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Thermal Deflection In A Semi-Infinite Solid Cylinder Subjected To Internal Heat Generation

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ABSTRACT

The present paper deals with the determination of thermal deflection in a semi-infinite circular cylinder defined as $0 \le r \le b,\ 0 \le z < \infty$ due to internal heat generation within it. A circular cylinder is considered having arbitrary initial temperature and subjected to time dependent heat flux at the fixed circular boundary $r = b$ whereas the zero temperature at the lower surface $(z = 0)$ of the semi-infinite circular cylinder. The governing heat conduction equation has been solved by using Integral transform method. The results are obtained in series form in terms of Bessel functions. The results for thermal deflection have been computed numerically and illustrated graphically.

Keywords: Thermoelastic problem, Non-homogeneous heat conduction equation, internal heat generation ,Semi-infinite circular cylinder.

I. INTRODUCTION

Boley and Weiner [1] studied the problems of thermal deflection of an axisymmetric heated circular plate in the case of fixed and simply supported edges. Roy choudhury [2] discussed the normal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face. This satisfies the time-dependent heat conduction equaton. Deshmukh and Khobragade [3] has determined a quasi-static thermal deflection in a thin circular plate due to partially distributed and axisymmetric heat supply on the outer curved surface with the upper and lower faces at zero temperature. Deshmukh et al. [5] has determined the thermal stresses in a hollow circular disk due to internal heat generation within it. Recently Deshmukh et al. [6] studied the thermal deflection in a thin circular plate subjected to heat generation within it.

In this paper the work of Deshmukh et al. [6] has been extended for two dimensional non-homogeneous boundary value problem of heat conduction and studied the thermal deflection of thin clamped semi-infinite solid circular cylinder defined as $0 \le r \le b,\ 0 \le z < \infty$ due to internal heat generation within it. A circular cylinder is considered having arbitrary initial temperature and subjected to time dependent heat flux at the fixed circular boundary $r = b$ whereas the zero temperature at the lower surface $(z = 0)$ of the semi-infinite circular cylinder. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of Bessel's functions. The results for thermal deflection have been computed numerically and are illustrated graphically. It is believe that this particular problem has not been previously considered.

The rotating hollow circular disk is having the applications in Aerospace engineering particularly in gas turbines and gears. The hollow circular disk is normally work under thermo-mechanical loads.

II. FORMULATION OF THE PROBLEM

A : HEAT CONDUCTION EQUATION

Consider a semi-infinite circular cylinder occupying space D defined by $0 \le r \le b,\ 0 \le z < \infty$. Initially the cylinder is at arbitrary temperature $F(r,z)$. The time dependent heat flux $f(z,t)$ is applied on the fixed circular boundary $(r=b)$ whereas the zero temperature at the lower surface $(z=0)$ of the semi-infinite circular cylinder. Heat generate within the semi infinite circular cylinder at the rate of $\frac{g(r,z,t)}{K}$. Under these conditions, the displacement and thermal stresses, in a semi-infinite circular cylinder due to heat generation are required to be determined.

The temperature of the semi-infinite circular cylinder satisfying the differential equation,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}+\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{g(r,z,t)}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{in } 0 \le r \le b,\ 0 \le z < \infty \tag{1}$$

with boundary conditions

$$k\frac{\partial T}{\partial r}=f(z,t) \quad \text{at } r=b \quad t>0 \tag{2}$$

$$T=0 \quad \text{at } z=0 \quad t>0 \tag{3}$$

and

the initial condition

$$T=F(r,z) \quad \text{when } t=0 \tag{4}$$

where K and α are thermal conductivity and thermal diffusivity of the material of the semi-infinite circular cylinder respectively.

