Inhomogeneous Spherical Symmetric Models with Quark and Strange Quark Matter and Varying Cosmological Term Λ

Rajani Shelote and Narendra Gharad *

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Abstract

In this paper we have studied a class of inhomogeneous spherical symmetric space-time possessing the varying cosmological term with quark and strange quark matter.

Key word: Cosmology, Strange quark matter

1 INTRODUCTION

It was suggested that the quark matter composed of comparable members of u, d and s quarks may be the true ground state of matter which is stable at zero pressure and temperature [Witten (1984), Farhi and Jaffe (1984), Modsen and Haensel (1991)] in which case some or all neutron stars can turn out to be so-called strange stars [Witten (1984), Farhi and Jaffe (1984), Modsen and Haensel (1991), Haensel et al. (1986), Alock et al. (1986)]. If on the other hand strange

*Department of Mathematics, Sant Gadge Maharaj Mahavidyalaya, Hingna Nagpur (INDIA) email:rajanishelote@gmail.com

Department of Physics, Jawaharlal Nehru College, Wadi Nagpur

matter is only metastable, the high pressure in the central regions of neutron stars may lead to formation of hybrid stars, having strange matter cores.

The possibility of the existence of quark matter dates back to early seventies Bodmer (1971) and Witten (1984) proposed two ways of formation of strange matter, the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities.

Typically, strange quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume [Farhi and Jaffe (1984)]. In this model, quark are thought as degenerate Fermi gases, which exist only in a region of space endowed with a vacuum energy density B_c (called as a bag constant). Also, in the framework of this model, the quark matter is composed of massless u, d quark, massive s quarks and electrons. In the simplified version of this model, on which our study is based, quarks are massless and noninteracting. Then we have quark pressure $p_q = \frac{\rho_q}{3}$ (ρ_q is the quark energy density), the total energy density $\rho = \rho_q + B_c$ and total pressure $p = p_q - B_c$. One therefore gets equation of state for strange quark matter [Kapusta (1994)]:

$$p = \frac{1}{3}(\rho - 4B_c). \tag{1}$$

In this study by considering the isotropic but inhomogeneous spherically symmetric Lemaitre-Tolman-Bondi (LTB) universe, we have found the coefficients of the LTB metric assuming the early universe possessed a time varying cosmological term with quark and strange quark matter.

2 4D cosmological model

Let us consider the isotropic but inhomogeneous spherically-symmetric Lemaitre-Tolman-Bondi metric of the form

$$ds^2 = -dt^2 + Y^{'2}dr^2 + Y^2(d\theta^2 + sin^2\theta d\phi^2), \quad (Y = Y(r,t)) \tag{2}$$

(the prime denotes partial derivative with respect to the radial coordinate r), the source of the metric being a material perfect fluid of energy density $\rho(r,t)$ and pressure $P(r,t)$ plus the quantum vacuum.

The energy momentum tensor reads

$$T_{ab} = (\rho + P)\mu_a\mu_b + \left(P - \frac{\Lambda}{8\pi G}\right) g_{ab}, \tag{3}$$

where $u_a = \delta^t_a$ is the fluid four velocity.

Einstein field equations of the LTB metric Eq.(2) with the help of Eq.(3) takes the form

$$\rho + \Lambda = \frac{1}{Y^2 Y'}(\dot{Y}^2 Y)', \tag{4}$$

$$P - \Lambda = -\frac{1}{Y^2 \dot{Y}}(\dot{Y}^2 Y)^{\cdot}, \tag{5}$$

$$\frac{\ddot{Y}}{Y} + \frac{\dot{Y}^2}{Y^2} - \frac{\ddot{Y}'}{Y'} - \frac{\dot{Y}\dot{Y}'}{YY'} = 0, \tag{6}$$

where the upper dot means partial derivative with respect to t and we have set $8\pi G = 1$.

Introducing the change of variables as

$$Y = f^{\frac{2}{3}},$$

Therefore Eq.(4) to Eq.(6) becomes

$$\rho + \Lambda = \frac{4\dot{f}\dot{f}'}{3ff'}, \tag{7}$$

$$P - \Lambda = -\frac{4\ddot{f}}{3f}, \tag{8}$$

and

$$\ddot{f}' f - f' \ddot{f} = 0, \tag{9}$$

respectively.

