

# A Systematic Investigation of Shape Transition in Hot Rotating 98Sr

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# ABSTRACT

A statistical theory incorporating deformation, collective and non collective rotational degrees of freedom, temperature and angular momentum is used to study the highly excited fused compound system. Temperature and spin induced shape transitions are investigated using statistical theory. We have carried out a systematic theoretical investigation of shape transition in <sup>98</sup>Sr. The prediction of spherical collective shape to triaxial deformed shape at low angular momentum and non collective oblate shape at higher angular momentum are very much significant in this work

Keywords: Shape transition Triaxial deformation.

## I. INTRODUCTION

Experimental investigation on the possibility of heating the nucleus to a finite temperature opens for us a new dimension in the study of nuclear structure. In Heavy ion fusion reactions hot (highly excited) residual nuclei with finite angular momentum can be formed leading to interesting problems of shape evolution in the dual space of angular momentum and temperatures. The main experimental techniques used in the formation of such highly excited nuclei are heavy-ion reactions. In heavy ion reactions, one of the main interests is the formation of fused systems in highly excited states [1-13]. The large energy dissipation of the relative kinetic energy into the intrinsic degrees of freedom, and the transfer of relative high angular momentum as collective rotation of the system are the salient features of these reactions. The behaviour of these highly excited fast rotating compound systems [9-13] provides important and interesting problems for the theoretical investigations. Theoretically such finite temperature effects have been studied using, mean field theories such as the microscopic Hartree-FockBogoliubov cranking theory

[14] and Landau theory [15] and Mottelson Nilsson and Nilsson Strutinsky [16].

There is considerable interest in the study of the structure of nuclei in the mass region  $A\approx100$  due to the onset of deformation in the neutron rich nuclei with N>60. These neutron rich nuclei are of special interest because they are just at the border between a rather spherical and a well deformed shape. Theoretically, the region has been studied using interacting boson model [17], Nilsson Strutinsky Cranking method [18], statistical theory[18-19] and and Hartree-Fock-Bogolyubov [20].

In the present work the statistical theory [21-26] incorporating deformation, collective and non collective rotational degrees of freedom, temperature and angular momentum is used to study the highly excited fused compound system. Temperature and spin induced shape transitions are investigated using statistical theory. We shall concentrate on the temperature range T = 1 - 3 MeV where the evolution of the nuclear shape with temperature and spin and in particular the possible shape transitions are of interest.

### **II. THEORETICAL FORMALISM**

The eigenvalues generated by Cranked Nilsson Hamiltonian with finite values of the collective frequency  $\omega$  are used here. The corresponding partition function is given by

$$Q(\omega, \alpha_z, \alpha_n, \beta) = \sum_{E_i N_i, Z_i} \exp(\alpha_z Z_i + \alpha_n N_i - \beta E_i(\omega))$$
 1

The Lagrangian multipliers  $\alpha_z$ , and  $\alpha_n$  conserve the proton and neutron numbers at a given temperature  $T = 1/\beta$ . The corresponding equations in terms of the single particle energies  $\varepsilon_i(\omega)$  are given by

$$\langle \mathbf{N} \rangle = \sum_{i} n_{i}^{n} = \sum_{i} \left\{ 1 + \exp(-\alpha_{n} + \beta \varepsilon_{i}^{n}(\omega)) \right\}^{-1}$$

$$\langle \mathbf{Z} \rangle = \sum_{i} z_{i}^{n} = \sum_{i} \left\{ 1 + \exp(-\alpha_{z} + \beta \varepsilon_{i}^{z}(\omega)) \right\}^{-1}$$

$$\mathbf{M} = \mathbf{M}_{z} + \mathbf{M}_{n}$$

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The total angular momentum is given by

 $\mathbf{F} - \mathbf{F} + \mathbf{F}$ 

where

$$\mathbf{M}_{n} = \mathbf{M}(\boldsymbol{\varepsilon}_{i}^{n}(\boldsymbol{\omega})) = \sum_{i} n_{i}^{n} m_{i}^{n} \text{ and } \mathbf{M}_{z} = \mathbf{M}(\boldsymbol{\varepsilon}_{i}^{z}(\boldsymbol{\omega})) = \sum_{i} n_{i}^{z} m_{i}^{z}$$

