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Interacting and Non-Interacting Holographic Gas Model of Dark energy in $f(G)$ Gravity

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ABSTRACT

In this paper, investigations of interaction and non-interaction between Holographic dark energy and dark matter within the frame work of $f(G)$ gravity using a spatially homogeneous and anisotropic Space-time are presented. A viable $f(G)$ model i.e. $f(G)=aG^{b+1}$ is used to explore the exact solutions of modified field equations. Some important cosmological parameters are calculated for the obtained solutions. Moreover, energy density and pressure of the universe is analysed for the model under consideration.

Keywords : Cosmological model, Holographic gas dark energy, $f(G)$gravity.

I. INTRODUCTION

The General Theory of Relativity is an astounding accomplishment: Together with quantum field theory, it is now widely considered to be one of the two pillars of modern physics. The theory itself is couched in the language of differential geometry, and was a pioneer for the use of modern mathematics in physical theories, One of the most striking facts about General Relativity is that, after almost an entire century, it remains completely unchanged: The field equations that Einstein communication are still our best description of how space-time behaves on macroscopic scales. These are

$$G_{ij}=\frac{8\pi G}{c^4}T_{ij},$$

where G_{ij} is the Einstein tensor, T_{ij} is the energy momentum tensor, G is Newton's constant, and c is the speed of light.

General Theory of relativity and the standard model of particle physics have both been extremely successful in describing our universe both on cosmological scales as well as on microscopic scales. Despite this amazing success, some observations cannot be explained within these otherwise extremely successful models. For example, the cosmic microwave background, the rotation curves of galaxies or the bullet cluster to quote a few, suggest that there is a new form of matter that does not shine in the electromagnetic spectrum. A central theme in cosmology is the perplexing fact that the Universe is undergoing an accelerating expansion [1]. Several candidates, responsible for this expansion, have been proposed in the literature, in particular, dark energy models and modified gravity. The reasons and motivations that lead to the consideration of alternatives to General Relativity are manifold and have changed over the years. Some theories are motivated by theoretical reasons while others are more phenomenological

Instead of considering GR, different kinds of modified gravity based on the curvature scalar have been performed in the recent years, as $f(R)$ [2-8], where R

is the curvature scalar, the $f(R,T)$, T being the trace of the energy-momentum tensor [9-14].

Another modified gravity so called $f(T)$ -gravity based on a space-time possessing absolute parallelism. A remarkable feature of $f(T)$ theories is that the dynamics of tetrads is described by second order equations, which is not usual in the context of modified gravity. The central piece of a Teleparallel Lagrangian is the Weitzenbock torsion. Jamil *at al.* [15] tried to resolve the dark matter problem in the light of $f(T)$ gravity and successfully obtained the flat rotation curves of galaxies containing dark matter as component with the density profile of dark matter in galaxies. Setare and Darabi [16] have studied the power-law solution when the universe enters in phantom phase and shown that such solutions may exist for some $f(T)$ solutions whereas Chirde and Shekh [17-19] investigated some cosmological models in the same gravity. Recently, Bhoyar *et al.* [20] discussed stability of accelerating Universe with linear equation of state in $f(T)$ gravity using hybrid expansion law.

Among the various modified gravity theories available in the literature, the one is Gauss Bonnet (GB) gravity which has received great attraction and is named as $f(G)$ gravity. The equation of motion for this gravity is required to be coupled with some scalar field or $f(G)$ must be some arbitrary function of *G*. This modified gravity could help out in the study of inflationary era, transition of acceleration from deceleration regimes, passing tests induced by solar system experiments and crossing phantom divide line for different viable $f(G)$ models [21, 22]. It is also seen that the GB gravity is less constrained than $f(R)$ gravity [23]. , The $f(G)$ gravity also provides an efficient platform to study various cosmic issues as an alternate to DE [24]. The $f(G)$ gravity could also be very helpful for the study of finite time future singularities as well as the universe acceleration during late time epochs [25]. Nojiri *et al.* [26] have discussed some fundamental cosmic issues, like inflation, late-time acceleration, bouncing cosmology and claimed that some modified theories of gravity, like $f(R)$, $f(G)$ and $f(T)$ theories (where T is the torsion scalar) could be used as a viable mathematical tool for analysing the clear picture of our universe. The general formalism for ECs are derived in $f(G)$ gravity by Garcia *et al.* [27]. Nojiri *et al.* [28] presented some specific realistic and viable $f(G)$ models by analysing the dynamical behaviour of WEC. Banijamali *et al.* [29] analysed the distribution of WEC for a class of consistent $f(G)$ models and claimed that power law model of the type $f(G) = \in G^m$ would satisfy WEC on setting $\in < 0$.