B : THERMAL DEFLECTION

The differential equation satisfying the deflection function $\omega(r,t)$ is given as

$$\nabla^4\omega=-\frac{\nabla^2 M_T}{D(1-\nu)} \tag{5}$$

where, M_T is the thermal moment of the disk defined as

$$M_T = a_t\, E\int_0^h T(r,z,t)\, z\, dz \tag{6}$$

D is the flexural rigidity of the disk denoted as

$$D = \frac{Eh^3}{12\,(1-\nu^2)} \tag{7}$$

a_t, E and ν are the coefficients of the linear thermal expansion, the Young's modulus and Poisson's ratio of the disk material respectively and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{8}$$

Since, the circular edge of the semi-infinite circular cylinder is fixed are clamped;

$$\omega = \frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = b \tag{9}$$

Initially $T = \omega = F(r,z)$ at $t = 0$

Equations (1) to (9) constitute the mathematical formulation of the thermoelastic problem in a semi-infinite solid circular cylinder.

SOLUTION

A : HEAT CONDUCTION EQUATION

To obtain the expression for temperature distribution function $T(r,z,t)$we introduce the Fourier transform and its inverse transform over the variable z in the range $0 \le z < \infty$ defined in [3] as

$$\overline{T}(r,\eta,t) = \int_0^\infty K(\eta,z')\,T(r,z',t)\,dz' \tag{10}$$

$$T(r,z,t) = \int_0^\infty K(\eta,z)\,\overline{T}(r,\eta,t)\,d\eta \tag{11}$$

where

Kernel $K(\eta,z) = \sqrt{\frac{2}{\pi}}\sin \eta z$.

Taking the integral transform of system (1-5) by applying the transform equation (10), one obtains

$$\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} - \eta^2 \overline{T} + \frac{\overline{g}}{K}(r,\eta,t) = \frac{1}{\alpha}\frac{\partial \overline{T}}{\partial t} \tag{12}$$

$$k\frac{\partial \overline{T}}{\partial r} = \overline{f}(\eta,t) \quad \text{at} \quad r = b \tag{13}$$

$$\overline{T} = \overline{F}(r,\eta) \quad \text{for} \quad t = 0 \tag{14}$$

Secondly, we define the finite Hankel transform and its inverse transform over the variable r in the range $0 \le r \le b$ as

$$\overline{\overline{T}}(\beta_m,\eta,t) = \int_0^b r' K_0(\beta_m, r')\, \overline{T}(r',\eta,t)\, dr' \tag{15}$$

$$\overline{T}(r,\eta,t) = \sum_{m=1}^{\infty} K_0(\beta_m, r)\, \overline{\overline{T}}(\beta_m,\eta,t) \tag{16}$$

where kernel is $K_0(\beta_0, r) = \dfrac{\sqrt{2}}{b}\dfrac{J_0(\beta_m r)}{J_0(\beta_m b)}$ (17)

and $\beta_1, \beta_2, \ldots$ are the positive roots of the transcendental equation

$$J_1(\beta_m b) = 0 \ . \tag{18}$$

Now we take the integral transform of the system (12-14) by applying the transform (15), one obtains

$$\frac{d\overline{\overline{T}}}{dt} + \alpha\left(\eta^2 + \beta_m\right)\overline{\overline{T}} = A(\beta_m,\eta,t) \tag{19}$$

$$\overline{\overline{T}}(\beta_m,\eta,t) = \overline{\overline{F}}(\beta_m,\eta) \quad \text{for} \quad t = 0 \tag{20}$$

where $A(\beta_m,\eta,t) = \dfrac{\alpha}{k}\overline{\overline{g}}(\beta_m,\eta,t).$ (21)

Solution of the equation (19) is obtained as

$$\overline{\overline{T}}(\beta_m,\eta,t) = e^{-\alpha\left(\beta_m^2+\eta^2\right)t}\left[\overline{\overline{F}}(\beta_m,\eta) + \int_{t'=0}^{t} e^{\alpha\left(\beta_m+\eta^2\right)t'} A(\beta_m,\eta,t)\,dt'\right] \tag{22}$$

