

# Systematic Study of $K^{*0}$ and $\varphi$ – Meson Spectra with Tsallis Non-Extensive Statistics

Bhaskar De

Department of Physics, Taki Government College, Taki, West Bengal, India

# ABSTRACT

The transverse momentum spectra of two heavier mesons, namely  $K^{*0}$  and  $\varphi$ -mesons, produced in different central lead-lead collisions at center of mass energy 2.76 TeV per nucleon at Large Hadron Collider have been analysed, systematically, in the light of a Tsallis non-extensive statistics. The study reveals the possible formation of a deconfined partonic matter in such central nuclear collisions at ultrarelativistic energy where the number of participant nucleons is comparatively large.

Keywords : Relativistic Heavy Ion Collision, Inclusive Cross Section

# I. INTRODUCTION

The ultra-relativistic heavy ion collisions provide a platform to study the mechanism of particle production in presence of a hot and dense medium. The fireball, produced in such nuclear collision, is supposed to consist of hadronic constituents in a deconfined phase. The existence of such a deconfined phase cannot be probed directly as the formation time --- just of the order of nuclear is very brief characteristic time i.e. of the order of 10<sup>-23</sup> seconds. So, one has to depend on the various indirect signatures to probe the fireball medium. The transverse momentum spectra of different secondaries produced in such nuclear collisions is one of the various indirect signatures. The constituent partons(quarks and gluons) of the various secondaries undergo interactions with the fireball medium before formation of different bound states; and these interactions leave their traits on the final shape of these spectra, especially at the high transverse momentum region. Hence, these spectra carry information on the thermodynamical properties of the created fireball.

However, to extract knowledge on thermodynamical properties of the fireball, one needs a suitable

theoretical model/approach to analyse these spectra. Tsallis generalized non-extensive statistics[1] can be a good choice for statistical analysis of these experimental data emanated from a non-equilibrated system having non-markovian nature due to possible presence of long-range correlations and/or having signatures of event-by-event fluctuations [2-11]. In our earlier studies we had taken up the analyses of transverse momentum spectra for pi-mesons, Kmesons and protons/anti-protons[8,9] at LHC(Large Hadron Collider) energies. In the present study, we will be dealing, systematically, with two heavier mesonic varities ---  $K^{*0}/\overline{K}^{*0}$  and  $\varphi$  --- in the relatively high transverse momentum region produced in different central Lead-Lead(Pb-Pb) interactions at LHC energy 2.76 TeV/nucleon.

The organization of the present work is as follows: a brief outline of the theoretical approach, to be used in the present study, has been presented in next section (**Outline of the Theoretical Approach**). The obtained results and a detailed discussion on it have been provided in Section 3(**Results and Discussions**). And the last section(**Conclusions**) is preserved for the concluding remarks.

### **II. OUTLINE OF THEORETICAL APPROACH**

The Tsallis-Boltzmann(TB) distribution (neglecting the chemical potential) is given by

$$n_i(E_i) = \left[1 + (q-1)\frac{E_i}{T}\right]^{\frac{1}{1-q}}$$
(1)

where  $n_i$  is the number of states available for energy  $E_i$ , T is the temperature and q is the non-extensive index. As  $q \rightarrow 1$  the above equation returns to the usual Boltzmann distribution for equilibrated system

$$n_i(E_i) = e^{-\frac{E_i}{T}} \tag{2}$$

The non-extensive distribution for fermions(+) and for bosons(-) are given by

$$n_i(E_i) = \left[ \left( 1 + (q-1)\frac{E_i}{T} \right)^{\frac{1}{q-1}} \pm 1 \right]^{-1}$$
(3)

However, for high energy or high transverse momentum the difference between eqn.(1) and eqn.(3) becomes negligible. So, eqn.(1) can be used for all the varieties in the present case.

For thermodynamical consistencies, the generalized distribution, given in eqn.(1), is constrained by[5]

$$\sum_{i} n_i^q = N \tag{4}$$

where N is the total number of the particles. In the large volume limit the summation on the left hand side of the last equation is replaced by

$$\sum_{i} \to V \int \frac{d^3 p}{(2\pi)^3}$$

where V is the volume of the system and p is the momentum of the particle.

Hence, total number of particles is given by

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[ 1 + (q-1)\frac{E}{T} \right]^{\frac{q}{1-q}}$$
(5)

with g being the degeneracy factor.