2. Field equations and $f(G)$ gravity:

Modified GB gravity is described by the action

$$S = \frac{1}{2\pi}\int d^4x\sqrt{-g}\,[R + f(G)] + S_M\left(g^{\mu\nu},\psi\right) \qquad (1)$$

where κ is the coupling constant, g is the determinant of the metric tensor $g_{\mu\nu}$, and $S_M\left(g^{\mu\nu},\psi\right)$ is the matter action, in which matter is minimally coupled to the metric tensor and ψ denotes the matter fields. This coupling of matter to the metric tensor suggests that $f(G)$ gravity is a purely metric theory of gravity. The $f(G)$ is an arbitrary function of the GB invariant G.

$$G \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\alpha}R^{\mu\nu\sigma\alpha}, \qquad (2)$$

where R is the Ricci scalar and $R_{\mu\nu}$ and $R_{\mu\nu\sigma\alpha}$ denote the Ricci and Riemann tensors. Gravitational field equations are obtained by varying the action in equation (1) with respect to the metric tensor:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + 8\left[R_{\mu\alpha\nu\sigma} + R_{\alpha\nu}g_{\sigma\mu} - R_{\mu\nu}g_{\sigma\alpha} - R_{\alpha\sigma}g_{\nu\mu} + R_{\mu\sigma}g_{\nu\alpha} + \frac{1}{2}\left(R_{\mu\nu}g_{\sigma\alpha} - R_{\sigma\alpha}g_{\nu\alpha}\right)\right]\times$$
$$\nabla^{\alpha}\nabla^{\sigma}F + \left(Gf_G - f\right)g_{\mu\nu} = \kappa T_{\mu\nu}, \qquad (3)$$

where ∇_μ denotes the covariant derivative and f_G represents the derivative of f with respect to G.

3. Holographic dark energy model in Bianchi type-I space-time:

The line element for a spatially homogeneous, anisotropic and LRS Bianchi type-I space-time is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 \left(dy^2 + dz^2\right), \tag{4}$$

Where A and $B$are the directional scale factors of cosmic time t.

The corresponding Ricci scalar and GB invariant for the space-time (5) are turn out to be

$$R = -2\left[\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}^2}{A^2}\right], \tag{5}$$

$$G = 8\left[\frac{\ddot{A}}{A}\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\frac{\ddot{B}}{B}\right], \tag{6}$$

where the dot denotes the differentiation with respect to t.

The energy momentum tensor for matter and the holographic dark energy is defined as

$$T^*_{\mu\nu} = T_{\mu\nu} + \overline{T}_{\mu\nu}, \tag{7}$$

wher $T_{\mu\nu} = \rho_m u_\mu u_\nu$ and $\overline{T}_{\mu\nu} = (\rho_\Lambda + p_\Lambda) u_\mu u_\nu - g_{\mu\nu} p_\Lambda$, ρ_m and ρ_Λ are the energy densities of matter and the holographic dark energy respectively and p_Λ is the pressure of the holographic dark energy.

