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A Brief Review of the Dynamical Universe in the Early and Late Stages

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ABSTRACT

A dynamical study of the universe is briefly reviewed in the early stage (inflation) as well as in the late era (cosmic doomsday). The cosmological model of the expanding universe, governed by the Friedman equations, is explored. It is indicated that the expansion is proved to be accelerated experimentally. In the early phase, just after the big bang, the expansion of the universe is rapid and approximately exponential, which is termed as inflation. The mathematical formulation and physical significance of the inflation is discussed briefly. A number of inflationary models are also listed. The ultimate fate of the universe is briefed in the scenario of dark energy and phantom energy. The possible singularities, arising in the phantom energy era, are also discussed.

Keywords : Expanding Universe, Inflation, Friedman Equations, Cosmological Singularities, Phantom Energy Era.

I. INTRODUCTION

It is believed that if the dynamical model of the universe is correct then it has been started to expand from a point, known as big bang. But it is not sure whether the universe will start to contract and will come back to the same point (big crunch) or it will expand forever. Let us go with a little history of the development of cosmology. It was a milestone in the cosmological study when Einstein introduced the concept of general theory of relativity and proposed the Einstein equation. In 1922 Friedmann derived a set of equations, known as Friedmann equation from Einstein equation [1], which shows that the universe expands. It was clearly a contrast of the static model of the universe advocated by Einstein at that time. In 1924 Hubble measured the great distance to the nearest spiral nebulae showing that these systems were indeed other galaxies. Hubble also proposed a correlation between distance and recession velocity [2], called Hubble's law. Lemaitre derived Friedmann equation independently and proposed that the inferred recession of the nebulae occurs as the universe expands [3]. He also suggested that the expansion in the forward time requires that the universe would contract backwards in time and continue until and unless it could bring all the mass into a single point, called a 'primeval atom', when and where the fabric time and space will come into existence. Alpher and Herman [4] predicted cosmic microwave background radiation (CMB) favouring big bang theory over steady state. Thus the concept of big bang theory was introduced. Lemaitre's big bang theory was later developed by Gammow. He and his collaborators introduced the concept of big bang nucleosynthesis [5]. Basically big bang theory depends on two major assumptions: the universality of physical law and cosmological principle. According to the cosmological principle the universe is homogeneous as well as isotropic.

II. EQUATION OF MOTION OF THE DYNAMICAL MODEL

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The big bang supposed to be a singular point at which the scalar factor a vanishes. At the big bang the density of the universe was extremely high. To formulate the scenario mathematically one must start with the Einstein's equation

$$R_{\mu\nu} - g_{\mu\nu}R = -\frac{8\pi G}{3}T_{\mu\nu} \qquad (1)$$

Here T stands for the energy momentum tensor including the vacuum energy arising from the cosmological constant. The FRW metric is given by

$$d\tau^2 = dt^2 - a^2[\frac{d\tau^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \qquad (2)$$

yields the well known Friedmann equations

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad (3)$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k \qquad (4)$$

where, $a(t)$ stands for the scale factor at the time t. The scale factor at the present moment is conventionally represented by a_0. k represents the nature of orientation of the universe. It takes the value 1, 0 or -1 accordingly the universe is closed, at or open. The conservation of energy yields another important equation

$$\dot{\rho} = -\frac{3\dot{a}}{a}(\rho + p) \qquad ()$$

The rate of expansion is denoted by the Hubble parameter $H = \frac{\dot{a}}{a}$.

In the very early stage of the universe the universe was supposed to be filled with positive vacuum energy represented by the physical condition $p = -\rho$ (> 0). The universe should be at the beginning and therefore

$$\dot{a}^2 = \frac{8\pi G}{3}\rho^{\Lambda} a^2 \qquad (6)$$

which implies an exponential expansion in the early stage. This is essentially the age of inflation, which was noticed by several authors. The energy density of the present universe can be divided into three parts: matter density ρ_M , radiation density ρ_R and the energy density due to vacuum. At the end of inflationary epoch the radiation epoch begins. It is quite easy to deduce the followings:

$$\rho_R \propto a^{-4}, \qquad \rho_M \propto a^{-3}, \qquad \rho_\Lambda = const \qquad (7)$$

The general Friedman equation takes the form

$$\dot{a}^2 = H_0^2\left[\Omega_\Lambda + \Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_M\left(\frac{a_0}{a}\right)^3\right]a^2 - k \qquad (8)$$

$$\ddot{a} = H_0^2\left[\Omega_\Lambda - \Omega_R\left(\frac{a_0}{a}\right)^4 - \frac{\Omega_M}{2}\left(\frac{a_0}{a}\right)^3\right]a \qquad (9)$$

where, all the terms Ω_Λ , Ω_R and Ω_M are fractions at the present universe.