The system of simultaneous non-linear Eqs. (2), (3) and (4) has to be solved to determine  $\alpha_z$ , and  $\alpha_n$  for each value of  $\omega$  and T. The total energy of the system is obtained as

Where

$$E = E_z + E_n$$
  

$$E_z = E_z(\varepsilon_i^z(\omega)) = \sum_i z_i^z \varepsilon_i^z(\omega) + \hbar \omega \sum_i n_i^z m_i^z$$

and

$$\mathbf{E}_{n} = \mathbf{E}_{n}(\boldsymbol{\varepsilon}_{i}^{n}(\boldsymbol{\omega})) = \sum_{i} n_{i}^{n} \boldsymbol{\varepsilon}_{i}^{n}(\boldsymbol{\omega}) + \hbar \boldsymbol{\omega} \sum_{i} n_{i}^{n} m_{i}^{n}$$
 5

The excitation energy of the system is given by

$$\mathbf{E}^* = \mathbf{E}_n^* + \mathbf{E}_z^*$$

Where

$$\mathbf{E}_{z}^{*} = \sum_{i} n_{i}^{z} \varepsilon_{i}^{z}(\omega) - \sum_{i} \varepsilon_{i}^{z}(0) \text{ and } \mathbf{E}_{n}^{*} = \sum_{i} n_{i}^{n} \varepsilon_{i}^{n}(\omega) - \sum_{i} \varepsilon_{i}^{n}(0)$$

The necessity of renormalizing the total energy does not arise here, since we are interested only in the energy difference between the excited and the ground states of the system and not in the actual magnitude of the energies.

#### **III. RESULTS AND DISCUSSION**

Numerical results are presented by performing systematic study of the Sr isotope within the framework of Statistical method [21-26]. The angular momentum has been generated by adjusting the cranking frequency  $\omega$  for different deformations. The necessity arises due to the fact that calculations are to be performed for both non-collective and collective states of the nuclei. Calculations are carried out for the deformation parameter  $\varepsilon = 0.0$  to 1 with  $\Delta \varepsilon = 0.1$ and for  $\gamma = -120^{\circ}$  to  $\gamma = -180^{\circ}$ . In our calculations the following convention is used: The deformation parameter =  $-180^{\circ}$  corresponds to the oblate shape rotating about the symmetry axis while =  $-120^{\circ}$ corresponds to the prolate shape rotating about an axis perpendicular to the symmetry axis. Calculations are carried out by minimizing the free energy for equilibrium deformation. The free energy of the

with respect to the deformation and shape parameter.



**Figure 1.** The deformation  $\beta$  as a function of angular momentum for different temperature for the nuclei <sup>98</sup>Sr

Figure 1 shows the changes in deformation as a function of angular momentum for different temperature for the nuclei 98Sr. The fluctuation in the plot indicates a shape transition less spherical to deformed shape. From the Fig. 1 for the nuclei 98Sr, for a given temperature T = 0.5 MeV, the fluctuation around the angular momentum 6-12h corresponds to the deformation change  $\varepsilon = 0.0$  to = 0.2 and the fluctuation around the angular momentum 14-16ħ corresponds to the deformation change  $\varepsilon = 0.2$  to = 0.0. The fluctuation around the angular momentum 18-20h corresponds to the deformation change  $\varepsilon$ = 0.0 to = 0.2, i.e. it is corresponds to spherical to less deformed shape. The variation at angular momentum 22-28ħ corresponds to a deformation change  $\varepsilon = 0.2$  to  $\varepsilon = 0.0$ , the variation around the angular momentum 3034ħcorresponds to a deformation change  $\varepsilon$ = 0.0 to  $\varepsilon$ = 0.2, i.e. it is corresponds to spherical to less deformed shape and the variation around the angular momentum 36-50ħ corresponds to a deformation change  $\varepsilon$ = 0.0 to  $\varepsilon$ = 0.3, it is corresponds to spherical to less deformed shape.For higher temperatures T≈1.0-2 MeV, similar trends of transition were observed. If the temperature increases further to 2.5 and 3MeV the fluctuations are reduced and we observe a sudden transition from spherical to deformed shape at angular momentum 36 ħ.