So, the Lorentz invariant momentum distribution will be of the form

$$E\frac{dN}{d^3p} = \frac{gV}{(2\pi)^3} E\left[1 + (q-1)\frac{E}{T}\right]^{\frac{q}{1-q}}$$
(6)

The energy, E, of the particle is given by

$$E = m_T \cosh y \tag{7}$$

where  $m_T = \sqrt{m_0^2 + p_T^2}$  is the transverse mass with  $m_0$  being the rest mass and  $p_t$  being the transverse momentum; whereas  $y = \frac{1}{2} \ln \frac{E + p_Z}{E - p_Z}$  is a Lorentz invariant variable, called rapidity, with  $p_Z$  being the longitudinal momentum. In the non-relativistic limit, y gives simply the velocity of the particle.

Since,  $d^3p = 2\pi p_T dp_T dy$ , the expression for invariant yield will be of the form

$$\frac{dN}{dy p_T dp_T} = \frac{gV}{(2\pi)^2} m_T \cosh y \left[1 + (q-1) \frac{m_T \cosh y}{T}\right]^{\frac{q}{1-q}}$$
(8)

In the central rapidity region (y = 0) the last equation will take the form

$$\frac{dN}{dy \, p_T dp_T} = \frac{gV}{(2\pi)^2} \, m_T \, \left[ 1 + (q-1) \, \frac{m_T}{T} \right]^{\frac{q}{1-q}} \tag{9}$$

The temperature is nothing but the effective local temperature of the interaction region. It has been observed that this temperature is correlated with the non-extensive parameter, q, and the correlation is given by[3]

$$T = T_0[1 - c(q - 1)]$$
(10)

where  $T_0$  is the critical temperature for phase transition or Hagedorn temperature and the parameter c depends on (i) the fluctuation of the temperature around  $T_0$  due to a stochastic process in any selected region of the system and (ii) some energy transfer between the selected region and the rest of the systems.

The combination of equation (9) alongwith equation(10) gives the final working formula to analyse the experimental data on transverse momentum spectra and it is given by

$$\frac{dN}{dy \, p_T dp_T} = C \, m_T \left[ 1 \qquad (11) + (q-1) \, \frac{m_T}{T_0 (1 - c(q-1))} \right]^{\frac{q}{1-q}}$$

with  $C = \frac{gV}{2\pi^2} = \text{constant.}$ 

## **III. RESULTS AND DISCUSSIONS**

As mentioned earlier, in the present study, we have dealt with, two heavier mesons ---  $K^{*0}/\overline{K}^{*0}$  and  $\varphi$  --produced in different central interactions of Pb-Pb interactions at centre-of-mass energy 2.76 TeV per nucleon  $(\sqrt{s_{NN}} = 2.76 \text{ TeV})$ . The centrality of nuclear interactions is measured as a function of impact parameter which is the distance between the centres of the two nuclei. So, the overlapped region, on the transverse plane, between two nuclei, will be almost circular for most central collisions whereas that for non-central interactions will be of almondshapes. As the volume of the overlapped region decreases gradually from most central to peripheral collisions, the number of participant nucleons  $(N_{part})$ or the number of wounded nucleons will also decrease from central to peripheral collisions. Those nucleons, which do not take part in the interactions, are called spectator nucleons. The average values of  $N_{\text{part}}$  for various centrality classes are given in Table I [8].

 Table 1. number of participant nucleons for different

centralities				
Centrality	Number of			
Class	Participant Nucleons			
0-5%	382.8			
5-10%	329.7			
10-20%	260.5			
20-30%	186.4			
30-40%	128.9			
40-50%	85			

The fits to the experimental data on  $K^{*0}$  and  $\varphi$ -meson spectra obtained on the basis of the present working formula [Eqn.(11)] are presented graphically in Fig.1 and in Fig. 2 respectively whereas the values of different parameters obtained from the fits are provided in Table II and in Table III respectively.



**Figure 1.** Plots of transverse momentum spectra of  $\frac{(K^{*0}+\bar{K}^{*0})}{2}$  produced in Lead-Lead interactions at centre-of-mass-energy 2.76 TeV/nucleon. The experimental data are taken from Ref.[12]. The solid lines are the fits obtained on the basis of equation (11).



**Figure 2.** Plots of transverse momentum spectra of  $\varphi$ mesons produced in Lead-Lead interactions at centreof-mass-energy 2.76 TeV/nucleon. The experimental data are taken from Ref.[12]. The solid lines are the fits obtained on the basis of equation (11).