In the present scenario the above two equations become

$$\Omega_\Lambda + \Omega_R + \Omega_M = \frac{k}{a_0^2 H_0^2} \qquad (10)$$

$$\ddot{a} = H_0^2\left[\Omega_\Lambda - \Omega_R - \frac{\Omega_M}{2}\right]a \qquad (11)$$

Now it is not quite clear what should be the sign of the double derivatives of a indicating acceleration or deceleration of the universe from the above equation. Observations of Type-Ia supernova determined the nature of the expansion in the present age. The Supernova Cosmology Project [6] estimated $\Omega_M = 0:28$ and $\Omega_\Lambda = 1 - \Omega_M$ (taking $\Omega_R = 0$ and $k = 0$) by analysing 42 Type Ia supernovae with redshift between 0.18 to 0.83. Fitting those values to the equation indicates the accelerated expansion of the universe. Another group, High-z Supernova Search Team [7, 8] in a at cosmology model estimated the same result, showing the expansion is accelerating. If the universe is open or closed the above interpretation remains same as the right hand side of the equation (11) does not contain k. One may now study the dynamics of the universe, especially in the early as well as end stage in more details.

III. DYNAMICS TOWARDS EARLY AGE: INFLATION

During the early age the universe was solely filled up with vacuum energy. The scale factor $a(t)$ grew exponentially so that the ratio of the vacuum energy density would rapidly be approaching towards the critical density at this era and thus a well known 'flatness problem' could be solved. Such phenomenon of exponential expansion is called inflation. It was A. Guth who first proposed such inflationary model [9, 10] to fix 'monopole problems', i.e., why today no magnetic monopole is present. It was soon observed that the inflation could solve other long-standing problems, such as 'flatness problem' and 'horizon problem'. Guth proposed that the scalar fields might get caught in the local minimum of some potential, and then rolled towards a true minimum of the potential. In 1980 Kazanas suggested [11] that an exponential expansion could eliminate the particle horizon. Sato proposed [12] that exponential expansion would be able to eliminate domain walls, another kind of exotic relic. In 1981 Einho et. al. [13] published a model that could solve 'magnetic monopole puzzle'. It was then realized that model of inflation had fatal problem as the transition from super cooled initial 'false vacuum' to the lower energy 'true vacuum' cannot occur everywhere simultaneously [14, 15]. Guth's version of inflation theory was replaced by 'new inflation' [16, 17, 18], which is termed as 'slow roll inflation' A scalar field, called 'inflation' was introduced to explain such phenomenon. Coleman and Weinberg introduced the symmetry breaking mechanism to the new inflation model [19]. In the inflation theory Linde added a new dimension by introducing the 'chaotic inflation [20] in which initially one or more inflaton fields vary in a random manner with positions. Linde also proposed the theory of 'hybrid inflation' [21, 22] theories by introducing more than one scalar fields.

In cosmology inflation is nothing but a rapid exponential expansion of the early universe by a factor of at least 10^{78} in volume, driven by negative pressure. The detailed particle physics mechanism behind such phenomenon is still unknown to the researchers. It is also a super cooled expansion, when the temperature drops down from 10^{27} K to 10^{22} K, although exact drop of temperature is model dependent. Such relatively low temperature is maintained throughout the entire inflationary phase and at the end of inflation the reheating occurs to attain the pre-inflationary temperature. The inflation is theorized through a number of models proposed by several researchers. In the present article a new kind of inflation model, based on the scalar potential with decreasing kinetic energy, is proposed. The vacuum energy is assumed to consist of a large constant part along with a small slowly varying part. The constant part would generate an exponential part, whereas the variable part of the vacuum energy density could be responsible for solving the flatness problem. The detailed mathematical formulations are carried out in the Section-2. In the Section-3 the physical significance of the proposed model is discussed. In this section it is also indicated how the dark energy is different from the vacuum energy, at least in the inflationary epoch.

It has already been mentioned that in the inflation theory a scalar field, known as the inflaton, along with a potential $V(\varphi)$ is assumed to take part to vary the vacuum energy. At this stage of expansion the universe is assumed to be filled up with such vacuum energy (which is also considered to be the 'dark energy'). The expression of the vacuum energy density as well as vacuum pressure in presence of a spatially homogeneous inflation in the Robertson-Walker spacetime becomes [23]

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad (12)$$

which must be derived from the expression of the generalized energy-momentum tensor

$$T_\phi^{\mu\nu} = -g^{\mu\nu}\left[\frac{1}{2}g^{\rho\sigma}\frac{\partial\phi}{\partial x^\rho}\frac{\partial\phi}{\partial x^\sigma} + V(\phi)\right] + g^{\mu\rho}g^{\nu\sigma}\frac{\partial\phi}{\partial x^\rho}\frac{\partial\phi}{\partial x^\sigma} \qquad (13)$$