Angular Momentum

**Figure 2.**The shape parameter as a function of angular momentum for different temperature for the nuclei <sup>98</sup>Sr

Figure 2 shows changes in deformation  $\gamma$  as a function of angular momentum for different temperature for the nuclei 98Sr. From the Fig. 2 for the nuclei 98Sr, it is found that the change in the deformation around the angular momentum 6-12h corresponds to a shape change from collective prolate to triaxial i.e., the deformation change from  $\gamma = -180^{\circ}$  to  $\gamma = -150^{\circ}$ . It shows fluctuations around the angular momentum I = 14-16ħ, its corresponds to shape change within from triaxial to collective prolate shape ( $\gamma = -140^{\circ}$  to  $\gamma = -$ 120°). It also shows fluctuations around the angular momentum I =  $18-20\hbar$ , i.e., corresponds shape change to triaxial shape. The fluctuation around the angular momentum 22-28ħ corresponds to the shape change from triaxial to collective prolate ( $\gamma = -170^{\circ}$  to  $\gamma = -$ 180°). The fluctuation around the angular momentum 30-42ħ corresponds to the deformation change  $\epsilon$ = 0.0 to = 0.2, i.e. it is corresponds collective prolate to

triaxial shape. The fluctuation at angular momentum 44-50ħcorresponds to a shape change from triaxial to non collective oblate. It is interesting to note that with increasing temperature the presence of triaxial shape vanish, the nuclei undergoes a shape transition from collective prolate to oblate shape.

#### **IV. CONCLUSION**

A systematic study on shape phase transitions between spherical and deformed in the framework of the statistical theory as a function of temperature and angular momentum for <sup>98</sup>Sr. The evolution from spherical to triaxially deformed and triaxially deformed to deformed oblate shape are very much significant in this work. At low temperature and angular momentum, triaxiality is predominant shape and at higher temperature and angular momentum[11]. Hens, A. Ruckelshausen, R.D. Fisher, W. Kuhn,shape transition to oblate.V. Metag, R. Novotny, R.V. F. Janssens, T.L.

# V. REFERENCES

- Ansari, Frontiers of Nuclear Structure Physics, (Indian Physics Association), edited by B.K. Jain 110, (1976)
- [2]. U. Hubel, R.M. Smilansky, R. Diamond and F. Stephens, Average Lifetimes of Collective Transitions in the Spin-(30-50) Region, Phys. Rev. Lett. 41, 791 (1978).
- [3]. V.S. Ramamoorthy, S.S. Kapoor and S.K. Kataria, Excitation Energy Dependence of Shell Effects on Nuclear Level Densities and Fission Fragment Anisotropies, Phys. Rev. Lett. 25, 386 (1970).
- [4]. S.K. Kataria and V.S. Ramamorthy, Macroscopic systematics of nuclear level densities, Nucl. Phys. A349, 10 (1980).
- [5]. H.J. Krappe, J.R. Nix and A.J. Sierk, Unified nuclear potential for heavy-ion elastic scattering, fusion, fission, and ground-state masses and deformations, Phys. Rev. 20, 992 (1979).
- [6]. J. Cerny, Nuclear Spectroscopy and Reactions, Vol. 1-3 (Academic Press, New York, 1974).
- [7]. Ragnarsson, T. Bengtsson, G. Leander and S. Aberg, Nilsson-Strutinsky Model of Very High Spin States, Nucl. Phys. A347, 287 (1980);
- [8]. Ragnarsson, S. Aberg, H.B. Hakansson and R.K. Sheline, Application of the cranked Nilsson model in some light nuclei: The super backbend in 11B and 11C? Nucl. Phys. A361, 1 (1981).
- [9]. W. Norenberg and H.A. Weidenmuller, Introduction to the Theory of Heavy Ion Collisions (Springer, Berlin) 51, 65 (1976).
- [10]. J. Ehlers, K. Hepp and H.A. Weidenmuller, Classical and Quantum Mechanical Aspects of Heavy Ion Collisions, (Springer Verlag, New York), 1975).

