© 2018 IJSRST | Volume 4 | Issue 2 | Print ISSN: 2395-6011 | Online ISSN: 2395-602X
Themed Section: Science and Technology

Influence of Thermal Stratification on Mhd Heat Transfer Flow Over an Exponentially Stretching Surface

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ABSTRACT

This study presents a mathematical analysis of MHD heat transfer flow over an exponentially stretching surface embedded in a thermally stratified medium. The similarity transformation technique is used to convert the governing partial differential equations into the ordinary differential equations and solved numerically using Finite difference scheme. The numerical solutions for dimensionless velocity and temperature profiles for various governing parameters are obtained and the results are discussed graphically.

Keywords : Thermal Stratification, Suction, Finite Difference Scheme, Magnetic parameter, Quasi-linearization.

I. INTRODUCTION

Flow of an incompressible viscous fluid and heat transfer phenomena over a stretching sheet have received great attention during the last decades owing to the abundance of practical applications in chemical and manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous casting of metals, wire drawing and glass blowing. Since the pioneering work of Sakiadis[1], various aspects of the problem have been investigated by many authors. Crane[2] studied a steady flow past a stretching sheet and presented a closed form solution to it. Gupta and Gupta [3] stressed that realistically, stretching surface is not necessarily continuous. Most of the available literature deals with the study of boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin. However, it is often argued that (Gupta and Gupta [3]) realistically, stretching of plastic sheet may not necessarily be linear. This situation was beautifully dealt by Kumaran and Ramanaiah [4] in their work on boundary layer fluid flow where, probably first time, a general quadratic stretching sheet has been assumed.

There has been a great interest in the study of magnetohydrodynamic flow and heat transfer in any medium due to the effect of a magnetic field on the boundary-layer flow control and on the performance of many systems using electrically-conducting fluids. This type of flow has attracted the interest of many researchers due to its application in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. Mahapatra and Gupta [5] analyzed steady orthogonal stagnation-point flow of an electrically-conducting fluid towards a stretching surface. Mahapatra, et al. [6] employed the homotopy analysis method to find analytical solutions for magnetohydrodynamic viscous stagnation-point flow of a power-law fluid over a stretching surface.

Heat transfer of the fluid has tremendous industrial applications for the thermal and moisture treatment of materials, cooling of fibers, paper production and metallurgical processes. Makinde and Aziz [7] analyzed the mixed convection from a convectively heated vertical plate to a fluid with internal heat generation. Khan and Pop [8] studied flow and heat transfer over a continuously moving flat plate in a porous medium. Fox *et al.* [9] studied the continuous moving flat plate with heat transfer and Pal [10] examined the combined effects of non-uniform heat transfer and thermal radiation on unsteady stretching permeable surface.

Sajid and Hayat [11] considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet by solving the problem analytically via homotopy analysis method (HAM). Recently, Bidin and Nazar [12] analysed the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet, which has been solved analytically by Sajid and Hayat [11]. Suction/injection (blowing) of a fluid through the bounding surface can significantly change the flow field. In general, suction tends to increase the skin friction, whereas injection acts in the opposite manner. The process of suction/blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery [13]. Suction is applied to chemical processes to remove reactants [14].

The flow due to a heated surface immersed in a stable stratified viscous fluid has been investigated experimentally and analytically in several studies such as Yang et al. [15], Jaluria and Gebhart [16] and Chen and Eichhorn [17]. Thermal stratification may arise when there is a continuous discharge of the thermal boundary layer into the medium, for example, a heated vertical surface embedded in a porous bed which is of limited extent in the direction of the plate. In such case, the thermal boundary layer eventually hits the ceiling, and at that point, it falls horizontally into the medium since it contains hotter fluid than the rest of the medium (hotter fluid is lighter than the colder fluid) [18]. Recently Swati [19] studied MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium.

The objective of this article is to study the effects of MHD by introducing the convective heat transfer past an exponentially stretching sheet embedded in a thermally stratified medium. Quasi-linearization method is employed to linearize the ordinary differential equations and then using finite difference scheme these equations are reduced to system of linear equations which are solved by Thomas algorithm. The effects of various parameters are examined and displayed through graphs.