The resulting double transform of temperature is inverted successively by means of the inversion formulas (16) and (11). Then we obtain the expression of temperature $T(r,z,t)$ as

$$T(r,z,t) = \int_0^\infty \sum_{m=1}^{\infty} K(\eta,z) K_0(\beta_m r) e^{-\alpha\left(\beta_m^2+\eta^2\right)t}\left[\overline{\overline{F}}(\beta_m,\eta) + \int_0^t e^{\alpha\left(\beta_m+\eta^2\right)t'} A(\beta_m,\eta,t)\,dt'\right] d\eta \tag{22}$$

$$T(r,z,t) = \sqrt{\frac{2}{\pi}}\int_0^\infty \sum_{m=1}^{\infty} \sin \eta z K_0(\beta_m r) e^{-\alpha\left(\beta_m^2+\eta^2\right)t}\left[\overline{\overline{F}}(\beta_m,\eta) + \int_0^t e^{\alpha\left(\beta_m+\eta^2\right)t'} A(\beta_m,\eta,t)\,dt'\right] d\eta \tag{23}$$

$$T(r,z,t)=\frac{2}{\sqrt{\pi b}}\int_0^\infty\sum_{m=1}^\infty \sin\eta z\,\frac{J_0(\beta_m r)}{J_0(\beta_m b)}e^{-\alpha(\beta_m^2+\eta^2)t}$$

$$\times\left[\frac{2}{\sqrt{\pi b}}\int_0^b\int_0^\infty r'\sin\eta z'\,\frac{J_0(\beta_m r)}{J_0(\beta_m b)}F(r',z')dr'dz'\right. \tag{24}$$

$$\left.+\int_{t'=0}^{t}e^{\alpha(\beta_m+\eta^2)t'}\,\frac{\alpha}{K}\frac{2}{\sqrt{\pi b}}\int_0^b\int_0^\infty r'\sin\eta z'\,\frac{J_0(\beta_m r)}{J_0(\beta_m b)}g(r'z't')dz'dr'dt'\right]d\eta.$$

B. THERMAL DEFLECTION

Using Eq. 23 into Eq. 6, one obtains

$$M_T=\sqrt{\frac{2}{\pi}}a_t E\int_0^\infty\sum_{m=1}^\infty K_0(\beta_m r)e^{-\alpha(\beta_m^2+\eta^2)t}$$
$$\left[\overline{\overline{F}}(\beta_m,\eta)+\int_0^t e^{\alpha(\beta_m+\eta^2)t'}A(\beta_m,\eta,t)dt'\right]d\eta\int_0^\infty z\sin\eta z dz \tag{26}$$

Assume the solution of Eq.5 satisfy condition 9 as

$$\omega(r,t)=\sum_{m=1}^\infty C_m(t)\left[J_0(\beta_m r)-J_0(\beta_m b)\right] \tag{27}$$

where β_m' are the positive roots of the transcendental equation

$J_1(\beta_m b)=0$.

It can be easily shown that

$\omega=\frac{\partial\omega}{\partial r}=0$ at $r=b$

Now,

$$\nabla^4\omega=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^2\sum_{m=1}^\infty C_m(t)\left[J_0(\beta_m r)-J_0(\beta_m b)\right] \tag{28}$$

Using the well known result

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)J_0(\beta_m r)=-\beta_m^2 J_0(\beta_m r) \tag{29}$$

Substitute this value in above equation

$$\nabla^4\omega=\sum_{m=1}^\infty C_m(t)\beta_m^4\left[J_0(\beta_m r)-J_0(\beta_m b)\right] \tag{30}$$

$$\nabla^2 M_T = \sqrt{\frac{2}{\pi}} a_t E\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\int_0^\infty \sum_{m=1}^{\infty} K_0(\beta_m r) e^{-\alpha(\beta_m^2+\eta^2)t}$$

$$\left[\overline{\overline{F}}(\beta_m,\eta) + \int_0^t e^{\alpha(\beta_m+\eta^2)t'} A(\beta_m,\eta,t)dt'\right] d\eta \int_0^\infty z \sin \eta z dz$$

$$\nabla^2 M_T = \sqrt{\frac{2}{\pi}} a_t E \beta_m^2 \int_0^\infty \sum_{m=1}^{\infty} K_0(\beta_m r) e^{-\alpha(\beta_m^2+\eta^2)t}$$

$$\left[\overline{\overline{F}}(\beta_m,\eta) + \int_0^t e^{\alpha(\beta_m+\eta^2)t'} A(\beta_m,\eta,t)dt'\right] d\eta \int_0^\infty z \sin \eta z dz \quad (31)$$

