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Nature of Singularities Arises in Five Dimensional Monopole Husain Space-Time

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ABSTRACT

We analyze the nature and occurrence of the singularities developed in the gravitational collapse of monopole Husain in five dimensional space-time. As a result both naked singularities and black holes are formed for final outcome of the collapse. Earlier work is generalized to monopole field to allow the effect of monopole field on the final outcome of the collapse in terms of black hole or a naked singularity. We will consider here both asymptotically flat as well as cosmological solution for this study.

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I. INTRODUCTION

The final outcome of massive star, when it collapses under its own gravity at the end of its life cycle, which is the most important issue in gravitation theory and relativistic astrophysics and cosmology today. A massive star more than five times the size of sun would undergo a continuous gravitational collapsing due to its self gravity, on exhausting its nuclear fuel, this phenomenon is called gravitational collapse. The main issue that remains open and on which currently much debate , [1] is the major question of stability of such naked singularities. The point that, even when naked singularities develop, if they are not stable or generic in suitable sense, it may not be physically realizable and physically realistic gravitational collapse will result into black hole. The problem however making any progress towards such a conclusion we have no well-defined notion of stability available presently in gravitation today.

It is known that the general theory of relativity predicts the developing of the spacetime singularities in gravitational collapse. At singularities, the spacetime curvature is enormously large and the classical general relativity theory breaks down laws there. Whether the singularities are visible or not to the observers at infinity has been debated. As physics at a spacetime singularity is not known, the existence of a naked singularity is usually expected to give serious problems as compared to a singularity which is not visible or black hole. For instance, there can be production of matter or radiation out of extremely high gravitational field and, as one knows, mechanism for that is not understood. Due to such type of reasons, naked singularities are abhorant to many physicists. The problem is observationally avoided if and only if it is assumed that no information can escape out of a singularity. Penrose [2], in a seminal review, asked, “Does there exist a cosmic censor who forbids the occurrence of naked singularities, clothing each one in an absolute event horizon?” The answer to this question is not known. Penrose [3] and also Hawking [4] considered this as the most important unsolved problem of classical general relativity theory.

The hypothesis that a physically realistic collapse will not lead to naked singularities is referred to as the cosmic censorship hypothesis (CCH) [2, 3]. Penrose [3] mentioned that unless the production of a naked singularity in a gravitational collapse is stable, the CCH remains valid. Till date there is no agreed and precise statement of a CCH. There exists in the literature some other formulations to CCH [5]. However, due to the lack of a sutaible mathematical formulation describing "a physically realistic system", no proof for any version of CCH is known.

From the point of view of CCH, one would like to know the effect of extra dimensions on the development of naked singularity. Is there any examples of a naked singularities in four dimensions go over to higher dimensions or not? Does the CCH holds in higher dimensional space time? Does the dimensions play major role in the formation of naked singularity? Such type of questions, which are to be studied in the higher dimensional gravitational physics. Number of research papers on the higher dimensional gravitational collapse have appeared so far, [6, 7, 8, 9] which proved that either naked singularities or black holes may form in the gravitational collapse depending upon the nature of initial data. It means that higher dimensional space-times are not so realistic, to verify CCH.

Husain solution of null fluid with $P = k\rho$ which has been used to study the development of a black hole with short hair[10] and can be considered as a generalization of Vaidya solution[11], also gravitational collapse of the Husain solution in four dimensional space-time[12] and found that this solution gives naked singularities under certain conditions on the mass function. Hence it would be interesting to see whether the five dimensional collapse of monopole Husain solution leads to a naked singularity or not. In the present work we generalize the earlier work in[13].

In the present paper section II and III, we give five dimensional solution to monopole Husain space time. In section IV, we discuss the nature of singularity (visible or invisible) in both the cases asymptotically flat and cosmological solution, by analyzing the outgoing radial null geodesics emanating from the central singularity when star collapsing. We summarise the paper in V section by some concluding remarks.

II. MONOPOLE HUSAIN SOLUTION IN FIVE-DIMENSIONAL SPACE-TIME

The general spherically symmetric line element in five dimensional space-time is given by[12, 14]

$$ds^2 = -\left[1 - \frac{m(u,r)}{r^2}\right]du^2 + 2dudr + r^2(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2) \tag{1}$$

as $m(u,r)$ is the mass function, and is related to gravitational energy within a given radius r. Null coordinate u represent the Eddington advanced time, as r decreases towards the future along a ray u =const., and $d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2$ is a line element on the unit 3-sphere.

Einstein field equations which governs the behaviour of the gravitational field are

$$G_{ij} = kT_{ij} \tag{2}$$

where k is the gravitational constant and T_{ij} is the energy momentum tensor of the matter.