II. MATHEMATICAL ANALYSIS

Let us consider the steady state, incompressible, boundary layer flow of an electrically conducting viscous fluid past a stretching sheet. Consider the origin is fixed, the positive $x-$ axis is along the direction of the flow and the $y-$ axis is measured normal to the sheet Two equal and opposite forces are applied along the X-axis, so that the wall is stretched keeping the origin fixed.

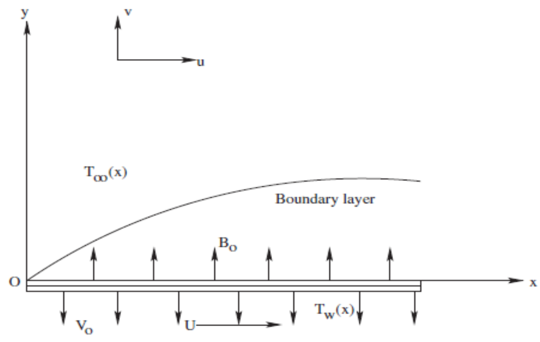

Figure 1 Sketch of the physical problem.

A variable magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$ is applied normal to the sheet, B_0 being a constant. The sheet is of temperature $T_w(x)$ and is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$ where $T_w(x) > T_\infty(x)$. It is assumed that $T_w(x) = T_0 + be^{\frac{x}{2L}}, T_\infty(x) = T_0 + ce^{\frac{x}{2L}}$ where T_0 is the reference temperature, $b > 0, c \geq 0$ are constants.

The continuity, momentum, and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{-- (1)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 \mu}{\rho} \quad \text{-- (2)}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad \text{-- (3)}$$

Here u and v are the velocity components in the *x*- and *y*-directions, respectively; $\nu = \frac{\mu}{\rho}$ is kinematic viscosity, ρ and μ are the density and viscosity of the fluid. C_p is the specific heat at constant pressure, k is the thermal conductivity of fluid.

The appropriate boundary conditions for the problem are given by

$$u = U, v = -V(x), T = T_w \; at \; y = 0 \quad \text{-- (4a)}$$

$$u \to 0, T = T_\infty \; as \; y \to \infty \quad (4b)$$

Introducing the suitable transformations as

$$\eta = \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y \, , \; u = U_0 e^{\frac{x}{L}} f'(\eta) \, , = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \{f(\eta) + \eta f'(\eta)\} \, , \;\; \theta = \frac{T - T_\infty}{T_w - T_0} \, , \quad \text{-- (5)}$$

and upon substitution of (5) in eqs. (2) and (3), the governing equations transform to

$$f''' + ff'' - 2\,(f')^2 - Mf' = 0 \quad \text{--(6)}$$

$$\theta'' + P_r(f\theta' - \theta f') - P_r St f' = 0 \quad \text{--(7)}$$

The corresponding boundary conditions are obtained as

$$f' = 1, f = S, \theta = 1 - St \, , \quad \text{at} \quad \eta = 0 \quad \text{-- (8)}$$

$$f' \to 0, \theta \to 0 \quad \text{as} \quad \eta \to \infty \quad \text{-- (9)}$$

where the primes denote differentiation with respect to η , $M = \frac{2\sigma B_0^2 L}{\rho U_0}$ is the magnetic parameter , $S = \frac{V_0}{\sqrt{\frac{U_0^x}{2L}}} > 0 (or < 0)$ is the suction (or blowing) parameter, $St = {}^{c}/_{b}$ is the stratification parameter and $P_r = \frac{\mu C_p}{k}$ is the Prandtl number. $St > 0$ implies a stably stratified environment, while $St = 0$ corresponds to an un stratified environment.