Such energy-momentum tensor is slightly different from that of the constant vacuum energy $T^{\mu\nu}$, which is proportional to the $g^{\mu\nu}$ in the general coordinate system according to the Lorentz invariance condition. The proportionality of $T^{\mu\nu}$ to $g^{\mu\nu}$ corresponds to the condition of negative pressure as $p = -\rho$, but by the introduction of inflation such condition is deviated as neither pressure nor density remains constant, even in the inflationary era. This is to be noted that the present article deals only with the period of inflation; the change of density due to radiation or matter has not been discussed. Under the circumstances it is assumed that the absolute value of the first derivative of φ is maximum at the beginning of the inflation, but of course the condition

$$\dot{\phi}^2 << V(\phi)$$

must be satisfied. Now the important question is what should be the expression of $V(\varphi)$. A number of expressions of $V(\varphi)$ has been taken into account. Some of those are given below:

$$\begin{aligned} V(\phi) &= g\phi^\alpha && \text{(Power law)} \\ &= ge^{-\lambda\phi} && \text{(Exponential potential)} \\ &= \lambda(\phi^2 - M^2) && \text{(Higgs potential)} \\ &= \frac{1}{2}m^2\phi^2 && \text{(Massive scalar field)} \\ &= \lambda\phi^4 && \text{(Self – interacting scalar field)} \\ &= 2H_i^2\left(3 - \frac{1}{s}\right)e^{\frac{-\phi}{\sqrt{s}}} && \text{(Dilaton scalar field)} \end{aligned}$$

The last expression for potential is related to string theory. Different forms of potential lead to different models of inflation. It is worth noting that the complete inflation model is still unknown and the whole idea of the inflation is nothing but a speculation. Eventually, apart from flatness problem the inflation not only could solve the horizon as well as magnetic monopole problem but it also successfully predicted some of the properties of the fluctuations in the cosmic microwave background and large scale structure.

IV. DYNAMICS TOWARDS END STAGE: SINGULARITIES

The cosmological singularities, at the end stage of the universe, bring attention to the re-searchers and scientists to understand the ultimate fate of the universe. For a decade it became a big puzzle whether the universe will re-collapse to a big crunch singularity or expand forever to become rip apart. This paper aims to study towards the end stage of the dynamics of our universe. In other words one may be interested to have the picture at the end of the expansion of the universe. In 1977 the possible fate of our universe was explored by Jamal Islam [24]. After that, in 1977 it was also studied by Dyson [25]. A high density vacuum energy, causing the expanding universe to get accelerated, is interpreted as the dark energy [26, 27]. It is characterized by the 'equation of state (EOS)' parameter $w = p / \rho$. It is quite interesting to note from the Freidman equations that $w < -1/3$ is sufficient to generate the cosmic acceleration, but $w = -1$ is taken into account to explain dark energy in terms of cosmological constant. Therefore, (–1/3, 1) may be considered as the dark energy region where the dominant energy condition is satisfied, although the strong energy condition is violated. The dark energy with $w = -1$ have a simple explanation by introducing the cosmological constant, which is eventually 120 orders of magnitude smaller than that expected in the quantum gravity theory. Thus a better understanding of this negative pres-sure is needed to explain such dark energy. A natural question comes what happens in the other side of the boundary $w = -1$. The region $w < -1$, typically considered to be phantom dark energy region [28, 29], brings the attention to the researchers due to some peculiar nature observed here. In this region not only the energy density increases with time, but both strong as well as dominant energy conditions are violated [30, 31]. As a consequence a number of possible singular fates are increased towards the end of the universe. Those singularities are classified into the following manners.

- Type 0 (Big-Crunch) : $a \rightarrow 0$, $\rho \rightarrow \infty$, $|p| \rightarrow \infty$ at $t \rightarrow t_s$
- Type I (Big-Rip) : $a \rightarrow \infty$, $\rho \rightarrow \infty$, $|p| \rightarrow \infty$ at $t \rightarrow t_s$
- Type II (Sudden-Rip) : $a \rightarrow a_s$, $\rho \rightarrow \rho_s$, $|p| \rightarrow \infty$ at $t \rightarrow t_s$
- Type III (Finite Scale Factor) : $a \rightarrow a_s$, $\rho \rightarrow \infty$, $|p| \rightarrow \infty$ at $t \rightarrow t_s$
- Type IV (Big-Separation) : $a \rightarrow a_s$, $\rho \rightarrow 0$, $|p| \rightarrow 0$ at $t \rightarrow t_s$ and higher derivatives of H diverge.
- Type V (w-Singularity) : $w \rightarrow \infty$, $\rho \rightarrow 0$, $|p| \rightarrow 0$ at $t \rightarrow t_s$

The mathematical frameworks have been designed to study that cosmic doomsday in the phantom energy regime. In addition to those there may exist few more in the phantom energy sea. Another two remarkable singularities, studied by Jambrina [32], are grand crunch and grand rip, which may exist at the border line $w = -1$.

V. CONCLUDING REMARKS

One may think that in case of the accelerated contraction the second law of thermodynamics, according to which the entropy increases towards forward direction of time, may be violated. But this is not true, since the negative sign of da/dt implies the contraction in space as well as time. Therefore, the entropy decreases towards the backward direction of time. It is also to be noted that in the accelerated contraction some existing physical laws, which are yet to be figure out, may be violated. That is subject to be verified. In the possible fate of the universe there may be situation for the universe as the ‘point of no return’. In other words it will attain either a ‘big freeze’ or ‘heat death’. It will reach to a stagnancy of space as well as time, and no existing physical law should hold. Literally the universe will come to an end, although it cannot be negated the possibility of the evolution of new physics at this point.

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