The components of energy momentum tensor are

$$T^*_{11} = T^*_{22} = T^*_{33} = -p_\Lambda \text{ and } T^*_{44} = (\rho_\Lambda + p_\Lambda). \tag{8}$$

From the equation of motion (3), the Bianchi type-I space-time (4) for the fluid of stress energy tensor (8) can be written as

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} - 24\frac{\dot{A}}{A}\frac{\dot{B}^2}{B^2}\dot{f}_G + G f_G - f = k(\rho_m + \rho_\Lambda), \tag{9}$$

$$-\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} - 16\frac{\dot{B}}{B}\frac{\ddot{B}}{B}\dot{f}_G + 8\frac{\dot{B}^2}{B^2}\ddot{f}_G - G f_G + f = k(p_\Lambda), \tag{10}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} + 8\left(\frac{\dot{A}}{A}\frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\frac{\ddot{A}}{A}\right)\dot{f}_G + 8\frac{\dot{A}}{A}\frac{\dot{B}}{B}\ddot{f}_G - G f_G + f = k(p_\Lambda). \tag{11}$$

Here afterwards the dot over the field variable represents ordinary differentiation with respect to t. Finally, here we have three differential equations with five unknowns namely A, B, f, p, ρ. The solution of these equations is discussed in next section. In the following we define some kinematical quantities of the space-time.

We define average scale factor and volume respectively as

$$a^3 = V = AB^2. \tag{12}$$

Another important dimensionless kinematical quantity is the mean deceleration parameter which tells whether the Universe exhibits accelerating volumetric expansion or not is

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right), \tag{13}$$

for $-1 \le q < 0$, $q > 0$ and $q = 0$ the Universe exhibit accelerating volumetric expansion, decelerating volumetric expansion and expansion with constant-rate respectively.

The mean Hubble parameter, which expresses the volumetric expansion rate of the Universe, given as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{14}$$

where H_1, H_2 and H_3 are the directional Hubble parameter in the direction of x, y and z-axis respectively.

Using equations (12) and (14), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}. \tag{15}$$

To discuss whether the Universe either approach isotropy or not, we define an anisotropy parameter as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{16}$$

The expansion scalar and shear scalar are defined as follows

$$\theta = u^\mu_{;\mu} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \tag{17}$$

$$\sigma^2 = \frac{3}{2}H^2 A_m. \tag{18}$$

4. Exact Matter Dominated Solution of the Field Equations

In order to solve the field equations completely, we first assume that the interaction between matter and holographic dark energy components i.e. the energy momentum tensors of the two sources interact / non-interact minimally and conserved separately.

In this case, the energy conservation equation of the matter leads to

$$(\dot{\rho}_m)+\frac{\dot{V}}{V}(p_m+\rho_m)=Q. \tag{19}$$

In this case, the energy conservation equation of holographic dark energy leads to

$$(\dot{\rho}_\Lambda)+\frac{\dot{V}}{V}(p_\Lambda+\rho_\Lambda)=-Q. \tag{20}$$

The quantity $Q \geq 0$, expresses the interaction (for $Q>0$) and non-interaction ($Q=0$) term between the matter and holographic dark energy components. It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction since we are interested to investigate the interaction between DE and matter. In our work we consider the interaction term in the form of $Q \propto H\rho_m$ which is already well-thought-out by Chirde and Shekh [30]

Secondly consider viable $f(G)$ model i.e.

$$f(G)=aG^{b+1}. \tag{21}$$

Subtracting equation (11) from (10), we get

$$\frac{\ddot{A}}{A}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}=0. \tag{22}$$

Integrating above equation, we find

$$\alpha t=\int \dot{A}B\,dt, \tag{23}$$

where α be the integration constant.

Also, the normal congruence to the homogeneous expansion satisfies the condition that σ/θ is constant, i.e., the expansion scalar is proportional to the shear scalar. This gives the relation between the metric potentials as

$$A=B^n, \tag{24}$$

Using equations (21) and (22), we get

$$A=(ct+d)^{\frac{n}{n+1}}, \tag{25}$$

$$B=(ct+d)^{\frac{1}{n+1}}. \tag{26}$$

Using equations (25) and (26), spatially homogeneous and anisotropic LRS Bianchi Type-I space-time with within the framework of $f(G)$ gravity becomes

$$ds^2=dt^2-(ct+d)^{\frac{2n}{(n+1)}}dx^2-(ct+d)^{\frac{2}{(1+n)}}\left(dy^2+dz^2\right).$$

Above equation represent a singular model and singularity exist at point $t=t_s=-d/c$.

Model for $Q=0$ (Non-interaction)

In this section we discussed the acts of non-interaction between matter and holographic dark energy with the changing aspects of physical behaviour of universe.