The corresponding energy momentum tensor can be written as[11, 14, 15]

$$T_{ij} = T_{ij}^{(n)} + T_{ij}^{(m)} \tag{3}$$

And

$$T_{ij}^{(n)} = \sigma l_i l_j \tag{4}$$

also

$$T_{ij}^{(m)} = (\rho + P)(l_i\eta_j + l_j\eta_i) + Pg_{ij} \tag{5}$$

where ρ and P are the energy density and thermodynamics pressure, σ is the energy density corresponding to Vaidya null radiation. l_i and η_j are linearly independent future pointing light-like vectors (null vectors) having components

$$l_i = \delta_i^0, \qquad \eta_i = \frac{1}{2}\left(1 - \frac{m}{r^2}\right)\delta_i^0 - \delta_i^1 \tag{6}$$

$$l_\lambda \eta^\lambda = -1 \qquad l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0 \tag{7}$$

We consider the general case of EMT of equation (5). Strong and weak energy conditions for ρ, P and σ are given by[11, 16]

(a) Weak and strong energy condition:

$$\sigma > 0\,, \quad \rho \geq 0\,, \quad P \geq 0. \tag{8}$$

(b) Dominant energy conditions:

$$\sigma > 0\,, \; P > 0, \quad \rho > P \quad . \tag{9}$$

for the metric (1) with matter field having stress energy given by (3) are

$$\rho = \frac{3m'}{2kr^3} \tag{10}$$

$$P = \frac{-m''}{2kr^2} \tag{11}$$

and

$$\sigma = \frac{3\dot{m}}{2kr^3} \tag{12}$$

where dot and dash stand for differentiation with respect to u and r respectively.

III. MONOPOLE HUSAIN SPACE-TIME IN FIVE DIMENSION

Mass function of A. Wang in 4D is given by

$$m(u,r) = ar \qquad 0 < a < 1$$

Where a is any arbitrary constant.

Mass function in five dimensional Monopole Husain space-time is given by

$$m(u,r) = ar^2 \qquad 0 < a < 1 \tag{13}$$

Monopoles are formed due to a guaze symmetry breaking and have many more properties of elementary particles. Most of their energy is concentrated in a small region near Monopole score.

Following references [14, 15], we define the mass function in 5D Husain solution as given below

$$m(u,r) = \begin{cases} f(u) - \frac{g(u)}{(3k-1)r^{3k-1}}, & k \neq \frac{1}{3}, \\ f(u) + g(u) \ln \quad r, & k = \frac{1}{3}, \end{cases} \tag{14}$$

$f(u)$ and $g(u)$ are arbitrary functions and which are restricted by the energy conditions.

In particular combining the mass functions (13) and (14) ,we obtain the mass function for Monopole Husain space-time as

$$m(u,r) = \begin{cases} ar^2 + f(u) - \frac{g(u)}{(3k-1)r^{3k-1}}, & k \neq \frac{1}{3}, \\ ar^2 + f(u) + g(u) \ln \quad r, & k = \frac{1}{3}, \end{cases} \tag{15}$$

Hence using mass function (15) , five dimensional monopole Husain space-time can be written as

$$ds^2 = -\left[1 - a - \frac{f(u)}{r^2} + \frac{g(u)}{(3k-1)r^{3k+1}}\right] du^2 + 2dudr + r^2\left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2\right) \tag{16}$$

The above metric may also be called as five , because if we set $g(u) = 0$ in equation (16) , then the solution reduces to five dimensional monopole Vaidya metric.

It can be observed that equation (16) is asymptotically flat for $k > \frac{1}{3}$ and cosmological for $< \frac{1}{3}$.

IV. NATURE OF THE SINGULARITY

We now investigate the nature of singularity in both the cases (i) asymptotically flat and (ii) cosmological solution. To investigate the nature of singularity, we follow the method given in references [17]. The central singularity is said to be naked, if the radial null geodesic equation admits at least one real and positive root [17, 18].

The outgoing radial null geodesic equation for the metric (16) is given by

$$\frac{dr}{du} = \frac{1}{2}\left[1 - a - \frac{f(u)}{r^2} + \frac{g(u)}{(3k-1)r^{3k+1}}\right] \tag{17}$$

The above equation does not yield analytic solution for $g(u)$.

However, if one select, $f(u) \propto u^2$, and $g(u) \propto u^{3k+1}$ then the equation (17) becomes homogeneous and can be solved in terms of the elementary function [19].

Therefore we take

$f(u) = \lambda u^2$, and $g(u) = \delta u^{3k+1}$

With the proper choice mass function, monopole Husain space-time metric (16) becomes

$$ds^2 = -\left[1 - a - \frac{\lambda u^2}{r^2} + \frac{\delta u^{3k+1}}{(3k-1)r^{3k+1}}\right] du^2 + 2dudr + r^2(d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3^2) \tag{18}$$

To analyze the structure of gravitational collapse, we need to consider radial null geodesics defined by $ds^2 = 0$, taking $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0$ into consideration.

