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Expressions of Some Complicated Integrals

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ABSTRACT

In this paper, we study two types of indefinite integrals. The analytic solutions of the two indefinite integrals can be obtained mainly using differentiation with respect to a parameter and integration term by term. In addition, we propose two examples to demonstrate the calculations. The research method adopted in this study is to find solutions through manual calculations and verify our answers using Maple. This method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking.

Keywords: Indefinite Integrals, Analytic Solutions, Differentiation with Respect to a Parameter, Integration Term by Term, Maple

I. INTRODUCTION

The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests.

In calculus and engineering mathematics, there are many methods to solve the integral problems, for example, change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. This paper considers the following two types of indefinite integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int (\ln x)^m x^a \cosh x^b dx \,, \tag{1}$$

$$\int (\ln x)^m x^a \sinh x^b dx \,, \tag{2}$$

where a, b, x are real numbers, $a > -1$, $b > 0$, $x > 0$ and m is a positive integer. We can obtain the analytic solutions of these two indefinite integrals mainly using differentiation with respect to a parameter and integration term by term; this is the major result of this study (i.e., Theorem A). Adams et al. [1], Nyblom [2], and Oster [3] provided some techniques to solve the integral problems. On the other hand, Yu [4-31], Yu and Chen [32], and Yu and Sheu [33-35] used complex power series method, integration term by term theorem, Parseval's theorem, area mean value theorem, and generalized Cauchy integral formula to evaluate some types of integral problems. In this paper, two examples are used to demonstrate the proposed calculations, and the manual calculations are verified using Maple.

II. MAIN RESULTS

First, some formulas used in this paper are introduced below.

2.1 $\cosh x=\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}$, where x is any real number.

2.2 $\sinh x=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1}$, where x is any real number.

2.3 Leibniz rule: Let m be a positive integer. If $f(x),g(x)$ are functions such that their p-th derivatives $f^{(p)}(x),g^{(p)}(x)$ exist for all $p=1,\ldots,m$. Then the m-th derivative of product function $f(x)g(x)$,

$$(fg)^{(m)}(x)=\sum_{k=0}^{m}\frac{(m)_k}{k!}f^{(m-k)}(x)g^{(k)}(x),$$

where $(m)_k=m(m-1)(m-2)\cdots(m-k+1)$ for $k=1,\cdots,m$, and $(m)_0=1$.

Next, we introduce two important theorems used in this study which can be found in ([36, p283]) and ([36, p269]) respectively.

2.4 Differentiation with respect to a parameter: Suppose that c,d,λ,β are real numbers and the function $f(a,x)$ is defined on $[c,d]\times[\lambda,\beta]$. If $f(a,x)$ and its partial derivative $\frac{\partial f}{\partial a}(a,x)$ are continuous functions on $[c,d]\times[\lambda,\beta]$. Then $F(a)=\int_{\lambda}^{\beta}f(a,x)dx$ is differentiable on the open interval (c,d). Moreover, $\frac{d}{da}F(a)=\int_{\lambda}^{\beta}\frac{\partial f}{\partial a}(a,x)dx$ for all $a\in(c,d)$.

2.5 Integration term by term: Assume that $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on I. If $\sum_{n=0}^{\infty}\int_I|g_n|$ is convergent, then $\int_I\sum_{n=0}^{\infty}g_n=\sum_{n=0}^{\infty}\int_I g_n$.

In the following, we determine the analytic forms of the indefinite integrals (1) and (2).

Theorem A *Suppose that a,b,x are real numbers, $a>-1$, $b>0$, $x>0$, m is a positive integer, and C is a constant, then*

$$\int(\ln x)^m x^a\cosh x^b dx$$

$$=m!\sum_{n=0}^{\infty}\sum_{k=0}^{m}\frac{(-1)^{m-k}}{k!(2n)!(2bn+a+1)^{m-k+1}}(\ln x)^k x^{2bn+a+1}+C, \tag{3}$$

and

$$\int(\ln x)^m x^a\sinh x^b dx$$

$$=m!\sum_{n=0}^{\infty}\sum_{k=0}^{m}\frac{(-1)^{m-k}}{k!(2n+1)!(2bn+a+b+1)^{m-k+1}}(\ln x)^k x^{2bn+a+b+1}+C. \tag{4}$$