The matter density in the Universe as

$$\rho_m=(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}. \tag{27}$$

We assume that the EoS parameter of the perfect fluid to be a constant (which is considered by Chirde and Shekh [30])

Pressure in the Universe is

$$p_m=\omega_m\rho_m=(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}. \tag{28}$$

Energy density of holographic dark energy

$$\rho_\Lambda=\frac{1}{k}\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2}+\frac{96anc^4(b-1)(b-2)(8\alpha)^{b-2}}{(n+1)^3(ct+d)^{4b-4}}+\frac{8(b-2)(8\alpha)^{b-1}}{(ct+d)^{4b-4}}\right\}-(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}, \tag{29}$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{\begin{array}{l}\frac{-nc-c}{(n+1)^2(ct+d)^2}+\frac{32ac^2(b-1)(b-2)(8\alpha)^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^3(ct+d)^{4b-4}}+\frac{a(2-b)(8\alpha)^{b-1}}{(ct+d)^{4b-4}}\\ +\frac{128ac^4(b-1)(b-2)(b-3)\alpha^{b-3}8^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^2(ct+d)^{4b-5}}\end{array}\right\}, \quad 30)$$

where $\alpha = \dfrac{3nc^4-3n^2c^3-3nc^3}{(n+1)^4}$.

Eos parameter for holographic dark energy

$$\omega_\Lambda = -\frac{\left\{\begin{array}{c}\frac{nc+c}{(n+1)^2(ct+d)^2}+\frac{32ac^2(1-b)(b-2)(8\alpha)^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^3(ct+d)^{4b-4}}+\frac{a(b-2)(8\alpha)^{b-1}}{(ct+d)^{4b-4}}\\ -\frac{128ac^4(b-1)(b-2)(b-3)\alpha^{b-3}8^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^2(ct+d)^{4b-5}}\end{array}\right\}}{\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2}+\frac{96anc^4(b-1)(b-2)(8\alpha)^{b-2}}{(n+1)^3(ct+d)^{4b-4}}+\frac{8(b-2)(8\alpha)^{b-1}}{(ct+d)^{4b-4}}-k(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}\right\}}, \quad (31)$$

Case-I: (linear: model for $a = b = 1$)

Energy density of holographic dark energy

$$\rho_\Lambda = \frac{1}{k}\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2}-8\right\}-(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}. \quad (32)$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{1-\frac{c(n+1)}{(n+1)^2(ct+d)^2}\right\}. \quad (33)$$

Eos parameter for holographic dark energy

$$\omega_\Lambda = \frac{\left\{1-\frac{c(n+1)}{(n+1)^2(ct+d)^2}\right\}}{\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2}-8\right\}-k(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}}. \quad (34)$$

Case-II: (Quadratic: model for $a = 1,\ b = 2$)

Energy density of holographic dark energy

$$\rho_\Lambda = \frac{1}{k}\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2}\right\}-(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}, \quad (35)$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{\frac{-nc-c}{(n+1)^2(ct+d)^2}\right\}, \quad (36)$$

Eos parameter for holographic dark energy

$$\omega_\Lambda = \frac{\left\{\dfrac{-c(n+1)}{(n+1)^2(ct+d)^2}\right\}}{\left\{\dfrac{c^2(1+2n)}{(n+1)^2(ct+d)^2}\right\} - k(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}}. \tag{37}$$

Case-III: (Inverse: model for $a = 1,\ b = 0$)

Energy density of holographic dark energy

$$\rho_\Lambda = \frac{1}{k}\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2} + \frac{192nc^4(ct+d)^4}{64\alpha^2(n+1)^3} - \frac{2(ct+d)^4}{\alpha}\right\} - (ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}, \tag{38}$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{\frac{-nc-c}{(n+1)^2(ct+d)^2} + \frac{c^2\left[2-5c^2(n+1)\right](ct+d)^4}{\alpha^2(n+1)^3} + \frac{(ct+d)^4}{4\alpha} - \frac{c^4\left[2-5c^2(n+1)\right](ct+d)^5}{3\alpha^3(n+1)^2}\right\}, \tag{39}$$

