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Flow Characteristic Through Convergent-Divergent Nozzle

S. Sathyapriya[1], R. Swathi[2], P. Bavya Seeli[3]
[1]Assistant Professor of Sri Krishna Arts And Science College. Coimbatore, TamilNadu, India
[2,3]Sri Krishna Arts and Science College, Coimbatore, TamilNadu, India

ABSTRACT

In this paper, A nozzle is a device designed to control the direction or characteristics of a fluid flow (especially to increase velocity) as it exits (or enter) an enclosed chamber or pipe via orifice. A numerical study has been carried out to analyze the mass flow rate in terms of Pressure ratio, area ratio and mach number in the convergent-divergent nozzle.
Keywords : Mach Number, Sub-Sonic, Super-Sonic, Sonic, Throat

I. INTRODUCTION

1.1 NOZZLE:

A nozzle is often a pipe or tube of varying cross sectional area, and it can be used to direct or modify the flow of a fluid (liquid or gas). Nozzles are frequently used to control the rate of flow, speed, direction, mass, shape, and/or the pressure of the stream that emerges from them. Three types of nozzle are convergent, divergent, convergent-divergent nozzle

1.2 CONVERGENT- DIVERGENT NOZZLE:

The nozzle was developed by Swedish inventor Gustaf de Laval in 1897 for use on an impulse steam turbine. A de Laval nozzle (or convergent-divergent nozzle, CD nozzle or con-di nozzle) is a tube that is pinched in the middle, making an hourglass-shape. The cross sectional area first decreases from its entrance to the throat and then again increases from throat to the exit. This case is used in the case where the back pressure is less than the critical pressure. Also, in present day application, it is widely used in many types of steam turbines and also in modern rocket engine and supersonic jet engines.

1.3 SPEED OF SOUND:

An important consequence of compressibility of the fluid is that the disturbances introduced at some point in the fluid propagate at finite velocity. The velocity at which these disturbances propagate is known as “acoustic velocity/speed of sound”. Mathematically, it is represented as below;

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_v}{\rho}}$$

$$E_v = p$$

$$c = \sqrt{\frac{p}{\rho}}$$

In an isothermal process,

$c = \sqrt{RT}$ (for an ideal gas medium)

$$E_v = \gamma p$$

$$c = \sqrt{\frac{\gamma p}{\rho}}$$

In isentropic process,

$c = \sqrt{\gamma RT}$ (for an ideal gas medium)

1.4 MACH NUMBER :

The Mach number is the ratio of flow velocity after a certain limit of the sounds speed. The formula that represents it is:

M = u/c

The Mach number is M,

Based on the limits the local flow velocity is u,

The speed of sound in that medium is c.

To explain it simply, the speed of sound can be equated to Mach 1 speed.

II. FLOW REGIME CLASSIFICATION

2.1 SUBSONIC FLOW:

The subsonic flow region is on the right of the incompressible flow region. In subsonic flow, fluid velocity (c) is less than the sound velocity (a) and the mach number in this region is always less than unity.

i.e. $m = c/a = 1$.

Eg: passenger air craft

2.2 SONIC FLOW:

If the fluid velocity (c) is equal to the sound velocity (a), that type of flow is known as sonic flow .In sonic flow Mach number value is unity.

i.e. $M = c/a = 1 \Rightarrow c = a$.

Eg: nozzle throat

2.3 SUPERSONIC FLOW:

The supersonic region is in the right of the transonic flow region. In supersonic flow, fluid velocity (c) is more than the sound velocity (a) and the Mach number in this region is always greater than unity.

i.e. $M = c/a > 1$.

Eg: military air crafts

2.4 HYPERSONIC FLOW:

In hypersonic flow region, fluid velocity (c) is much greater than sound velocity (a).In this flow, Mach number value is always greater than 5.

i.e. $M = c/a > 5$.

Eg: rockets

2.5 TRANSONIC FLOW:

If the fluid velocity close to the speed of sound, that type of flow is known as transonic flow .In transonic flow, Mach number value is in between 0.8 and 1.2.

i.e. $0.8 < M < 1.2$.

III. OPERATION OF C-D NOZZLE:

For understanding the working principle of convergent-divergent type of nozzles, first we need to look the working principle of only convergent type of nozzles. In these type of nozzles the area of the nozzle reduces gradually in the direction of flow. The pressure at intake is called stagnation pressure and the pressure at exit is called back pressure. The value of back pressure can never be more than 1 in case of a nozzle. As we start reducing the back pressure we observe that flow velocity and mass flow rate also starts increasing, but this will happen upto a certain limit, after which no increase in velocity or mass flow rate takes place. This situation is known as choked i.e no further increase in mass flow rate takes place whatever be the back pressure now. This situation takes place at a particular mach number i.e at mach number '1'.

