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Construction of Control Chart Based on Six Sigma Initiatives for Regression

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ABSTRACT

A control chart is a statistical device used for the study and control of repetitive process. W.A. Shewhart [21] of Bell Telephone Laboratories suggested control charts based on the 3 sigma limits. Now the companies in developed and developing countries started applying Six Sigma initiatives in their manufacturing process, which results in lesser number of defects. The companies practicing Six Sigma initiatives are expected to produce 3.4 or less number of defects per million opportunities, a concept suggested by Motorola [22]. If the companies practicing Six Sigma initiatives use the control limits suggested by Shewhart, then no point fall outside the control limits because of the improvement in the quality of the process. In this paper an attempt is made to construct a control chart based on six sigma initiatives for Regression specially designed for the companies applying Six Sigma initiatives in their organization. Suitable Table – 2 is also constructed and presented for the engineers to take quick decisions.

Keywords: Control Chart, Process control, Six Sigma, Six Sigma Quality Level.

I. INTRODUCTION

The concept of Six Sigma was introduced by Motorola [22] by the engineer M.Harry who analyzed variations in outcomes of the company's internal procedures and realized that by measuring variations it will be possible to improve the working of the system. The procedure was aimed at taking action to improve the overall performance. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities. Radhakrishnan and Sivakumaran [15-20] used the concept of Six Sigma in the construction of sampling plans such as single, double and repetitive group sampling plans indexed through Six Sigma Quality Levels (SSQLs) with Poisson distribution as the base line distribution. Radhakrishnan [2] suggested single sampling plan indexed through Six Sigma quality levels (SSQLs) based on Intervened Random Effect Poisson Distribution and Weighted Poisson Distribution as the base line distributions. Radhakrishnan and Balamurugan [3-14] constructed control charts based on six sigma initiatives for defects, mean, average fraction defectives, number of defectives, X bar using standard deviation, Exponentially Weighted Moving Average (EWMA), proportion defectives – number of defectives, Fraction defectives, Standard deviation with variable sample size, average number of nonconformities per multiple units and number of defects - average number of defects per unit. The control charts originated by W.A. Shewhart [21] was based on 3 sigma control limits. If the same charts are used for the products of the companies which adopt six sigma initiatives in the process, then no point will fall outside the control limits because of the improvement in the quality. So a separate control chart is required to monitor the outcomes of the companies, which adopt six sigma initiatives.

In this paper an attempt is made to construct a control chart based on six sigma initiatives for Regression. Suitable Table 3 is also constructed and presented for the engineers to take quick decisions.

II. CONCEPTS AND TERMINOLOGIES

Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

Tolerance level (T_L)

It is the difference between USL and LSL, TL = USL-LSL

Process capability (Cp)

This is the ratio of tolerance level to six times standard deviation of the process.

$$c_p = \frac{TL}{6\sigma} = \frac{USL-LSL}{6\sigma}$$

Parameters for Regression (a & b):

The parameter 'a' is an intersection point of the fitted center line with the vertical axis and the parameter 'b' is the slope of the fitted center line.

Quality Control Constant ($C_{6\sigma}$)

The constant $C_{6\sigma}$ introduced in this paper to determine the control limits based on six sigma initiatives for Regression chart.

Subgroup size (n & N)

It is the choice of the sample size n and the frequency of sampling and 'N' is the total number of samples.

III. CONSTRUCTION OF CONTROL CHART BASED ON SIX SIGMA INITIATIVES FOR REGRESSION

Fix the tolerance level (TL) and process capability (C_P) to determine the process standard deviation ($\sigma_{6\sigma}$). Apply the value of $\sigma_{6\sigma}$ in the control limits $\left[a+\{C_{6\sigma}\times\sigma_{6\sigma}\}\right]+b_i$ & $\left[a-\{C_{6\sigma}\times\sigma_{6\sigma}\}\right]+b_i$, to get the control limits based on six sigma initiatives for Regression chart. The value of $C_{6\sigma}$ is obtained using $p(\text{z} \le \text{z}_{6\sigma}) = 1-\alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard normal variate. For a specified TL and C_P of the process, the value of σ (termed as $\sigma_{6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ using a C program and presented in Table 3 for various combinations of TL and C_P. Further the value of $C_{6\sigma}$ is also obtained using the procedure given above and presented in Table 3 for various combinations of TL and CP. The control limits based on six sigma initiatives for Regression chart are

$$\text{UCL}_{6\sigma} = \left[a+\{C_{6\sigma}\times\sigma_{6\sigma}\}\right]+b_i$$

$$\text{Central Line CL}_{6\sigma} = a+b_i$$

$$\text{LCL}_{6\sigma} = \left[a-\{C_{6\sigma}\times\sigma_{6\sigma}\}\right]+b_i$$