- [11]. Hens, A. Ruckelshausen, R.D. Fisher, W. Kuhn, V. Metag, R. Novotny, R.V. F. Janssens, T.L. Khoo, D. Habs, D. Schwalm, D. Freeman, G. Duchene, B. Hass, F. Hass, S. Hlavac and R.S. Simon, Measurement of the Nuclear Level Density at High Spins, Phys. Rev. Lett. 60, 11 (1988).
- [12]. F. Hannachi, G. Bastin, M.G. Porquet, C. Shcuck, J.P. Thibaud, C. Bourgeois, L. Hildingsson, D. Jerrestam, N. Perrin, H. Sergolle, F.A. Beck, J.C. Merdinger, T. Byrski and J. Dudek, High-spin excitations of 187, 188Hg, Nucl. Phys. A481, 135 (1988).
- [13]. J.J. Kolata, R.A. Kryger, P.A. De Young and F.W. Prosser, Neutron-Emission Spectra and Superdeformation in Light Nuclei, Phys. Lett. 61, 1178 (1988).
- [14]. K.Sugawara-Tanabe, K.Tanabe and H.J.Mang, Application of the cranked temperature dependent Hartree-Fock-Bogoliubov theory to the excited states in the deformed nucleus, Nucl. Phys. A357, 45 (1981).
- [15]. Y. Alhassid, J. Zingman, S. Levit, Landau theory of shape transitions in hot rotating nuclei, Nucl. Phys. A469, 205 (1987).
- [16]. G. Shanmugam and V. Selvam Shape Transitions in Hot Rotating Strontium and Zirconium Isotopes, Phys. Rev. C62, 014302 (2000).
- [17]. M. M. Sirag, Investigation of Nuclear Shape Phase Transitions in the A~100 Region Using the Interacting Boson Model, Chinese Journal Of Physics Vol. 53, 7 (2015)
- [18]. Mamta Aggarwal, Shape Transition to a Rare Shape Phase of Prolate Non-collective in A=100 Isobars Journal of Nuclear Physics, Material Sciences, Radiation and Applications -3(2), 179 (2016).
- [19]. S. Santhosh Kumar, A. Victor Babu, P. Preetha and T. R. Rajasekaran, Structural Changes of Hot Rotating Neutron Deficient Doubly Magic Nucleus 100Sn, Asian Journal of Science and Applied Technology, 4(1), 21 (2015).

- [20]. E. Kirchuk and P. Federman, Physical Nuclear deformation in the mass-80 and mass-100 regions, Phys. Review C, 47(2), 2 (1993).
- [21]. M. Rajasekaran, T. R. Rajasekaran, and N. Arunachalam, Nuclear Level Density Parameter
  Its Dependence on Spin and Temperature, Phys. Rev. C37, 307 (1988).
- [22]. M. Rajasekaran and V. Devanathan, A New Formula for Nuclear Level Density Phys. Lett. 113B, 433 (1982).
- [23]. M. Rajasekaran and V. Devanathan, Nuclear Level Density and the Mass Distribution of Fission Fragments, Phy. Rev. C24(6), 2606 (1981).
- [24]. M. Rajasekaran, N. Arunachalam and V. Devanathan, Effect of high spin states on fusion in heavy ion collisions Phys. Rev. 36, 1860 (1987).
- [25]. M. Rajasekaran, N. Arunachalam, T. R. Rajasekaran, and V. Devanathan Shell effects in hot isobaric nuclei Phys. Rev. C38, 1926 (1988).
- [26]. M. Rajasekaran, T. R. Rajasekaran, N. Arunachalam and V. Devanathan, Neutron Separation Energy and Emission Probability at High SpinsPhys. Rev. Lett. 61, 2077 (1988).