For metric (18), the radial null geodesics must satisfy the null condition

$$\frac{du}{dr} = \frac{2}{\left[1 - a - \frac{\lambda u^2}{r^2} + \frac{\delta u^{3k+1}}{(3k-1)r^{3k+1}}\right]} \tag{19}$$

If we observed the equation (19) that this equation admits a singularity at $r \to 0$, and $u \to 0$. In order to discuss the radial and non-radial outgoing non-space like geodesics terminating at this singularity in the past, we need to consider the limiting value of $X = \frac{u}{r}$ along a singular geodesic as the singularity is approached [18].

Thus we must have $$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{u}{r} = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} \tag{20}$$

$$= \lim_{\substack{u\to 0\\ r\to 0}} \frac{2}{1 - a - \frac{\lambda u^2}{r^2} + \frac{\delta u^{3k+1}}{(3k-1)r^{3k+1}}}$$

$$X_0 = \frac{2}{1 - a - \lambda X_0^2 + \frac{\delta X_0^{3k+1}}{(3k-1)}}$$

i.e. $$\frac{\delta X_0^{3k+2}}{(3k-1)} - \lambda X_0^3 - aX_0 + X_0 - 2 = 0$$

$$\frac{\delta X_0^{3k+2}}{(3k-1)} - \lambda X_0^3 + X_0(1 - a) - 2 = 0, \qquad 0 < a < 1 \tag{21}$$

Case I: Let us take $k > \frac{1}{3}$, for a proper choice we take $k = \frac{3}{4}$ in equation (21) we gate

$$\frac{4\delta}{5} X_0^{17/4} - \lambda X_0^3 + X_0(1 - a) - 2 = 0 \tag{22}$$

For fix $\lambda = 0.001$ and we take different values for the parameter a and δ then the roots of equation (22) will be obtained for five dimensional monopole Husain space-time are given in the following table.

Table 1. Values of X_0 for different values of δ

	X_0			
δ	**a=0.1**	**0.25**	**0.5**	**0.75**
0.01	1.415	1.435	1.462	1.483
0.02	1.363	1.381	1.405	1.425
0.03	1.334	1.35	1.373	1.392
0.04	1.313	1.329	1.351	1.369
0.05	1.297	1.313	1.334	1.351
0.06	1.284	1.299	1.32	1.337
0.07	1.274	1.288	1.308	1.325
0.08	1.264	1.279	1.298	1.315
0.09	1.256	1.27	1.29	1.306

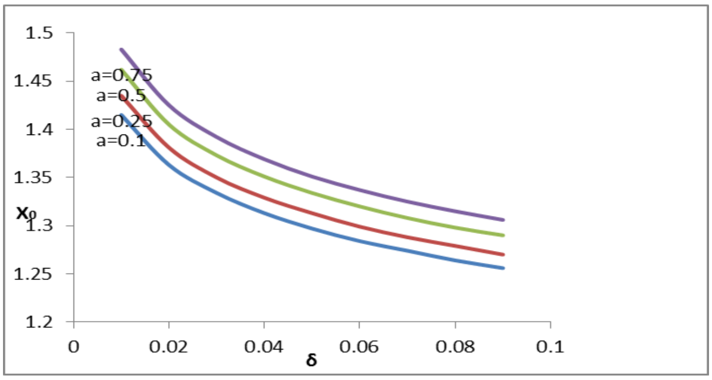

Figure 1. Graph of the values of X0 against the parameter δ for fixed values of λ.

From the graph we may observe that for $= 0.1$, if we increase the value of δ, the value of X_0 decreases exponentially and then decrease gradually. Also for $a = 0.5$ and $a = 0.75$, X_0 is at peak and values of X_0 decreases very smoothly. It is observed that peak shifted towards higher values of δ.

For fix $\delta = 0.1$ and we take different values for the parameter a and λ then the roots of equation (22) will be obtained for five dimensional monopole Husain space-time are given in the following table.

Table 2. Values of X_0 for different values of λ

	X_0			
λ	a=0.1	0.25	0.5	0.75
0.1	1.263	1.275	1.293	1.307
0.2	1.276	1.287	1.303	1.316
0.3	1.288	1.298	1.312	1.324
0.4	1.299	1.308	1.321	1.332
0.5	1.309	1.317	1.329	1.339
0.6	1.318	1.325	1.336	1.346
0.7	1.327	1.333	1.343	1.353
0.8	1.334	1.341	1.35	1.359
0.9	1.342	1.348	1.357	1.365