Substituting eq 28 and eq 31 in eq 5 we get

$$\sum_{m=1}^{\infty} C_m(t)\beta_m^4 J_0(\beta_m r) = -\sqrt{\frac{2}{\pi}} a_t E \frac{1}{D((1-\nu))} \beta_m^2 \int_0^\infty \sum_{m=1}^{\infty} K_0(\beta_m r) e^{-\alpha(\beta_m^2+\eta^2)t}$$

$$\left[\overline{\overline{F}}(\beta_m,\eta) + \int_0^t e^{\alpha(\beta_m+\eta^2)t'} A(\beta_m,\eta,t)dt'\right] d\eta \int_0^\infty z \sin \eta z dz \quad (32)$$

$$C_m(t) = -\frac{2}{\sqrt{\pi b}} a_t E \frac{1}{D((1-\nu))} \int_0^\infty \frac{1}{\beta_m^2 J_0(\beta_m b)} e^{-\alpha(\beta_m^2+\eta^2)t}$$

$$\left[\overline{\overline{F}}(\beta_m,\eta) + \int_0^t e^{\alpha(\beta_m+\eta^2)t'} A(\beta_m,\eta,t)dt'\right] d\eta \int_0^\infty z \sin \eta z dz \quad (33)$$

Putting the value of eq 36 in eq 27 we get

$$\omega(r,t) = -\frac{2}{\sqrt{\pi b}} a_t E \frac{1}{D((1-\nu))} \int_0^\infty \frac{(J_0(\beta_m r) - J_0(\beta_m b))}{\beta_m^2 J_0(\beta_m b)} e^{-\alpha(\beta_m^2+\eta^2)t}$$

$$\left[\overline{\overline{F}}(\beta_m,\eta) + \int_0^t e^{\alpha(\beta_m+\eta^2)t'} A(\beta_m,\eta,t)dt'\right] d\eta \int_0^\infty z \sin \eta z dz \quad (34)$$

SPECIAL CASE

Setting:

$F(r,z) = z^2 \times (r^2 - b^2)^2$

$f(z,t) = z^2 \times e^{-\omega t}, \omega > 0$

$g(r,z,t) = g_{pi}.\delta(r - r_1).\delta(z - z_1).\delta(t - \tau)$

with $\omega = 10, t \to \tau = 5$, $g_{pi} = 50$

where r is the radius measured in meter, δ is the Dirac-delta function, $\omega > 0$.

The heat source $g(r,z,t)$ is an instantaneous point heat source of strength $g_{pi} = 50\ J/m$ situated at the center of the semi-infinite circular cylinder along radial direction and releases its heat instantaneously at the time $t \to \tau = 5$.

DIMENSIONS :

Radius of a semi-infinite circular plate $b = 1\ m$

Central circular paths of semi-infinite circular cylinder $r_1 = 0.5m$.

MATERIAL PROPERTIES:

The numerical calculation has been carried out for a Copper (Pure) thin circular cylinder with the material properties as,

Thermal diffusivity $\alpha = 112.34 \times 10^{-6} (m^2 s^{-1})$.

Thermal conductivity $K = 386 (W / mk)$.

Density $\rho = 8954\, kg / m^3$.

Specific heat $c_p = 383\, J / kgK$,

Poisson ratio $\nu = 0.35$,

Coefficient of linear thermal expansion $a_t = 16.5 \times 10^{-6} \dfrac{1}{K}$,

Lamé constant $\mu = 26.67$.