Eos parameter for holographic dark energy

$$\omega_\Lambda = \frac{\left\{\dfrac{-nc-c}{(n+1)^2(ct+d)^2} + \dfrac{c^2\left[2-5c^2(n+1)\right](ct+d)^4}{\alpha^2(n+1)^3} + \dfrac{(ct+d)^4}{4\alpha} - \dfrac{c^4\left[2-5c^2(n+1)\right](ct+d)^5}{3\alpha^3(n+1)^2}\right\}}{\left\{\dfrac{c^2(1+2n)}{(n+1)^2(ct+d)^2} + \dfrac{192nc^4(ct+d)^4}{64\alpha^2(n+1)^3} - \dfrac{2(ct+d)^4}{\alpha}\right\} - k(ct+d)^{\left(\frac{(1+\omega_m)(n+2)}{3(n+1)}\right)}}, \tag{40}$$

Model for $Q \neq 0$ (Interaction)

In this section we discussed the acts of interaction between matter and holographic dark energy with the changing aspects of physical behaviour of universe.

The matter density in the Universe as

$$\rho_m = (ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}. \tag{41}$$

We assume that the EoS parameter of the perfect fluid to be a constant (which is considered by Chirde and Shekh [30])

Pressure in the Universe is

$$p_m = \omega_m \rho_m = (ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}. \tag{42}$$

Energy density of holographic dark energy

$$\rho_\Lambda = \frac{1}{k}\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2} + \frac{96anc^4(b-1)(b-2)(8\alpha)^{b-2}}{(n+1)^3(ct+d)^{4b-4}} + \frac{8(b-2)(8\alpha)^{b-1}}{(ct+d)^{4b-4}}\right\} - (ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}. \tag{43}$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{\begin{array}{l}\dfrac{-nc-c}{(n+1)^2(ct+d)^2} + \dfrac{32ac^2(b-1)(b-2)(8\alpha)^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^3(ct+d)^{4b-4}} + \dfrac{a(2-b)(8\alpha)^{b-1}}{(ct+d)^{4b-4}} \\ + \dfrac{128ac^4(b-1)(b-2)(b-3)\alpha^{b-3}8^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^2(ct+d)^{4b-5}}\end{array}\right\}, \tag{44}$$

where $\alpha = \dfrac{3nc^4 - 3n^2c^3 - 3nc^3}{(n+1)^4}$

Eos parameter for holographic dark energy

$$\omega_\Lambda = -\frac{\left\{ \begin{array}{c} \dfrac{nc+c}{(n+1)^2(ct+d)^2} + \dfrac{32ac^2(1-b)(b-2)(8\alpha)^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^3(ct+d)^{4b-4}} + \dfrac{a(b-2)(8\alpha)^{b-1}}{(ct+d)^{4b-4}} \\ -\dfrac{128ac^4(b-1)(b-2)(b-3)\alpha^{b-3}8^{b-2}\left[2-5c^2(n+1)\right]}{(n+1)^2(ct+d)^{4b-5}} \end{array} \right\}}{\left\{ \dfrac{c^2(1+2n)}{(n+1)^2(ct+d)^2} + \dfrac{96anc^4(b-1)(b-2)(8\alpha)^{b-2}}{(n+1)^3(ct+d)^{4b-4}} + \dfrac{8(b-2)(8\alpha)^{b-1}}{(ct+d)^{4b-4}} - k(ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)} \right\}}, \tag{45}$$

Case-I: (linear: model for $a = b = 1$)

Energy density of holographic dark energy

$$\rho_\Lambda = \frac{1}{k}\left\{ \frac{c^2(1+2n)}{(n+1)^2(ct+d)^2} - 8 \right\} - (ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}. \tag{46}$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{ 1 - \frac{c(n+1)}{(n+1)^2(ct+d)^2} \right\}. \tag{47}$$

Eos parameter for holographic dark energy

$$\omega_\Lambda = \frac{\left\{ 1 - \dfrac{c(n+1)}{(n+1)^2(ct+d)^2} \right\}}{\left\{ \dfrac{c^2(1+2n)}{(n+1)^2(ct+d)^2} - 8 \right\} - k(ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}}. \tag{48}$$