But the case is not the same when we use a divergent nozzle just after the convergent. Actually the principle reverses i.e when we attach a divergent nozzle just after the convergent nozzle our flow speed starts increasing with the decrease in back pressure and also the mass flow rate. And therefore in this type of nozzles we can reach to the speeds above sonic i.e supersonic.

IV. FLOW THROUGH C-D NOZZLE

The usual configuration for a converging diverging (CD) nozzle is shown in the figure. Gas flows through the nozzle from a region of high pressure (usually referred to as the chamber) to one of low pressure (referred to as the ambient or tank). The chamber is usually big enough so that any flow velocities here are negligible. The pressure here is denoted by the symbol pc. Gas flows from the chamber into the converging portion of the nozzle, past the throat, through the diverging portion and then exhausts into the ambient as a jet. The pressure of the ambient is referred to as the 'back pressure' and given the symbol pb.

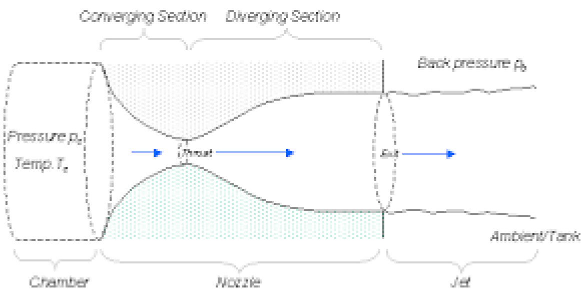

Figure 1. Converging Diverging Nozzle Configuration

Total Temperature:

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2$$

Total Pressure:

$$\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Total Velocity:

$$\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}$$

We have

$$M = 1 \quad , \; T = T^*, P = P^*, \rho = \rho^*$$

$$\frac{T_0}{T^*} = \frac{\gamma+1}{2}$$

$$\frac{P_0}{P^*} = \left[\frac{\gamma+1}{2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho^*} = \left[\frac{\gamma+1}{2}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{T}{T^*} = \frac{T_0/_{T^*}}{T_0/_{T}} = \frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M^2}$$

$$\frac{P}{P^*} = \frac{P_0/_{P^*}}{P_0/_{P}} = \frac{\left[\frac{\gamma+1}{2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{\rho}{\rho^*} = \frac{\rho_0/_{\rho^*}}{\rho_0/_{\rho}} = \frac{\left[\frac{\gamma+1}{2}\right]^{\frac{1}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}}$$

Reference mach number

$$M^{*2} = \frac{\left(\frac{\gamma+1}{2}\right)M^2}{1+\left(\frac{\gamma-1}{2}\right)M^2}$$

$$M^2 = \frac{\left(\frac{2}{\gamma+1}\right)M^{*2}}{1-\left(\frac{\gamma+1}{\gamma-1}\right)M^{*2}}$$

W.K.T

Area ratio as function of mach number

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{(\gamma+1)} + \left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{A^*} \times \frac{P_0}{P} = \frac{\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left(1+\frac{(\gamma-1)M^2}{2}\right)^{\frac{1}{2}}}$$

V. MASS FLOW RATE

5.1 MASS FLOW RATE IN TERMS OF PRESSURE RATIO:

From continuity equation the mass flow ratio is

$$m = \rho A C$$

For pressure and density relation

$$\frac{\rho_0}{\rho} = \left[\frac{P_0}{P}\right]^{\frac{1}{\gamma}}$$

$$\rho = \frac{\rho_0}{\left[\frac{P_0}{P}\right]^{\frac{1}{\gamma}}}$$

$$\rho = \rho_0\left[\frac{P}{P_0}\right]^{\frac{1}{\gamma}}$$

Temperature

$$T_0 = T + \frac{C^2}{2C_P}$$

$$\left(C_P = \frac{\gamma}{\gamma-1}R\right)$$

$$C^2 = 2\frac{\gamma}{\gamma-1}R(T_0 - T)$$

$$= 2\frac{\gamma}{\gamma-1}RT_0\left(1 - \frac{T}{T_0}\right)$$

W.K.T

$$\frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$C^2 = 2\frac{\gamma}{\gamma-1}RT_0\left[1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$C = \sqrt{2\frac{\gamma}{\gamma-1}RT_0\left[1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Substitute equation 3 and 4 in 2

$$m = \rho_0\left[\frac{P}{P_0}\right]^{\frac{1}{\gamma}} \times A \times \sqrt{2\frac{\gamma}{\gamma-1}RT_0\left[1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