III. NUMERICAL PROCEDURE

The set of governing Equations (6) and (7) is highly nonlinear, so it is difficult to obtain closed form solution. Hence these equations together with the boundary conditions (8) and (9) are solved numerically using finite difference technique. The momentum equations is first linearized using Quasi-linearization technique [20] and then these linear ordinary differential equations are transformed into a system of linear equations by using implicit Finite difference Scheme. Now the computation procedure is employed to obtain the numerical solutions in which first the momentum equation is solved to obtain the values of f using which the solution of energy equation is solved under the given boundary conditions using Thomas algorithm for various parameters entering into the problem and computations were carried out by using The numerical solutions of f are considered as (n+1)th order iterative solutions and F are the nth order iterative solutions. After each cycle of iteration the convergence check is performed, and the process is terminated when $|F - f| < 10^{-4}$

IV. RESULTS AND DISCUSSIONS

The numerical results were discussed for the various values of the parameters such as suction parameter (S), stratification parameter (St), magnetic parameter (M) and Prandtl number (Pr) graphically.

The influence of suction parameter S on velocity and shear stress profiles, respectively are shown in figures 2 and 3, for exponentially stretching sheet. It is clear from the Figure 2, that the horizontal velocity decelerates with an increase in suction parameter. It is evident from Figure 3, that the shear stress decreases near the wall and a reverse phenomenon can be observed away from the wall. It is observed that, when the wall suction (S> 0) is considered, this causes a decrease in the boundary layer thickness and the velocity field is reduced.

The effect of variable suction on temperature and temperature gradient is plotted in figures 4 and 5. It is noticed that the effect of suction is to decrease the temperature. Whereas the effect of suction is to reduce the temperature near the wall and enhance the temperature away from the wall. Hence suction has a tendency to reduce both the hydrodynamic and thermal boundary layer thicknesses near the surface. The influence of magnetic field on horizontal velocity of the fluid in shown in figure 6. It is evident from the graph that the velocity decreases with the magnetic field which is due the Lorentz force which opposes the motion of fluid increases with the increase in M.

The effect of thermal stratification on temperature with and without suction are presented in figures 7 and 8. It is observed that the temperature falls as the stratification parameter St increases in both the cases (in the presence and absence of suction) which is obvious. Since increase in stratification means increase in free-stream temperature or decrease in surface temperature and hence thermal boundary layer thickness decreases with an increase in St values. Similarly figs. 9 and 10 are respectively the graphical representations of temperature gradient for several values of stratification parameter for non-porous (for S =0) and porous (for S > 0) sheet. It is seen from the plots that the temperature gradient increases considerably with an increase in stratification, St for both cases.

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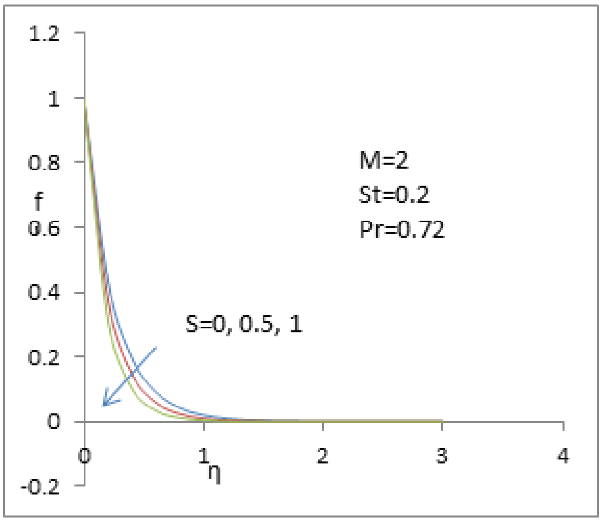

Figure 2. Variation of horizontal velocity for several values of suction parameter S