Centrality	Non-Extensive Parameter	Hagedorn	с	Reduced chi-
	(q)	Temperatur		square
		e		
		(T₀ in GeV)		
0-5%	1.137±0.004	0.133±0.008	$0.0040 \pm 0.000$	1.957
			3	
5-10%	1.138±0.006	$0.136 \pm 0.008$	$0.063 \pm 0.003$	1.901
10-20%	1.134±0.005	$0.164 \pm 0.007$	$0.329 \pm 0.007$	1.955
20-30%	1.130±0.007	0.19±0.02	$0.698 \pm 0.003$	1.787
30-40%	1.127±0.007	0.18±0.02	0.871±0.002	1.507
40-50%	1.124±0.008	0.183±0.009	$0.887 \pm 0.003$	0.993

Table 3. Values of Different Parameters from the Fits of  $\phi$ -meson spectra

Centrality	Non-Extensive Parameter	Hagedorn	с	Reduced chi-
	(q)	Temperatur		square
		e		
		(T₀ in GeV)		
0-5%	1.132±0.003	0.13±0.01	0.007±0.001	1.959
5-10%	1.132±0.003	0.13±0.02	0.047±0.002	1.900
10-20%	1.130±0.004	0.14±0.02	0.097±0.002	2.154
20-30%	1.127±0.003	0.14±0.02	0.119±0.003	1.964
30-40%	1.124±0.003	0.16±0.02	0.131±0.003	1.656
40-50%	1.122±0.007	0.17±0.03	0.506±0.004	1.691

The moderate values of the reduced chi-square ensure the applicability of the present statistical approach to the heavier secondaries, even at high transverse



**Figure 3.** The plot of the Hagedorn Temperature, obtained from fits, as a function of number of participant nucleons.

momenta. The centrality-dependence of different parameters,  $T_0$ , c and q are depicted pictorially in Figure 3 to Figure 5.



**Figure 4.** Centrality dependence of the parameter, *c*, obtained from fits.



**Figure 5.** Centrality dependence of the non-extensive index, q, obtained from fits.

It is observed from Figure 3 as well as from the Table-II and Table-III that the values of Hagedorn Temperature, obtained for both the varieties and for all the centralities, lie in the neighbourhood of the mass of the pi-meson ( $m_{\pi} \sim 0.140 \text{ GeV}$ ) and the average value of it is 0.151 GeV which is in agreement with the findings from the analysis of  $e^+e^$ collisions and lattice-QCD interactions, pp calculations[10,11,13,14]. This temperature is to be recognised as the critical temperature at which the hadronic boundary vanishes and the QCD(Quantum Chromo Dynamics) medium of the free partons or the QGP(quark-gluon plasma) evolves.

Figure 4 depicts the centrality dependence of the parameter, c. This parameter exhibits strong dependence on the centrality in terms of number of participant nucleons. As one moves from peripheral to central collisions, the number of participant nucleons increases and consequently the number of spectator nucleons decreases. As the number of spectator nucleons decreases, the possibility of transfer of energy from the interaction region to the spectator nucleons decreases, and hence this decrement in the value of c from peripheral to central collisions.

Figure 5 exhibits the nature of centrality dependence of the most important parameter, q, the non-extensive

index. q also exhibits strong centrality dependence. It shifts from it's equilibrium value 1 more and more from peripheral to central region, which indicates at the evolution of more non-equilibrated systems at most central interactions. In the central nuclear reactions at LHC energy, the particle production mechanism is governed by pQCD (perturbative QCD) hard scatterings of the high- $p_T$  partons followed by fragmentation and hadronization. Besides, multigluon fluctuations alongwith initial state fluctuations also lead to non-equilibrium nature of the central collisions[15,16]. And, hence, the high q –values at central collisions hints at the possible existence of the decoupled partonic medium, namely, QGP.

#### **IV. CONCLUSION**

In the present analysis we have dealt, systematically, with the spectra of two heavier mesons produced in various central Lead-Lead interactions at LHC centreof-mass energy 2.76 TeV/nucleon in the light of Tsallis non-extensive statistics. Earlier we had studied some lighter mesons and baryons, mainly in the low and intermediate transverse momentum region. It is observed from the present analyses that Tsallis nonextensive formalism reproduces experimental data even for heavier secondaries and at high transverse momenta where pQCD dominates the particle mechanism processes. Besides, the parameters associated with this formalism successfully extract the value of critical temperature for phase transition and hint at the possible formation of a weakly coupled hot and dense partonic matter in the most central interactions. So, in brief, Tsallis non-extensive statistics can be treated as a competent theoretical too to understand the thermo-dynamical evolution process of the QGP medium in a high energy nuclear reactions.

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