IV. CONDITIONS FOR APPLICATION

- ✓ Human involvement should be less in the manufacturing process
- ✓ The company adopts Six sigma quality initiatives in its processes

V. EXAMPLE

The example provided by Amitava Mitra [1] is considered here. The following data are the course of machining the diameter of steel hubs, tool wear is gradual and the sample $\overline{X}$ and the range **R** for 25 such samples with subgroup size 4:

Table 1. Sample mean and Range for tool wear data (in mm)

Sample	Average $\bar{X}$	Range R	Sample	Average $\bar{X}$	Range R	Sample	Average $\bar{X}$	Range R
1	36.2	8.0	10	52.6	7.8	19	64.2	13.5
2	42.4	11.8	11	50.4	11.3	20	61.4	9.4
3	38.6	6.2	12	59.5	15.1	21	66.7	16.6
4	45.5	14.3	13	60.5	11.7	22	63.2	12.2
5	53.1	16.2	14	53.8	8.8	23	62.1	10.5
6	46.7	9.5	15	54.5	12.8	24	64.5	12.6
7	55.4	10.2	16	61.2	14.5	25	69.6	14.7
8	42.8	12.0	17	60.4	12.0			
9	57.3	13.9	18	63.8	10.4			

Table 2. Calculations for determining Trend chart for tool wear data (in mm)

Sample i	Average $\bar{X}$	$\bar{X}_i$	i^2	R
1	36.2	36.2	1	8.0
2	42.4	84.8	4	11.8
3	38.6	115.8	9	6.2
4	45.5	182.0	16	14.3
5	53.1	265.5	25	16.2
6	46.7	280.2	36	9.5
7	55.4	387.8	49	10.2
8	42.8	342.4	64	12.0
9	57.3	515.7	81	13.9
10	52.6	526.0	100	7.8
11	50.4	554.4	121	11.3
12	59.5	714.0	144	15.1
13	60.5	786.5	169	11.7
14	53.8	753.2	196	8.8
15	54.5	817.5	225	12.8
16	61.2	979.2	256	14.5
17	60.4	1026.8	289	12.0
18	63.8	1148.4	324	10.4
19	64.2	1219.8	361	13.5
20	61.4	1228.0	400	9.4
21	66.7	1400.7	441	16.6
22	63.2	1390.4	484	12.2
23	62.1	1428.3	529	10.5
24	64.5	1548.0	576	12.6
25	69.6	1740.0	625	14.7

$\sum i = 325$	$\sum \bar{X} = 1386.4$	$\sum \bar{X}_i = 19,471.6$	$\sum i^2 = 5525$	$\sum R = 296$

$$a=\frac{(\sum \bar{X})(\sum i^2)-(\sum \bar{X}_i)(\sum i)}{N\sum i^2-(\sum i)^2}=\frac{(1386.4)(5525)-(19,471.6)(325)}{25(5525)-(325)^2}=40.972$$

$$b=\frac{N(\sum \bar{X}_i)-(\sum \bar{X})(\sum i)}{N\sum i^2-(\sum i)^2}=\frac{25(19,471.6)-(1386.4)(325)}{25(5525)-(325)^2}=1.114$$

$$\bar{R}=\frac{\sum_{i=1}^{N} R_i}{N}=\frac{296}{25}=11.84$$

Three Sigma Control limits for Regression chart

The 3σ control limits suggested by Shewhart (1931) are $\left(a+A_2\bar{R}\right)+b_i$ & $\left(a-A_2\bar{R}\right)+b_i$

$$UCL_{6\sigma} = \left[a+A_2\bar{R}\right]+b_i = \left[40.972+0.729(11.84)\right]+1.114i = 49.6034+1.114i$$

$$\text{Central Line } CL_{6\sigma} = a+b_i = 40.972+1.114i$$

$$LCL_{6\sigma} = \left[a-A_2\bar{R}\right]+b_i = \left[40.972-0.729(11.84)\right]+1.114i = 32.341+1.114i$$

From the resulting Figure 1, it is clear that the process is in control, since the entire subgroup numbers lie inside the control limits.

