© 2018 IJSRST | Volume 4 | Issue 5 | Print ISSN: 2395-6011 | Online ISSN: 2395-602X
Themed Section: Science and Technology

Reliability Analysis of Structural Beam Section Using First Order Reliability Method (FORM)

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ABSTRACT

Reliability is the ability of the structure to meet the construction requirements set under specified conditions during the service life, according which it is designed. It refers to the capacity, serviceability and durability of construction and according to them different techniques of reliability can be defined. In this paper the reliability index β is used to present the size of uncertainty in the theory of reliability. Two types of techniques are used to obtain the reliability index, which are First Order Reliability Method and Hosofer-Lind Method. The design of rectangular reinforced concrete section of beam is considered. The llablty and flexural failure caused by tension of the beam section are discussed. The methodology involved in determining the reliability to the design variables which are bending moment, cross sectional area, specified yield strength, specified compressive strength of concrete.

Keywords: Structural reliability, First order reliability method, Hosofer-Lind method, Rectangular reinforced concrete section of the beam, Bending moment, Moment capacity.

I. INTRODUCTION

Reliability is defined as the probability that a system performs its intended function without failure during a specified time period. The subject of structural reliability provides the tools and methodology to explicitly determine the probability of such failures by taking into account all relevant uncertainties. These techniques can be used to design new structures with specified reliabilities, and to maintain existing structures at or above specified reliabilities. The study of structural reliability is concerned with the calculation and prediction of the probability of limit state for engineering structures at any stage. Reliability is therefore the branch of structural engineering which is concerned with the analysis and probabilistic assessment of design random variables in order to predict whether specified limit state would be violated.

This paper determines the reliability index for the rectangular reinforced concrete section of the beams in flexural limit state by failure due to tension. The problem is formulated as reliability problem and concerns the reliability of the beam in flexure. Various parameters like, bending moment, cross sectional area, specified yield strength, specified compressive strength of concrete are modeled as random variables. First Order Reliability Method (FORM) and Hosofer-Lind Method are used to determine the reliability Index.

There are several papers, which considered structural reliability problems. Gunjan Agarwal and Baidurya Bhattacharya (2010) studied partial safety factor design of rectangular partially prestressed concrete beams in ultimate flexural limit state. Antanas Kudzys and Romualdas Kliukas(2010) discussed the reliability index design in reinforced concrete structures of annular cross sections. S.Choi, R.V.Gradnhi and

R.A.Canfield (2007) discussed reliability based structural design. Robert C.Wetherhold and Anthony M.Ucci used probability techniques for reliability analysis of composite materials. Stefan Conic, NemanjaMarkovic, Nikola Velimirovic and Nikola Stojic(2014) analysed reliability for timber structures. Abubakar Idris and Pius Edache(2007) evaluated the reliability of simple supported steel beams.

II. STATISTICAL METHODOLOGY

If the system has a deterministic resistance (strength) *X*, and a random applied load(stress) *Y* then the reliability of the system is the probability that the resistance is greater than the load, *P(X>Y).* Failure probability is the probability that the load is greater than the strength *(Y>X).* In the specific case of a Gaussian random variable the load *Y* can be reduced into standard normal variable y,

$$Y = \mu_Y + y\sigma_Y \Rightarrow y = \frac{Y - \mu_Y}{\sigma_Y} \quad (1)$$

Where μ_Y is the mean of Y and σ_Y is the standard deviation of Y, then the Reliability of the system

$$R = P(X > \mu_Y + y\sigma_Y)$$
$$\Rightarrow R = P((X - \mu_Y)/\sigma_Y > y)$$
$$\Rightarrow R = P(\beta > y) \quad (2)$$

where β is the reliability index

In the single variable case, this inequality is a safe region. This region is the set of values of y for which the structure will not fail. The probability of failure is the complement of the reliability.

$$P_f = 1 - R = 1 - P(\beta > y) \quad (3)$$

The probability of failure P_f is the probability that the load is greater than the resistance, P(Y>X).Let $f_X(x)$ and $f_Y(y)$ are the probability density functions of strength X and load Y then the distribution function F is defined by

$$F_Y(y) = P(Y \le y) = \int_{-\infty}^{y} f_Y(u)du \quad (4)$$

The probability of failure becomes

$$P_f = P(Y > X) = \int_0^{\infty} F_X(y) f_Y(y)dy$$
$$= \int_0^{\infty}\int_{-\infty}^{y} f_X(u) f_Y(y)dudy \quad (5)$$

A. First Order Reliability Technique

Let $(X_1, X_2, ...X_n)$ be the set of random variables (structural design variables) .The limit state equation for the failure surface of the structure is

$$g(X_1, X_2, ...X_n) = 0 \quad (6)$$

Collapse of the structure or failure is defined by the failure condition as $g(X_1, X_2, ...X_n) < 0$. Probability of failure is $P_f = P(g(X_1, X_2, ...X_n) < 0)$ and the reliability of the system is $P(g(X_1, X_2, ...X_n) < 0)$.

