sum_{i=1}^{t} m_i k_i, h, 1), (-\rho-\sum_{i=1}^{t} n_i k_i, g; 1), (a_j)_{1,p} \\ (b_j)_{1,q}, (-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1, h+g; 1) \end{matrix} \right] .$$

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