Eq.(9) can be expressed as

$$\ddot{f} - F(t)f = 0, \tag{10}$$

where $F(t)$ is a function of t and does not depend on the radial coordinate. Introducing f of the form

$$f(r,t) = R(r)T(t), \tag{11}$$

in Eq.(10) we get

$$\ddot{T} - FT = 0. \tag{12}$$

By substituting the value of f from Eq.(11), Eq.(7) and Eq.(8) becomes

$$\rho = \frac{4}{3}\left(\frac{\dot{T}}{T}\right)^2 - \Lambda, \tag{13}$$

and

$$P = -\frac{4}{3}\frac{\ddot{T}}{T} + \Lambda, \tag{14}$$

respectively.

To incorporate to this system, we used equation of state in the form of strange quark matter Eq.(1).

To get an equation for T we assume that the equation of state Eq.(1) with the help of Eq.(13) and Eq.(14) we get

$$T\ddot{T} + \frac{1}{3}\dot{T}^2 - [\Lambda + B_c]T^2 = 0. \tag{15}$$

To solve Eq.(15) by considering the change of variables as

$$T = Z^{\frac{3}{4}}, \tag{16}$$

we get

$$\ddot{Z} - \frac{4}{3}[\Lambda + B_c]Z = 0, \tag{17}$$

To solve Eq.(17) we consider following two cases for Λ: Case (i) $\Lambda = constant$, Case (ii) $\Lambda = \Lambda(t)$.

3 Case (i): Λ = constant

For this case from Eq.(17) we get

$$Z_1 = C_1 \cosh\left(2\sqrt{\frac{\Lambda + B_c}{3}}t + \psi_1\right), \tag{18}$$

$$Z_2 = C_2 \sinh\left(2\sqrt{\frac{\Lambda + B_c}{3}}t + \psi_2\right), \qquad (19)$$

where C_1, C_2, ψ_1 and ψ_2 are arbitrary constants.

And therefore

$$Y_1 = R^{\frac{2}{3}}(r)C_1^{\frac{1}{2}} \cosh^{\frac{1}{2}}\left(2\sqrt{\frac{\Lambda + B_c}{3}}t + \psi_1\right), \qquad (20)$$

$$Y_2 = R^{\frac{2}{3}}(r)C_2^{\frac{1}{2}} \sinh^{\frac{1}{2}}\left(2\sqrt{\frac{\Lambda + B_c}{3}}t + \psi_2\right). \qquad (21)$$

It is observed that Eq.(20) does not present initial singularity, but solution Eq.(21) has a singularity at $t_0 = \frac{-\sqrt{3}\psi_2}{2\sqrt{\Lambda + B_c}}$. However both sets of solutions have a final inflationary stage.

4 Case (ii): $\Lambda = \Lambda(t)$

Here we consider four different values for $\Lambda(t)$: $\Lambda(t) \propto t^{-2}$, $\Lambda(t) = \lambda_0 + ct^{n-2}$, $\Lambda(t) = \lambda_0 + ce^{-\alpha t}$ and $\Lambda(t) \propto \rho$.

4.1 Case (a): $\Lambda(t) \propto t^{-2}$

We take

$$\Lambda(t) = \frac{\lambda_0}{t^2}, \qquad (22)$$

where λ_0 is constant of proportionality.

By using Eq.(22), Eq.(17) become

$$\ddot{Z} - \frac{4}{3}[B_c + \lambda_0 t^{-2}]Z = 0, \qquad (23)$$

and the general solution can be expressed as a combination of Bessel functions

$$Z = t^{\frac{1}{2}}\left[C_3 J_{a_0}\left(2\sqrt{\frac{-B_c}{3}}t\right) + C_4 J_{-a_0}\left(2\sqrt{\frac{-B_c}{3}}t\right)\right], \qquad (24)$$

where $a_0 = \sqrt{\frac{1}{4} + \frac{4\lambda_0}{3}}$, C_3 and C_4 are arbitrary constants.

The behavior at the asymptotic limits depends on a_0.