Methods for the determination of this probability depend on the complexity of the limit state function. The limit state function is the limit at which the performance transits from acceptable to unacceptable. There are different types of limit states: strength or ultimate limit states, serviceability limit states, fatigue limit states, and extreme event limit states. The limit state function g is given as:

$$g = S - Q \quad (7)$$

In the above expression S and Q are random variables. S is the resistance (strength of the system) and Q is the load (stress).If g<0 it leads to breakage of the structure i.e failure and if $g \ge 0$ then the structural reliability is high. In case of bending of beams S represents the ultimate moment capacity and Q represents the bending moment. In the case of serviceability limit state of deflection, S represents the maximum acceptable deflection and Q represents the deflection caused by the load.

Hasofer- Lind performed the transformation limit state function in the standard normal variable. The random variables S and Q are reduced into SNV S_1 and Q_1:

$$S_1 = \frac{S-\mu_S}{\sigma_S} \Rightarrow S = S_1\sigma_S + \mu_S \quad (8)$$

$$Q_1 = \frac{Q-\mu_Q}{\sigma_Q} \Rightarrow Q = Q_1\sigma_Q + \mu_Q \quad (9)$$

$$g = S - Q = (S_1\sigma_S + \mu_S) - (Q_1\sigma_Q + \mu_Q) \Rightarrow$$
$$g = (\mu_S - \mu_Q) + S_1\sigma_S - Q_1\sigma_Q \quad (10)$$

The line in reliability analysis is the line corresponding to $g(S_1, Q_1) = 0$ because this line separates the safe and failure region in the normal space. From the above the reliability index β is the shortest distance from origin of reduced variables to the line of $g(S_1, Q_1) = 0$ (Hasofer-Lind method, 1974).

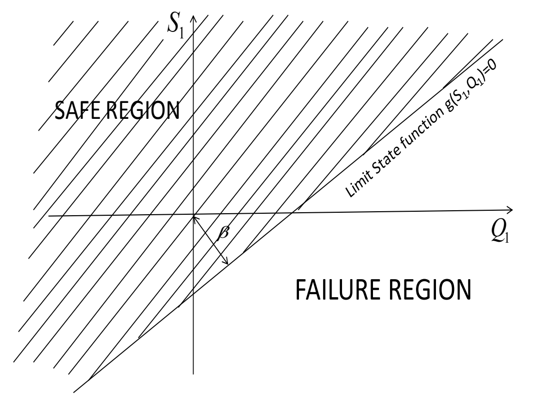

Fig 1 : Reliability index by Hasofer –Lind Method

$$\beta = \frac{\mu_S - \mu_Q}{\sqrt{\sigma_S^2 + \sigma_Q^2}} = \frac{\overline{S} - \overline{Q}}{\sqrt{\sigma_S^2 + \sigma_Q^2}} \quad (11)$$

Where μ_S and μ_Q are the mean values of S and Q respectively; σ_S^2 and σ_Q^2 are their variance values.

If the random variables S and Q have the log- normal distribution, then the reliability index is given by:

$$\beta = \frac{\overline{wS} - \overline{wQ}}{\sqrt{\sigma_{wS}^2 + \sigma_{wQ}^2}} \quad (12)$$

where $wS = \ln S$ and $wQ = \ln Q$. $\overline{wS}$ and $\overline{wQ}$ are the mean values of wS and wQ; σ_{wS}^2 and σ_{wQ}^2 are their variance values.

Then the probability of failure is

$$P_f = P((S-Q) < 0) = P(g < 0) \quad (13)$$

$$P_f = \phi(-\beta) \text{ and } \beta = -\phi^{-1}(P_f) \quad (14)$$

and reliability = $1 - P_f$, Where ϕ and ϕ^{-1} are the standard normal cumulative distribution function and its inverse.