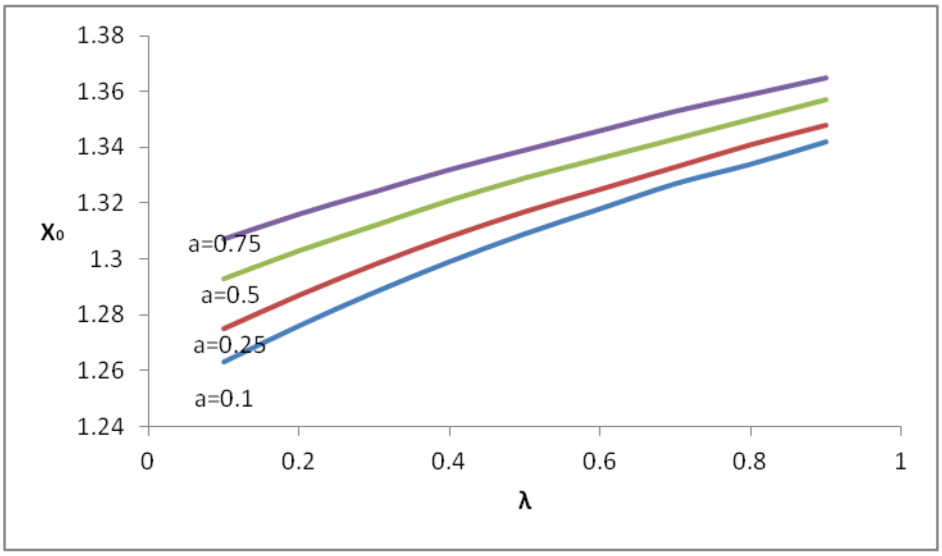

Figure 2. Graph of the values of X0 against the parameter λ for fixed values of δ.

From the graph we may observe that for $= 0.1$, if we increase the value of λ, the value of X_0 increase exponentially . Also for $a = 0.5$ and $a = 0.75$, X_0 is at peak and values of X_0 increases very smoothly. It is observed that lower values of X_0 shifted towards higher values of λ.

Case-II: for $k < \frac{1}{3}$, then the space-time will become cosmological

Let us take $= \frac{1}{4}$, to get analytical solution,

we choose $f(u) = \lambda u^2$, and $g(u) = \delta u^{7/4}$ with this proper choices, the mass function (22) becomes

$$m(u,r) = ar^2 + \lambda u^2 + \frac{1}{4}\delta u^{7/4} r^{1/4} \tag{23}$$

using this mass function the five dimensional cosmological monopole Husain solution can be written as

$$ds^2 = -\left[1 - a - \frac{\lambda u^2}{r^2} - \frac{1}{4}\frac{\delta u^{7/4}}{r^{7/4}}\right] du^2 + 2du dr + r^2(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2) \tag{24}$$

for the space-time (24) the outgoing radial null geodesics must satisfy the null condition i.e. $ds^2 = 0$, taking $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0$ into consideration.

Therefore, $$\frac{du}{dr} = \frac{2}{\left[1 - a - \frac{\lambda u^2}{r^2} - \frac{1}{4}\frac{\delta u^{7/4}}{r^{7/4}}\right]} \tag{25}$$

To determine the nature of singularity at $r = 0,\ u = 0$

Therefore

$$X_0 = \lim_{\substack{u \to 0 \\ r \to 0}} \frac{u}{r} = \lim_{\substack{u \to 0 \\ r \to 0}} \frac{du}{dr} = \frac{2}{1 - a - \lambda X_0^2 - \frac{1}{4}\delta X_0^{7/4}} \tag{26}$$

$$\lambda X_0^3 + \frac{1}{4}\delta X_0^{11/4} - (1 - a)X_0 + 2 = 0 \tag{27}$$

In particular $\lambda = 0.01,\quad \delta = 0.001$ and $a = 0.5$ then equation (27) has solution $X_0 = -2.676$ this is non-positive real root, It ensures that singularity develop into black hole and hence which supports to the CCH.

V. CONCLUDING REMARKS

Gravitational collapse of a star is one of the most interesting subjects in gravitational physics. It is well known that formation of singularity is inevitable in complete gravitational collapse. It was conjectured that such type of a singularity should be hidden by horizons if it is formed from generic initial data with physically reasonable matter fields [2].

In the present work we finding the nature of singularity in presence of monopole field with 5D Husain space time. Here we consider both the cases asymptotically flat and cosmological solution for this study by imposing some conditions on mass function. It has been found that naked singularities developed in the case of asymptotically flat solution it implies that this case violates the CCH. On the other hand when we studied the second case cosmological solution of 5D monopole Husain space-time it is found that the formation of singularities are a black hole i.e. non positive real roots will be obtained for this case.

Thus we may conclude that there is no effect of monopole charge on 5D Husain space-time in the case of asymptotically flat solution, whereas the monopole charge will effect in the case of cosmological solution, hence monopole charge plays important role in cosmological solution. Occurrence of naked singularities in 5D monopole Husain space-time suggest that monopole Husain solution violates the cosmic censorship hypothesis.

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