W.K.T $\rho_0 = \frac{P_0}{RT_0}$

$$m = \frac{P_0}{RT_0} \times \left[\frac{P}{P_0}\right]^{\frac{1}{\gamma}} \times A \times \sqrt{2\frac{\gamma}{\gamma-1}RT_0\left[1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{AP_0}{\sqrt{RT_0}} \times \left[\frac{P}{P_0}\right]^{\frac{1}{\gamma}} \times \sqrt{2\frac{\gamma}{\gamma-1}\left[1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{AP_0}{\sqrt{RT_0}} \times \sqrt{2\frac{\gamma}{\gamma-1}\left[\left(\frac{P}{P_0}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$= \frac{AP_0}{\sqrt{RT_0}} \times \sqrt{\gamma} \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P}{P_0}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$m = \frac{AP_0}{\sqrt{T_0}} \times \frac{\sqrt{\gamma}}{\sqrt{R}} \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P}{P_0}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$\frac{\mathbf{m \times \sqrt{T_0}}}{\mathbf{AP_0}} \times \frac{\sqrt{\mathbf{R}}}{\sqrt{\boldsymbol{\gamma}}} = \sqrt{\frac{\mathbf{2}}{\boldsymbol{\gamma}-\mathbf{1}}\left[\left(\frac{\mathbf{P}}{\mathbf{P_0}}\right)^{\frac{2}{\gamma}} - \left(\frac{\mathbf{P}}{\mathbf{P_0}}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

This equation gives mass flow rate in terms of pressure ratio.

For maximum flow rate condition $m = m_{max}$ and $A = A^*$

W.K.T $\left(\frac{P}{P_0}\right)_{max} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

$$\frac{\mathbf{m_{max} \times \sqrt{T_0}}}{\mathbf{A^* P_0}} \times \frac{\sqrt{\mathbf{R}}}{\sqrt{\boldsymbol{\gamma}}} = \left(\frac{\mathbf{2}}{\boldsymbol{\gamma}+\mathbf{1}}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

5.2 MASS FLOW RATE IN TERMS OF AREA RATIO:

Mass flow rate $m = \rho AC = \rho^* A^* C^*$

$\div$ by A

$$\frac{m}{A} = \rho C = \rho^* \frac{A^*}{A} C^*$$

W.K.T

$$\rho^* = \frac{P^*}{R^* T^*}, \quad C^* = a^* = \sqrt{\gamma RT^*}$$

Substitute ρ^* and C^* value in Equation 6

$$\frac{m}{A} = \frac{P^*}{R^* T^*} \times \frac{A^*}{A} \times \sqrt{\gamma RT^*}$$

$$\frac{m}{A} = \frac{P^*}{\sqrt{T^*}} \times \frac{A^*}{A} \times \sqrt{\frac{\gamma}{R}}$$

W.K.T

$$\frac{T_0}{T^*} = \frac{\gamma+1}{2} \Rightarrow T^* = \left(\frac{2}{\gamma+1}\right) T_0$$

$$\frac{P_0}{P^*} = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P^* = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} P_0$$

Sub equation T^* and P^* in 7

$$\frac{m}{A} = \frac{A^*}{A} \times \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} P_0}{\sqrt{\left(\frac{2}{\gamma+1}\right) T_0}} \times \sqrt{\frac{\gamma}{R}}$$

$$\frac{m}{A} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \sqrt{\left(\frac{\gamma+1}{2}\right)} \times \frac{P_0}{\sqrt{T_0}} \times \sqrt{\frac{\gamma}{R}} \times \frac{A^*}{A}$$

$$\frac{m\sqrt{T_0}}{AP_0} \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{\gamma+1}{2}\right)^{\frac{1}{2}} \times \frac{A^*}{A}$$

$$\frac{m\sqrt{T_0}}{AP_0} \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{A^*}{A}$$

For maximum flow rate condition, $m = m_{max}$

$$\frac{\mathbf{m_{max}\sqrt{T_0}}}{\mathbf{A^* P_0}} \sqrt{\frac{\mathbf{R}}{\boldsymbol{\gamma}}} = \left(\frac{\mathbf{2}}{\boldsymbol{\gamma}+\mathbf{1}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