B. The Hasofer-Lind Method (limit state function as a linear expression)

In some situations the failure surface is expressed as linear combination of random variables.

$$g(X) = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \ldots\ldots\ldots + \alpha_n X_n$$

$$g(X) = \alpha_0 + \sum_{i=1}^{n} \alpha_i X_i \quad (15)$$

Where α_i 's are constants. After normalization of variables, the equation of limit state (failure surface) becomes

$$\sum_{i=1}^{n} \alpha_i \sigma_{X_i} x_i + \sum_{i=1}^{n} \alpha_i \overline{x_i} + \alpha_0 = 0, \quad (16)$$

where SNV , $x_i = \frac{X_i - \overline{x_i}}{\sigma_{X_i}}$, $\overline{x_i}$ is the mean value and σ_{X_i} is the standard deviation of the random variable X_i.The reliability index β can be determined from the geometry

$$\beta = \frac{\sum_{i=1}^{n} \alpha_i \overline{x_i} + \alpha_0}{\sqrt{\sum_{i=1}^{n} \alpha_i^2 \sigma_{X_i}^2}} = \frac{\overline{g}}{\sqrt{\sum_{i=1}^{n} \alpha_i^2 \sigma_{X_i}^2}} \quad (17)$$

Where $\overline{g} = \alpha_0 + \sum_{i=0}^{n} \alpha_i \overline{x_i}$ is the mean value of g. (18)

C. The Hasofer-Lind Method (limit state is a nonlinear)

This is the simplest first order second moment reliability method (FOSMRM) used to evaluate reliability. This technique approximates the failure surface by a hyperplane tangent to the failure surface. Consider a limit state (failure surface) equation of random variables $X_1, X_2, ...X_n$.

$$g(X_1, X_2, ...X) = \bar{g} + \sum_{i=1}^{n}\left[(X_i - \bar{x}_i)\frac{\partial g}{\partial X_i}\right] = 0 \quad (19)$$

Where $\bar{g}$ the mean value of g: $\bar{g} = g(\bar{x}_i, \bar{x}_2, ...\bar{x}_n)$

The reliability index , $\beta = \frac{\bar{g}}{\sqrt{\sum_{i=1}^{n}(\alpha_i X_i)^2}}$ with

$$\alpha_i = \frac{\partial g}{\partial X_i} \quad (20)$$

The reliability index β is a measure of reliability of the system. Table 1 shows the values of R for selected values of β , given in the limits that are applied in engineering.

Table 1. Probability of success for selected value of reliability index

S.No	β	*R*
1	1	0.81
2	1.5	0.9332
3	2	0.9772
4	2.5	0.99379
5	3	0.99865
6	3.5	0.999767
7	4	0.9999683
8	4.5	0.9999966
9	5	0.999999713
10	5.5	0.999999981

III. FLEXURAL FAILURE OF A RECTANGULAR REINFORCED CONCRETE SECTION OF THE BEAM

The design of different rectangular reinforced concrete section of beam is considered. The main task of a structural engineer is the analysis and design of structures. The design of a section implies that the external moment is known, and it is required to compute the dimensions of an adequate concrete section and the amount of steel reinforcement. Concrete strength and yield of steel used are assumed. The dimensions and steel used in the section are given. It is required to calculate the ultimate moment capacity of the section.

The assumptions are made in flexural theory are

(i) Plane section is remain same after bending.

(ii) Strain in concrete is the same as in reinforcing bars, provided that the bond between the steel and concrete is sufficient to keep them acting together under the different load stages i.e., no slip can occur between the two materials.

(iii)The stress-strain curves for the steel and concrete are known.

(iv)The tensile strength of concrete may be neglected.

(v) At ultimate strength, the maximum strain at the extreme compression fiber is assumed to 0.003.The assumption of plane sections remaining plane (Bernoulli's principle) means that strains above and below the neutral axis NA is proportional to the distance from the neutral axis.

A. Tension Failure

If the steel content of the section is small (an under-reinforced concrete section), the steel will reach its yield strength before the concrete reaches its maximum capacity. The flexural strength of the section is reached when the strain in the extreme compression fiber of the concrete is approximately 0.003, Fig.2. With further increase in strain, the moment of resistance reduces, and crushing commences in the compressed region of the concrete. This type of failure, because it is initiated by yielding of the tension steel, could be referred to as a tension failure.