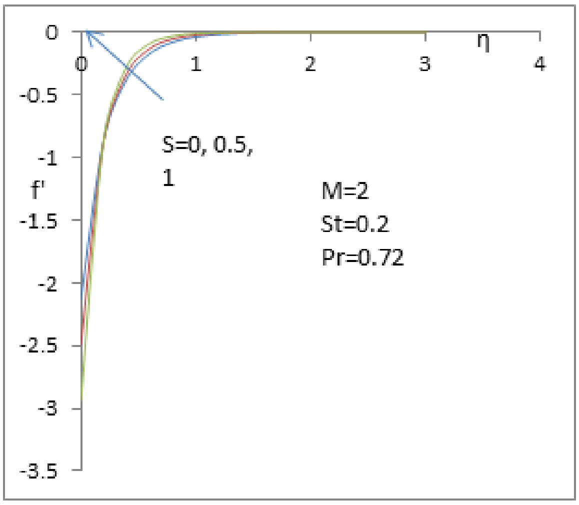

Figure 3. Variation of Shear stress for several values of suction parameter S

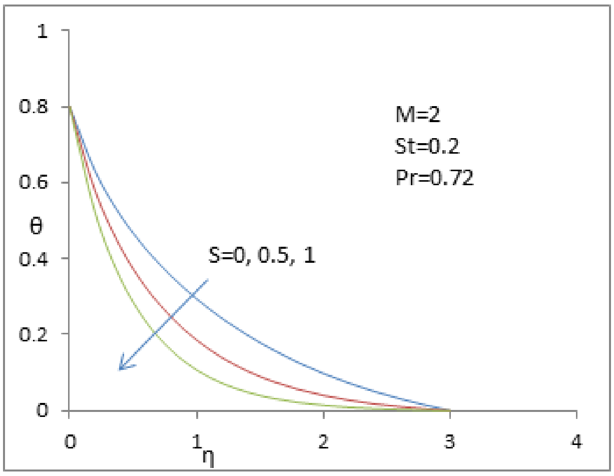

Figure 4. Variation of temperature for several values of suction parameter S

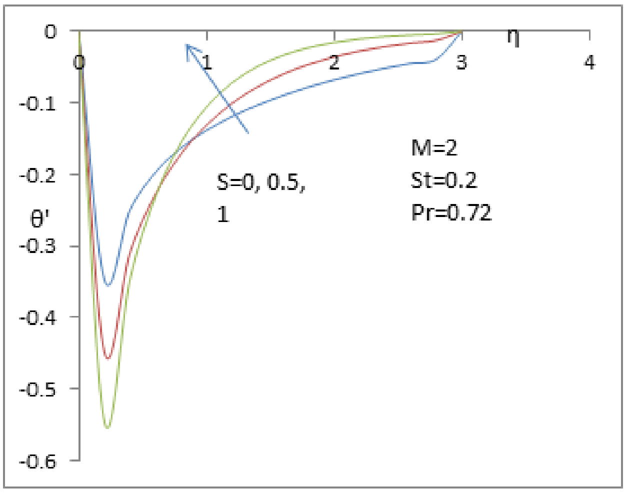

Figure 5. Variation of temperature gradient for several values of suction parameter S

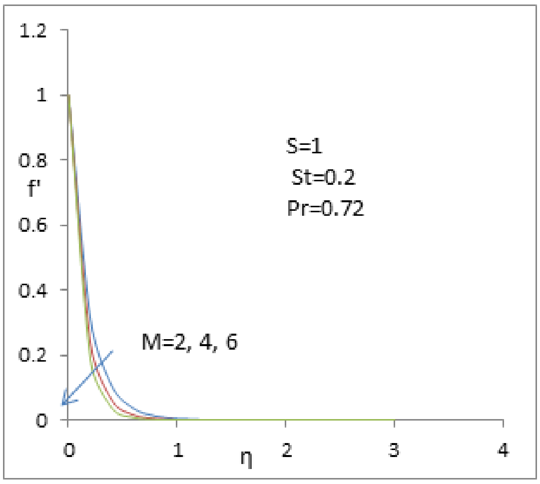

Figure 6. Variation of horizontal velocity for several values of Magnetic parameter M

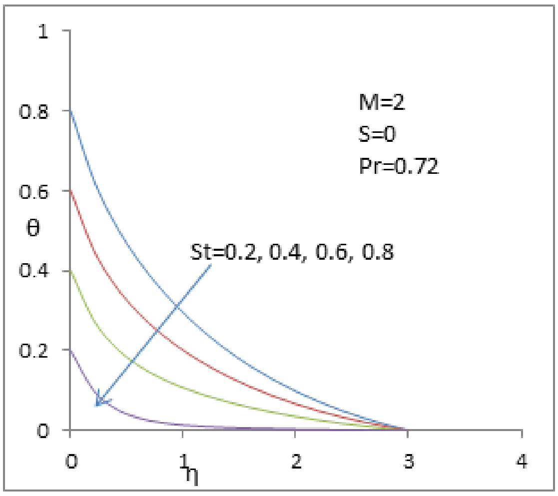

Figure 7. Variation of temperature for several values of thermal stratification parameter St in absence of suction