5.3 MASS FLOW RATE IN TERMS OF MACH NUMBER:

From equation 1

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Substitute Equation 9 in 8

$$\frac{m\sqrt{T_0}}{AP_0} \sqrt{\frac{R}{\gamma}} = \frac{\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{\frac{1}{M}\left[\frac{2}{\gamma+1} + \left(1+\frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times M}{\left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(1+\frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{M}{\left[1+\frac{\gamma-1}{2} M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{M}{\left[1+\frac{\gamma-1}{2} M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\frac{\mathbf{m\sqrt{T_0}}}{\mathbf{AP_0}} \sqrt{\frac{\mathbf{R}}{\boldsymbol{\gamma}}} = \frac{\mathbf{M}}{\left[\frac{\gamma+1}{2} \mathbf{M^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

This equation gives the mss flow rate in terms of Mach number for maximum mass flow rate condition, $m = m_{max}$

M=1

$$\frac{\mathbf{m\sqrt{T_0}}}{\mathbf{A^* P_0}} \sqrt{\frac{\mathbf{R}}{\boldsymbol{\gamma}}} = \frac{\mathbf{1}}{\left[\frac{\gamma+1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

VI. PROBLEM

1. Gas passes through a CD Nozzle with the pressure at inlet is 235 kN/m², pressure at the outlet is 55 kN/m², and area of the inlet is 0.5 m².Assume temperature at the outlet is 823 k Find the mass flow through the nozzle by using pressure ratio equation while specific heat ratio is 1.4 and consider the mach number as 1.

Given:

Pressure P_0= 235 kN/m²=235× 10^3N/m2

Pressure $P = 55\ kN/m2 = 55 \times 10^3 N/m2$

Area A=0.5m^2

Temperature T_0=823K

Specific Heat Ratio $\gamma = 1.4$

Mach number M=1

Universal Gas Constant

$R = 8314.5\,\mathrm{J/Kmol.k}$

Solution:

$$\frac{m\times\sqrt{T_0}}{AP_0}\times\frac{\sqrt{R}}{\sqrt{\gamma}}=\sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P}{P_0}\right)^{\frac{2}{\gamma}}-\left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$m=\frac{AP_0}{\sqrt{T_0}}\times\frac{\sqrt{\gamma}}{\sqrt{R}}\sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P}{P_0}\right)^{\frac{2}{\gamma}}-\left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$=\frac{0.5\times235\times10^3}{\sqrt{823}}\times\sqrt{\frac{1.4}{8314.5}}\sqrt{\frac{2}{1.4-1}\left[\left(\frac{55\times10^3}{235\times10^3}\right)^{\frac{2}{1.4}}-\left(\frac{55\times10^3}{235\times10^3}\right)^{\frac{1.4+1}{1.4}}\right]}$$

$=4095.7925\times0.01237\times0.60611$

$m=30.7085\,\mathrm{kg/s}$

2. In a nozzle gas flow at a temperture298k, pressure at inlet is 300 kN/m^2 and area of inlet is 1.2 m^2 and area of the throttle is 0.6 m^2 assume specific heat ratio is 1.4. Find the mass flow rate through the nozzle by using area ratio equation.

Given:

Pressure P_0= 300 kN/m^2=$300\times10^3 N/m^2$

Inlet Area A=$1.2m^2$

Area $A^* = 0.6m^2$

Temperature T_0=298

Specific Heat Ratio $\gamma = 1.4$

Universal Gas Constant

$R = 8314.5\,\mathrm{J/Kmol.k}$

Solution:

$$\frac{m\sqrt{T_0}}{AP_0}\sqrt{\frac{R}{\gamma}}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\times\frac{A^*}{A}$$

$$m=\frac{AP_0}{\sqrt{T_0}}\times\sqrt{\frac{\gamma}{R}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\times\frac{A^*}{A}$$

$$=\frac{1.2\times300\times10^3}{\sqrt{298}}\times\sqrt{\frac{1.4}{8314.5}}\left(\frac{2}{1.4-1}\right)^{\frac{1.4+1}{2(1.4-1)}}$$

$=20854.2400\ (0.01297)\ (59.4891)$

$m=16090.5\,\mathrm{kg/s}$

3. Find the mass flow rate and maximum mass flow rate through the nozzle while inlet pressure is 156 kN/m^2, At the temperature 550 k in the area $0.4m^2$ with the mach number 1. consider specific heat ratio is 1.3.