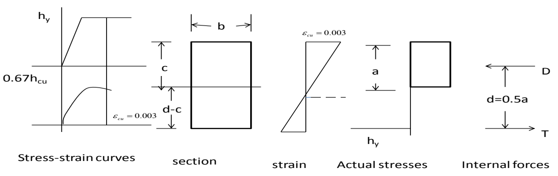

Fig 2: Reinforced section when tension failure

In tensile failure $h_s = h_y$; D=T;

$T = B_s h_y$ and $D = 0.67 h_{cu} ba$ (21)

From the above equation

$$B_s h_y = 0.67 h_{cu} ba$$

$$a = \frac{B_s h_y}{0.67 h_{cu} b} \tag{22}$$

The nominal strength of the cross section N_u (capacity of the cross section) is

$$N_u = T(d - 0.5a) = B_s h_y \left[d - \frac{B_s h_y}{1.34 h_{cu} b} \right] \tag{23}$$

where B_s =cross sectional area, h_y =specified yield strength, h_{cu} =specified compressive strength of concrete, N_u = Ultimate moment capacity of the cross section, d=distance between the steel reinforcement and the extreme compressive fiber, b= width of the section.

B. Reliability Calculations for Flexural Failure of a Rectangular Reinforced Concrete Section

The flexural limit state function for rectangular reinforced concrete section is

$$g = S - Q \tag{24}$$

Where S is the strength of the cross section (the Ultimate moment capacity that it can carry) and Q is the bending moment.

$$g = B_s h_y \left[d - \frac{B_s h_y}{1.34 h_{cu} b} \right] - Q \tag{25}$$

Design parameter's variability: The application of the reliability model assumes that the covariances of the parameters are known so that the design parameter's variability can be estimated. The coefficient of variance of specified yield strength is 10% to 12%.The cross sectional area has a coefficient of variance of 2 to 5%. The coefficient of variance of specified compressive strength of concrete is 10% -15%.The coefficient of variance of bending moment is 8%-10%.Table 2 shows the coefficient of variance of the design parameters.

Table 2. Summary of Design Parameter and Coefficient of variance

Design parameter	Cov (%)
Cross sectional area	2.17
Specified yield strength	11.5
Specified compressive strength of concrete	14.28
Bending moment	8

Consider the cross section area $2.3 \times 10^{-3} m^2$, specified yield strength is 350MPa, specified compressive strength 28MPa, bending moment is $250 \times 10^3 Nm$, width of the section is 0.25 and d=0.42. B_s, h_s, h_{cu}, Q are the random variables normally distributed having mean and SDvalues: $\overline{B_s} = 2.3 \times 10^{-3} m^2$; $\sigma_{B_s} = 0.00005 m^2$; $\overline{h_y} = 350 MPa$; $\sigma_{h_y} = 40 MPa$; $\overline{h_{cu}} = 28 MPa$; $\sigma_{h_{cu}} = 4 MPa$; $\overline{Q} = 200 \times 10^3 Nm$; $\sigma_Q = 16 \times 10^3 Nm$.

The mean value of the limit state function

$$\overline{g} = \overline{B_s}\,\overline{h_y} \left[d - \frac{\overline{B_s}\,\overline{h_y}}{1.34 \overline{h_{cu}} b} \right] - Q = 69.0142 \times 10^3 Nm$$

The partial derivatives at the mean values are

$$\alpha_1 = \frac{\partial g}{\partial X_1} = \frac{\partial g}{\partial B_s} = 86925373 N/m$$

$$\alpha_2 = \frac{\partial g}{\partial X_2} = \frac{\partial g}{\partial h_y} = 571.2\times10^{-6}m^3$$

$$\alpha_3 = \frac{\partial g}{\partial X_3} = \frac{\partial g}{\partial h_{cu}} = 2467\times10^{-6}m^3$$

$$\alpha_4 = \frac{\partial g}{\partial X_4} = \frac{\partial g}{\partial Q} = -1$$

The reliability index result is $\beta = \frac{\bar{g}}{\sqrt{\sum_{i=1}^{n}(\alpha_i X_i)^2}}$ =2.3077

The reliability of the flexural rectangular concrete section and probability of failure are

Reliability=0.9896, $P_f = 10.4\times10^{-3}$.

Table 3, shows the values of reliability index and probability of failure when cross sectional area, specified yield strength, specified compressive strength of concrete, distance and width of the section are constant and bending moment changing. It is observed that reliability decreasing as shown in fig 3. So that for the design of the structure, the bending moment should lies between 180000Nm and 280000Nm.