Proof Since $\int x^a\cosh x^b dx$

$$=\int\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2bn+a}dx$$

(by Formula 2.1)

$$=\sum_{n=0}^{\infty}\frac{1}{(2n)!}\int x^{2bn+a}dx$$

(by integration term by term)

$$=\sum_{n=0}^{\infty}\frac{1}{(2n)!(2bn+a+1)}x^{2bn+a+1}+C. \tag{5}$$

Using differentiation with respect to a parameter, differentiating m times with respect to a on both sides of Eq. (5), then

$$\int(\ln x)^m x^a\cosh x^b dx$$

$$=\sum_{n=0}^{\infty}\frac{1}{(2n)!}\sum_{k=0}^{m}\frac{(m)_k}{k!}\left(\frac{1}{2bn+a+1}\right)^{(m-k)}(x^{2bn+a+1})^{(k)}+C$$

(by Leibniz rule)

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{m}\frac{(m)_k}{k!}\frac{(-1)^{m-k}(m-k)!}{(2n)!(2bn+a+1)^{m-k+1}}(\ln x)^k x^{2bn+a+1}+C$$

$$=m!\sum_{n=0}^{\infty}\sum_{k=0}^{m}\frac{(-1)^{m-k}}{k!(2n)!(2bn+a+1)^{m-k+1}}(\ln x)^k x^{2bn+a+1}+C.$$

Similarly, by Formula 2.2, integration term by term, and differentiation with respect to a parameter, Eq.(4) is easily obtained. q.e.d.

III. EXAMPLES

Next, for the integral problems discussed in this study, two examples are proposed and we use Theorem A to determine their analytic solutions. Additionally, we employ Maple to calculate the approximations of some definite integrals to verify our answers.

Example 1 Using Eq. (3) yields

$$\int (\ln x)^3 x^2 \cosh x^4 dx = 6\sum_{n=0}^{\infty}\sum_{k=0}^{3}\frac{(-1)^{3-k}}{k!(2n)!(8n+3)^{4-k}}(\ln x)^k x^{8n+3} + C. \quad (6)$$

Therefore, the definite integral

$$\int_2^3 (\ln x)^3 x^2 \cosh x^4 dx = 6\sum_{n=0}^{\infty}\sum_{k=0}^{3}\frac{(-1)^{3-k}}{k!(2n)!(8n+3)^{4-k}}\left[(\ln 3)^k 3^{8n+3} - (\ln 2)^k 2^{8n+3}\right] \quad (7)$$

Next, we use Maple to verify the correctness of Eq. (7).

> evalf(int((ln(x))^3*x^2*cosh(x^4),x=2..3),18);

$$8.2760244498\ 0242495 \cdot 10^{33}$$

> evalf(6*sum(sum((-1)^(3-k)/(k!*(2*n)!*(8*n+3)^(4-k))*((ln(3))^k*3^(8*n+3)-(ln(2))^k*2^(8*n+3)),k=0..3), n=0..infinity),18);

$$8.2760244498\ 0242492 \cdot 10^{33}$$

Example 2 By Eq. (4), we have

$$\int (\ln x)^7 x^3 \sinh x^2 dx = 7!\sum_{n=0}^{\infty}\sum_{k=0}^{7}\frac{(-1)^{7-k}}{k!(2n+1)!(4n+6)^{8-k}}(\ln x)^k x^{4n+6} + C. \quad (8)$$

Hence, the definite integral

$$\int_3^4 (\ln x)^7 x^3 \sinh x^2 dx = 7!\sum_{n=0}^{\infty}\sum_{k=0}^{7}\frac{(-1)^{7-k}}{k!(2n+1)!(4n+6)^{8-k}}\left[(\ln 4)^k 4^{4n+6} - (\ln 3)^k 3^{4n+6}\right]. \quad (9)$$

Using Maple to verify the correctness of Eq. (9) as follows:

> evalf(int((ln(x))^7*x^3*sinh(x^2),x=3..4),18);

$$2.8293478110\ 3768031 \cdot 10^{8}$$

> evalf(7!*sum(sum((-1)^(7-k)/(k!*(2*n+1)!*(4*n+6)^(8-k))*((ln(4))^k*4^(4*n+6)-(ln(3))^k*3^(4*n+6)),k=0..7),n=0..infinity),18);

$$2.8293478110\ 3768031 \cdot 10^{8}$$

IV. CONCLUSION

In this paper, some techniques: differentiation with respect to a parameter and integration term by term are used to evaluate two complicated integrals. In fact, the applications of the two methods are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topics to other calculus and engineering mathematics problems and solve these problems using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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