For $0 < a_0$ one has the following

(i) when $t \to 0$ one obtains

$$Z \sim C_3 t^{\frac{1}{2}+a_0} + C_4 t^{\frac{1}{2}-a_0}. \tag{25}$$

(ii) when $t \to \infty$, one obtains the following asymptotic behaviour

$$Z \sim \cos(t + \psi_1). \tag{26}$$

4.2 Case (b): $\Lambda(t) = \lambda_0 + ct^{n-2}$, ($\lambda_0 = -B_c$ (say), $n \neq 2$)

By using above, case Eq.(17) becomes

$$\ddot{Z} - \frac{4c}{3} t^{n-2} Z = 0, \tag{27}$$

and the general solution can be expressed as a combination of Bessel functions

$$Z = t^{\frac{1}{2}} \left[C_5 J_{\frac{1}{n}} \left(\frac{4}{n} \sqrt{\frac{-c}{3}} t^{\frac{n}{2}} \right) + C_6 J_{\frac{-1}{n}} \left(\frac{4}{n} \sqrt{\frac{-c}{3}} t^{\frac{n}{2}} \right) \right], \tag{28}$$

where C_5 and C_6 are arbitrary constants.

The behavior at the asymptotic limits depends on n.

For $0 < n < 2$ one has the following

(i) when $t \to 0$ one obtains

$$Z \sim C_5 t + C_6. \tag{29}$$

One can choose $C_6 = 0$ to have initial singularity at $t = 0$.

(ii) When $t \to \infty$, there follows

$$Z \sim t^{\frac{1}{2}-\frac{n}{4}} \cos(t^{\frac{n}{2}} + \psi). \tag{30}$$

For $n < 0$ one has the following:

(i) when $t \to 0$ one obtains $Z \sim t^{\frac{1}{2}-\frac{n}{4}} \cos(t^{\frac{n}{2}} + \psi)$.

(ii)when $t \to \infty$ one obtains $z \sim t$.

4.3 Case (c): $\Lambda(t) = \lambda_0 + ce^{-\alpha t}$, ($\lambda_0$, c and α are constant)

In this case, Eq.(17) becomes

$$\ddot{Z} - \frac{4}{3}\left[(B_1 + ce^{-\alpha t}\right] Z = 0, \tag{31}$$

where $B_1 = B_c + \lambda_0$.

The general solution can be expressed as a combination of Bessel functions

$$Z = C_7 J_{\frac{4}{\alpha}\sqrt{\frac{B_1}{3}}}\left(\frac{4}{\alpha}\sqrt{\frac{-c}{3}}e^{\frac{-\alpha t}{2}}\right) + C_8 J_{\frac{-4}{\alpha}\sqrt{\frac{B_1}{3}}}\left(\frac{4}{\alpha}\sqrt{\frac{-c}{3}}e^{\frac{-\alpha t}{2}}\right), \tag{32}$$

where C_7 and C_8 are arbitrary constants, with

$$C_8 = \frac{-C_7 J_{\frac{4}{\alpha}\sqrt{\frac{B_1}{3}}}\left(\frac{4}{\alpha}\sqrt{\frac{-c}{3}}\right)}{J_{\frac{-4}{\alpha}\sqrt{\frac{B_1}{3}}}\left(\frac{4}{\alpha}\sqrt{\frac{-c}{3}}\right)}, \tag{33}$$

in order to fix the initial singularity at t = 0.

The asymptotic behavior near the initial singularity, when $t \to 0$, is given by

$$Z \sim t. \tag{34}$$

When $t \to \infty$ and $\Lambda \to \lambda_0$, one obtains the following asymptotic behavior

$$Y \approx R^{\frac{2}{3}}(r)e^{Xt}, \tag{35}$$

where $X = \sqrt{\frac{B_c+\lambda_0}{3}}$.

Besides, from Eq.(21) we recover the same result in the far future.

For the particular case $\lambda_0 = -B_c$ the general solution of Eq.(31) is given by

$$Z = C_7 J_o\left(\frac{4}{\alpha}\sqrt{\frac{-c}{3}}e^{\frac{-\alpha t}{2}}\right) + C_8 Y_0\left(\frac{4}{\alpha}\sqrt{\frac{-c}{3}}e^{\frac{-\alpha t}{2}}\right), \tag{36}$$

where Y_0 is the Weber function of the second kind of zero order.

In the limit $t \to \infty$ and $\Lambda \to 0$ the final behavior of the solutions, obtained from Eq.(34) are

$$Y \approx R^{\frac{2}{3}}(r)t^{\frac{1}{2}}. \tag{37}$$

The same result can be easily obtained from (17) by setting $\Lambda = -B_c$.

4.4 Case (d): $\Lambda(t) \propto \rho$

we consider

$$\Lambda = \alpha_1 \rho, \tag{38}$$

where α_1 is constant of prportionality.