Table 3

Bending Moment is Increasing and other parameters keeping constant
Bs=0.0025,hs=350*10^6,hcu=28*10^6,d=0.42,b=0.25

Q	$\bar{g}$	β	P_f	R
180000	105876.8657	3.5137	1E-04	0.9999
190000	95876.86567	3.1411	0.0002	0.9998
200000	85876.86567	2.7766	0.0028	0.9972
210000	75876.86567	2.4203	0.0078	0.9922
220000	65876.86567	2.0725	0.0192	0.9808
230000	55876.86567	1.7333	0.0418	0.9582
240000	45876.86567	1.403	0.0808	0.9192
250000	35876.86567	1.0814	0.1401	0.8599
260000	25876.86567	0.7687	0.2216	0.7784
270000	15876.86567	0.4647	0.3228	0.6772
280000	5876.86567	0.1694	0.433	0.567

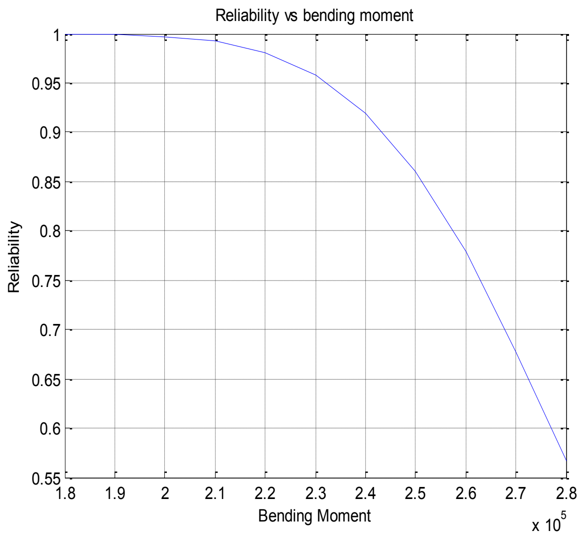

Figure 3. Reliability and bending moment

From table 4, it is obtained the reliability index and probability of failure when bending moment, specified yield strength, specified compressive strength of concrete, distance and width of the beam are changing and cross sectional area is changing. It is observed that reliability is increased as shown in fig4.

Table 4

Cross sectional area is changing and other parameters keeping constant
hs=350*10^6,hcu=28*10^6,d=0.42,b=0.25,Q=220*10^3

Bs	$\bar{g}$	β	P_f	R
0.0018	2286.5671	0.0816	0.4681	0.5319
0.0019	12154.4776	0.4248	0.336	0.664
0.002	21761.19	0.7457	0.228	0.772
0.0021	31106.71	1.046	0.148	0.852
0.0022	40191.044	1.3276	0.0926	0.9074
0.0023	49014.17	1.5918	0.0559	0.9441
0.0024	57576.1194	1.8397	0.033	0.967
0.0025	65876.8656	2.0725	0.0192	0.9808
0.0026	73916.41	2.2908	0.011	0.989
0.0027	81694.77	2.4953	0.0064	0.9936
0.0028	89211.9403	2.6866	0.0037	0.9963

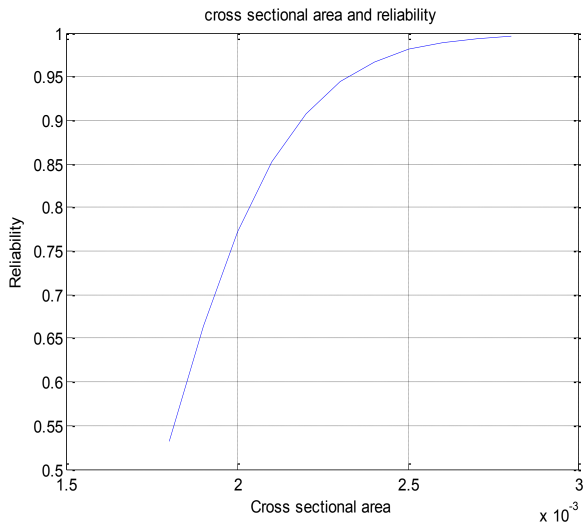

Figure 4. Reliability vs cross sectional area

In table 5, when cross sectional area, specified compressive strength of concrete, distance and width of the section keeping constant and specified yield strength is increasing, it is observed that the reliability is increasing as shown in figure 5.