Given:

Pressure $P_0 = 156\,kN/m^3 = 156\times10^3 N/m^2$

Area A=$0.4m^2$

Temperature T_0=550K

Specific Heat Ratio $\gamma = 1.3$

Mach number M=1

Universal Gas Constant

$R = 8314.5\,\mathrm{J/Kmol.k}$

Solution:

$$\frac{m\sqrt{T_0}}{A^*P_0}\sqrt{\frac{R}{\gamma}}=\frac{1}{\left[\frac{\gamma+1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$m=\frac{AP_0}{\sqrt{T_0}}\times\sqrt{\frac{\gamma}{R}}\frac{M}{\left[\frac{\gamma+1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$=\frac{0.4\times156\times10^3}{\sqrt{550}}\times\sqrt{\frac{1.4}{8314.5}}\frac{1}{\left[\frac{1.4+1}{2}(1)\right]^{\frac{1.4+1}{2(1.4-1)}}}$$

$=(2660.74)\ (0.01297)\left(\frac{1}{1.9361}\right)$

$m=17.8243\,\mathrm{kg/s}$

4. Dry air at a pressure of 12 bar and 300 °C is expanded isentropically through nozzle at a pressure of 2 bar. If the specific heat ratio is 1.5. Determine the maximum mass discharge through the nozzle of $150mm^2$.

Given:

Pressure $P_0 = 12\text{ bar} = 12\times10^5 N/m^2$

Pressure P=2 bar $= 2\times10^5 N/m^2$

Area A=$150mm^2 = 150\times10^{-6}m^2$

Temperature is 300°C

=300+273=573k

$T_0 = 573k$

Specific heat ratio $\gamma = 1.5$

Universal Gas Constant

$R = 8314.5\,\mathrm{J/Kmol.k}$

Solution:

$$\frac{m_{max}\times\sqrt{T_0}}{A^*P_0}\times\frac{\sqrt{R}}{\sqrt{\gamma}}=\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

Area A=A^*

$$m_{max}=\frac{A^*P_0}{\sqrt{T_0}}\times\sqrt{\frac{\gamma}{R}}\times\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$= \frac{150\times10^{-6}(12\times10^{5})}{\sqrt{573}} \times \sqrt{\frac{1.5}{8314.5}} \times \left(\frac{2}{1.5+1}\right)^{\frac{1}{2}\left(\frac{1.5+1}{1.5-1}\right)}$$

=7.5196 (0.01343) (4.47213)

m_{max}=0.4516kg/s

5. In rocket engine the air flows at a velocity of $500\,{}^{m}/_{s}$ at room temperature the density of the air $1.225\,{}^{kg}/_{m^3}$ and area of the nozzle is $0.5m^2$. If the specific heat ratio is 1.4 and consider mach number as 1. Find out the pressure through the nozzle.

Given:

Velocity C = $500\,{}^{m}/_{s}$

Room temperature is 25°C

0°C = 273k ∴25+273=298k

T_0=298K

Density $\rho = 1.225\,{}^{kg}/_{m^3}$

Specific Heat Ratio $\gamma = 1.4$

Mach number M=1

Area A=$0.5m^2$

Universal Gas Constant

$R = 8314.5\,J/Kmol.k$

Solution:

Mass flow rate m=ρAC

= (1.225)(500)(0.5)

$m = 306.25\,{}^{kg}/_{s}$

$$\frac{m\sqrt{T_0}}{AP_0}\sqrt{\frac{R}{\gamma}} = \frac{M}{\left[\frac{\gamma+1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\frac{1}{P_0} = \frac{\sqrt{\frac{\gamma}{R}}A\times M}{m\sqrt{T_0}\left[\frac{\gamma+1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$P_0 = \frac{m\sqrt{T_0}}{A\times M\sqrt{\frac{\gamma}{R}}\left[\frac{\gamma+1}{2}M^2\right]^{-\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{(306.25)\sqrt{298}}{(0.5)(1)\sqrt{\frac{1.4}{8.314}}\left[\frac{1.4+1}{2}(1^2)\right]^{-\frac{1.4+1}{2(1.4-1)}}}$$

$$= \frac{5286.6946}{(0.5)(0.4104)\left(\frac{2.4}{2}\right)^{-3}}$$

$$= \frac{5286.6946}{(0.2052)(0.5787)}$$

= 44538.24

$P_0 = 44.538\,kN/m^2$

VII. CONCLUTION

A numerical study has been carried out by using our derived equation the results of mass flow rate in terms of Pressure ratio, area ratio and mach number in the convergent-divergent nozzle.

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