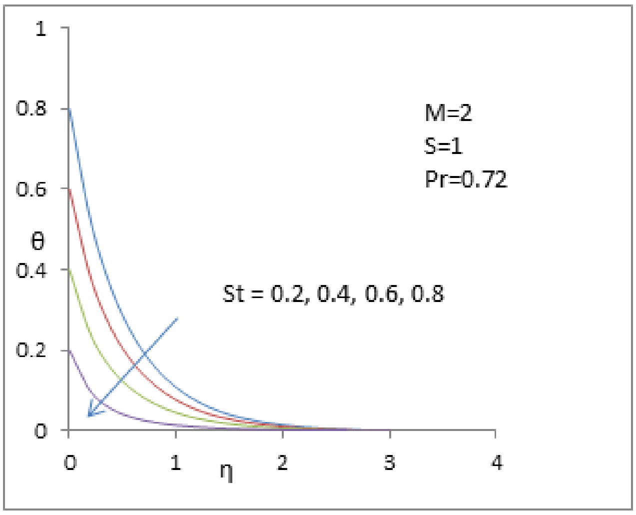

Figure 8. Variation of temperature for several values of thermal stratification parameter St in presence of suction

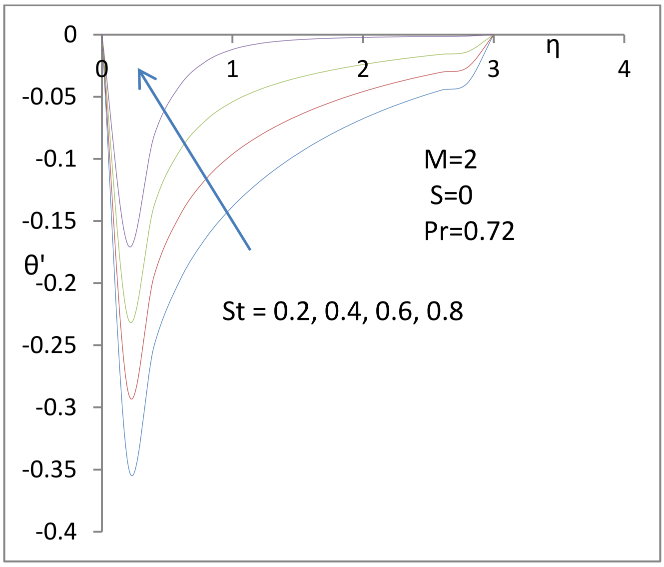

Figure 9. Variation of temperature gradient for several values of thermal stratification parameter St in absence of suction

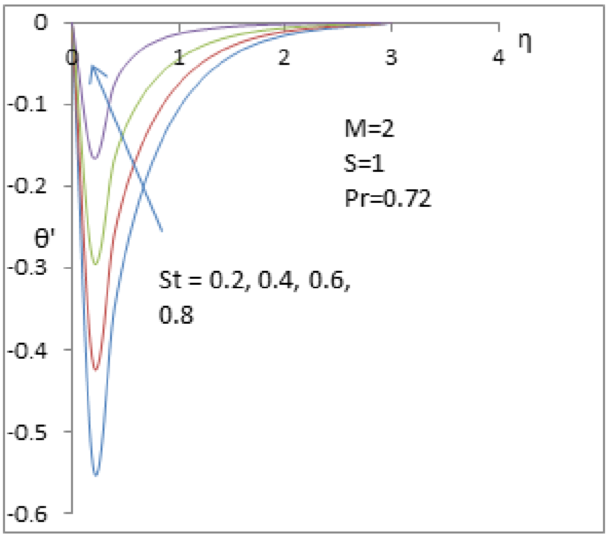

Figure 10. Variation of temperature gradient for several values of thermal stratification parameter St in presence of suction