Case-II: (Quadratic: model for $a = 1,\ b = 2$)

Energy density of holographic dark energy

$$\rho_\Lambda = \frac{1}{k}\left\{ \frac{c^2(1+2n)}{(n+1)^2(ct+d)^2} \right\} - (ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}, \tag{49}$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{ \frac{-nc-c}{(n+1)^2(ct+d)^2} \right\}, \tag{50}$$

Eos parameter for holographic dark energy

$$\omega_\Lambda = \frac{\left\{ \dfrac{-c(n+1)}{(n+1)^2(ct+d)^2} \right\}}{\left\{ \dfrac{c^2(1+2n)}{(n+1)^2(ct+d)^2} \right\} - k(ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}}. \tag{51}$$

Case-III: (Inverse: model for $a = 1,\ b = 0$)

Energy density of holographic dark energy

$$\rho_\Lambda = \frac{1}{k}\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2} + \frac{192nc^4(ct+d)^4}{64\alpha^2(n+1)^3} - \frac{2(ct+d)^4}{\alpha}\right\} - (ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}, \quad (52)$$

Pressure of holographic dark energy

$$p_\Lambda = \frac{1}{k}\left\{\frac{-nc-c}{(n+1)^2(ct+d)^2} + \frac{c^2\left[2-5c^2(n+1)\right](ct+d)^4}{\alpha^2(n+1)^3} + \frac{(ct+d)^4}{4\alpha} - \frac{c^4\left[2-5c^2(n+1)\right](ct+d)^5}{3\alpha^3(n+1)^2}\right\} \quad (53)$$

Eos parameter for holographic dark energy

$$\omega_\Lambda = \frac{\left\{\frac{-nc-c}{(n+1)^2(ct+d)^2} + \frac{c^2\left[2-5c^2(n+1)\right](ct+d)^4}{\alpha^2(n+1)^3} + \frac{(ct+d)^4}{4\alpha} - \frac{c^4\left[2-5c^2(n+1)\right](ct+d)^5}{3\alpha^3(n+1)^2}\right\}}{\left\{\frac{c^2(1+2n)}{(n+1)^2(ct+d)^2} + \frac{192nc^4(ct+d)^4}{64\alpha^2(n+1)^3} - \frac{2(ct+d)^4}{\alpha}\right\} - k(ct+d)^{\left(\frac{(1+\omega_m-\sigma)(n+2)}{3(n+1)}\right)}}, \quad (54)$$

II. Conclusion

In the investigations of interaction and non-interaction between dark energy and dark matter within the frame work of $f(G)$ gravity using a spatially homogeneous and anisotropic Space-time following results are obtained.

From equations (12) to (18), It is observed that Spatial volume of the Universe starts with constant value at $t \to 0$ and with big bang at t_s, with the increase of time it always expands. Thus, inflation is possible in this model. This shows that the Universe starts evolving with zero volume and expands with time *t*.

At the initial epoch, the Hubble parameter and expansion scalar both are constant and approaches to zero monotonically at $t \to \infty$, but at t_s both are infinitely large.

The Hubble parameter, scalar expansion and shear are the functions of time and decreases as *t* increases and approaches null at later time. This suggested that at initial stage of the Universe, the expansion of the model is much faster and then slow down for later time this shows that the evolution of the Universe starts with infinite rate and with the expansion it declines.

It is observed that the spatial volume is zero at $t = t_s$, where $t_s = -d/c$ and expansion scalar is infinite, which shows that at that point the universe starts evolving from zero volume with an infinite rate of expansion, the scale factors also vanish at $t = t_s$ and hence the model has a point singularity at the initial epoch. Hubble's factors and shear scalar diverge at the singularity. The universe exhibits the power law expansion after the big bang impulse. As *t* increases the scale factor and spatial volume increase but the expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Shear scalar decrease as *t* increases. As $t \to \infty$, scale factors and volume become infinite. The anisotropy parameter is constant throughout the universe. Hence model does not approaches to isotropy. Thus the model represents shearing, non-rotating and expanding model of the universe with a big-bang start but not approaching to isotropy at late times.

III. REFERENCES

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