Then Eq.(13) and Eq.(14) reduces to

$$\rho = \frac{4}{3(1+\alpha_1)} \left(\frac{\dot{T}^2}{T^2} \right) \tag{39}$$

and

$$p = -\frac{4}{3}\frac{\ddot{T}}{T} + \frac{4\alpha_1}{3(1+\alpha_1)}\frac{\dot{T}^2}{T^2}. \tag{40}$$

By using Eq.(1), we reduce Eq.(39) and Eq.(40) as

$$T\ddot{T} + \frac{1-3\alpha_1}{3(1+\alpha_1)}\dot{T}^2 - B_c T^2 = 0. \tag{41}$$

We solve this equation by change of variables

$$T = Z^{\frac{3(\alpha_1+1)}{4}}, \tag{42}$$

By using Eq.(42), Eq.(41) becomes

$$\ddot{Z} - \frac{4B_c}{3(\alpha+1)} Z = 0. \tag{43}$$

On solving Eq.(43), one obtains

$$Z_1 = C_9 cosh \left(2\sqrt{\frac{B_c}{3(\alpha+1)}} t + \psi_1 \right) \tag{44}$$

or

$$Z_2 = C_{10} sinh \left(2\sqrt{\frac{B_c}{3(\alpha+1)}} t + \psi_2 \right) \tag{45}$$

where C_9 and C_{10} are arbitrary constants.

And therefore

$$Y_1 = R^{\frac{2}{3}}(r) C_9^{(\frac{\alpha+1}{2})} cosh^{(\frac{\alpha+1}{2})} \left(2\sqrt{\frac{B_c}{3(\alpha+1)}} t + \psi_1 \right), \tag{46}$$

$$Y_2 = R^{\frac{2}{3}}(r)C_{10}^{(\frac{\alpha+1}{2})}sinh^{(\frac{\alpha+1}{2})}\left(2\sqrt{\frac{B_c}{3(\alpha+1)}}t+\psi_2\right). \tag{47}$$

It is observed that Eq.(46) does not present initial singularity, but solution Eq.(47) has a singularity at $t_0 = -\psi_2\sqrt{\frac{3(\alpha+1)}{4B_c}}$. However both sets of solutions have a final inflationary stage.

To investigate the singular structure of the LTB metric Eq.(2), we can easily calculate the curvature scalar $\Re$ by resorting to change of variables $Y = f^{\frac{2}{3}}$ we get

$$\Re = 2\frac{\ddot{f}}{f} + \frac{4\dot{f}\dot{f}'}{3ff'} + 2\frac{\ddot{f}'}{f'}, \tag{48}$$

and we solve it at the points where the coefficients of the metric $Y^{'2}$ and/or Y^2 vanish. To do this we insert Einstein equation Eq.(9) along with Eq.(11), Eq.(12) and Eq.(13) in Eq.(48), obtaining

$$\Re = 4(\Lambda + B_c). \tag{49}$$

5 Conclusion

In this paper we have found the coefficient of LTB metric by assuming the quark and strange quark matter. We have discussed the behavior of strange quark matter for the different values of cosmological term Λ i.e. Λ is constant and Λ is variable.

For the case (i) $\Lambda =$ constant, it is observed that Eq.(20) does not present initial singularity but Eq.(21) has singularity at $t_0 = \frac{-\psi_2\sqrt{3}}{2\sqrt{B_c+\Lambda}}$ and gives asymptotically to exponential inflation - see Eq.(20), Eq.(21).

For the case (ii) $\Lambda = \Lambda(t)$, we have discussed the four different cases: Case (a) to Case (d). For the Case (c) and Case (d), it is observed that the solutions gives asymptotically to exponential inflation - see Eq.(35), Eq.(46), Eq.(47). For the case (c), it is also observed that for $\lambda_0 = -B_c$ their exist solution of the form $Y \propto t^{\frac{1}{2}}$. Again for the Case (d), it is observed that Eq.(47) does not present

initial singularity but Eq.(48) has singularity at $t_0 = -\psi_2\sqrt{\frac{3(\alpha+1)}{4B_c}}$.
It is observed that all the solutions we have derived for the above both the cases contains an arbitrary function of the radial coordinate. All the solutions have singularity at $t = 0$, i.e. big-bang singularity except Eq.(20) and Eq.(47). Constant as well as varying cosmological terms give rise asymptotically to exponential inflation i.e. Eq.(20), Eq.(21), Eq.(35), Eq.(46) and Eq.(47). None of the solutions found has aspatially-homogeneous limit for $t \to \infty$.

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