Control limits based on six sigma initiatives for Regression chart

For a given TL = 10.4 (USL-LSL =16.6-6.2) & C_p = 1.5, it is found from the Table-3 that the value of $\sigma_{6\sigma}$ is 1.16. The control limits based on six sigma initiatives for Regression chart for a specified TL and $C_{6\sigma}$ are $\left[a+\{C_{6\sigma}\times\sigma_{6\sigma}\}\right]+b_i$ & $\left[a-\{C_{6\sigma}\times\sigma_{6\sigma}\}\right]+b_i$ with

$$UCL_{6\sigma} = \left[a+\left\{\left(\frac{3}{\sqrt{n}}\right)\times\sigma_{6\sigma}\right\}\right]+b_i = \left[40.972+\{2.4155\times1.16\}\right]+1.114_i = 43.77+1.114i$$

$$\text{Central Line } CL_{6\sigma} = a+b_i = 40.972+1.114i$$

$$LCL_{6\sigma} = \left[a-\left\{\left(\frac{3}{\sqrt{n}}\right)\times\sigma_{6\sigma}\right\}\right]+b_i = \left[40.972-\{2.4155\times1.16\}\right]+1.114_i = 38.17002+1.114i$$

From the resulting Figure 1, the sample numbers **5, 7, 9, 12** and **13** goes above the upper control limit and the sample numbers **1, 3, 8, 23** and **24** goes below the lower control limit. Therefore the process does not exhibit statistical control.

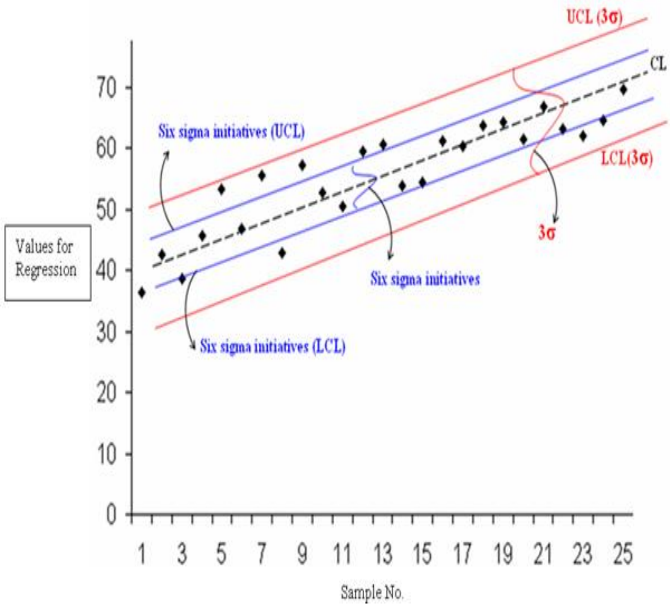

Figure 1. Comparison of the process: 3σ limits and control limits using six sigma initiatives

VI. CONCLUSION

In this paper, a procedure is given to construct a control chart based on six sigma initiatives for Regression with an example. It is found that the process was not in control even when six sigma initiatives are adopted. It is very clear from the comparison that when the process is centered with reduced variation many points fall outside the control limits than the 3 sigma control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. The charts suggested in this paper will be very useful for the companies practicing Six Sigma initiatives in their process. These charts will replace the existing Shewhart [21] control charts in future when all the companies started implementing six sigma initiatives in their organization.

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Table 3. $\sigma_{6\sigma}$ Values for a specified C_p and TL

C_p \ TL	10.1	10.2	10.3	10.4	10.5
1.0	1.68	1.7	1.72	1.73	1.75
1.1	1.53	1.55	1.56	1.58	1.59
1.2	1.40	1.42	1.43	1.44	1.46
1.3	1.29	1.31	1.32	1.33	1.35
1.4	1.20	1.21	1.23	1.24	1.25
1.5	1.12	1.13	1.14	**1.16**	1.17
1.6	1.05	1.06	1.07	1.08	1.09
1.7	0.99	1.00	1.01	1.02	1.03
1.8	0.94	0.94	0.95	0.96	0.97
1.9	0.89	0.89	0.90	0.91	0.92
2.0	0.84	0.85	0.86	0.87	0.88
2.1	0.80	0.81	0.82	0.83	0.83
2.2	0.77	0.77	0.78	0.79	0.80
2.3	0.73	0.74	0.75	0.75	0.76
2.4	0.70	0.71	0.72	0.72	0.73
2.5	0.67	0.68	0.69	0.69	0.70