ROOTS OF TRANSCENDENTAL EQUATION:

The $\beta_1 = 3.8317$, $\beta_2 = 7.0156$, $\beta_3 = 10.1735$, $\beta_4 = 13.3237$, $\beta_5 = 16.470$,
$\beta_6 = 19.6159$, $\beta_7 = 22.7601$, $\beta_8 = 25.9037$, $\beta_9 = 29.0468$, $\beta_{10} = 32.18$

are the roots of transcendental equation $J_1(\beta b) = 0$.

We set for convenience,

$$X = \frac{2}{10^8 \sqrt{\pi b}},\ Y = \frac{2(1+\nu)a_t}{10^7 \sqrt{\pi b}} \text{ and } Z = \frac{4(1+\nu)a_t \mu}{10^7 \sqrt{\pi b}}$$

The numerical calculation has been carried out with the help of computational mathematical software Mathcad-2000 professional and the graphs are plotted with the help of Excel (MS office-2007).

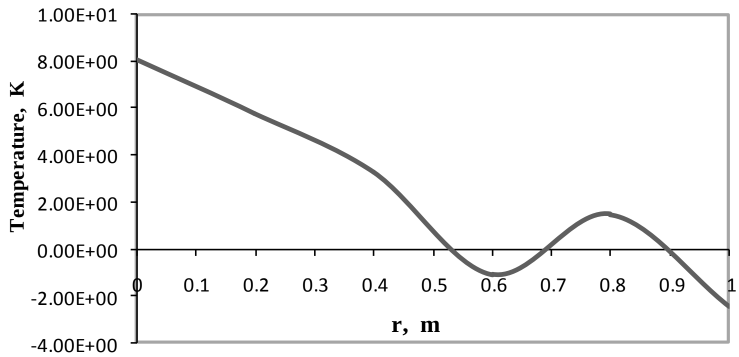

Figure 1. Temperature distribution

From fig. 1; it is observed that, the temperature is maximum at the centre of the cylinder due to arbitrary initial heat supply and goes on decreasing towards the mid point of the radius and it fluctuate due to point heat source towards the outer circular edge of the cylinder

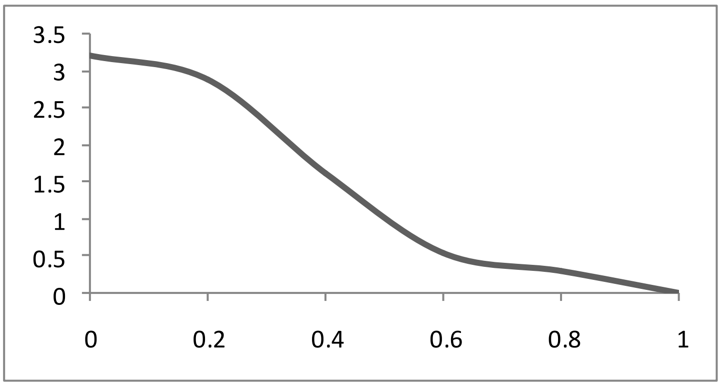

Figure 2. Deflection

From fig. 1; it is observed that, the deflection is maximum at the center and goes on decreasing towards end due to internal heat generation

III. CONCLUSION

In this paper we consider a semi infinite solid cylinder and discusses the quasi-static's thermal deflection at the outer surface .Thermal deflection is studied due to instantaneous point heat source of strength $g_{pi} = 50\ J/m$ situated at the center of the semi-infinite circular cylinder along radial direction and releases its heat instantaneously at the time $t \to \tau = 5$.

Exact analytical solution have been develop for thermal deflection in a semi infinite circular cylinder experiencing internal heat generation and subject to arbitrary initial .heat supply on its outer surface while the radial and the tangential component can be compressed or tensile in nature depending on the level of heat generation and the temperature distribution ,the axial stresses compressive throughout the cylinder because of built in edge imposed

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