Table 5

Specified yield strength is increasing and other parameters keeping constant				
Bs=0.0025,hcu=28*10^6,d=0.42,b=0.25,Q=220*10^3				
hs	$\bar{g}$	β	P_f	R
250*10^6	855.54371	0.0306	0.488	0.512
275*10^6	18360.20789	0.63357	0.2643	0.7357
300*10^6	35031.9829	1.1689	0.1215	0.8785
325*10^6	50870.8688	1.6462	0.05	0.95
350*10^6	65876.8656	2.0725	0.0192	0.9808
375*10^6	80049.9733	2.4527	0.0071	0.9929
400*10^6	93390.1919	2.79	0.0026	0.9974
425*10^6	105897.5213	3.0865	0.001	0.999
450*10^6	117571.9616	3.34279	0.0005	0.9995
475*10^6	128413.5128	3.5588	0.0002	0.9998
500*10^6	138422.1748	3.7341	1E-04	0.9999

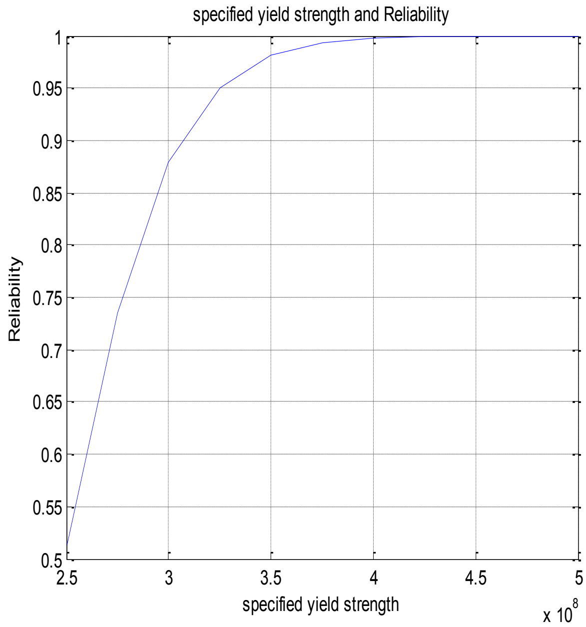

Figure 5. Reliability vs specified yield strength

In table 6, When Distance between the steel reinforcement and the extreme compressive fiber is increasing and bending of beam, cross sectional area, specified yield strength, specified compressive strength of concrete keeping constant thenthe Reliability index is increasing as shown in figure 6.

Table 6

Distance between the steel reinforcement and the extreme compressive fiber is changing and other parameters keeping constant Bs=0.0025,hs=350*10^6,hcu=28*10^6,b=0.25,Q=220*10^3				
d	$\bar{g}$	β	P_f	R
0.35	4626.8656	0.1721	0.4325	0.5675
0.37	22126.8657	0.7851	0.2177	0.7823
0.39	39626.8657	1.3399	0.0901	0.9099
0.41	57126.8657	1.8408	0.0329	0.9671
0.43	74626.8657	2.2924	0.011	0.989
0.45	92126.8657	2.6993	0.0035	0.9965
0.47	109626.866	3.0664	0.0013	0.9987
0.49	127126.866	3.3979	0.0012	0.9988
0.51	144626.866	3.6978	1E-04	0.9999
0.53	162126.866	3.9698	1E-04	0.9999

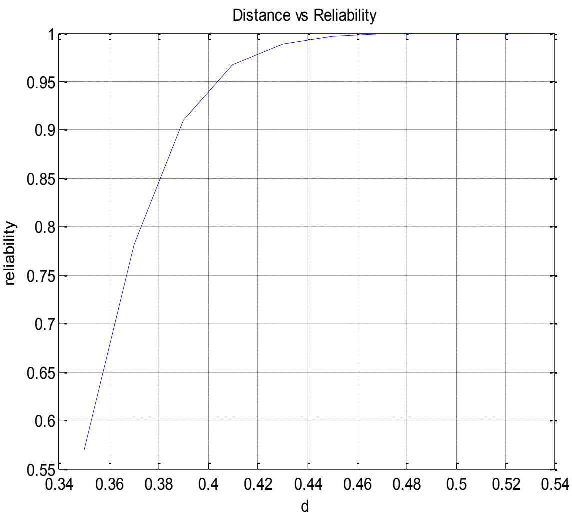

Figure 6. Reliability vs d

In table 7, when width of the section is increasing and rest are keeping constant, it is observed that the reliability index is increasing as shown in figure 7.

Table-7

Width of the section is changing and other parameters keeping constant				
Bs=0.0025,hs=350*10^6,hcu=28*10^6,d=0.42,Q=220*10^3				
d	$\bar{g}$	β	P_f	R
0.15	11461.4428	0.4025	0.3446	0.6554
0.2	45471.0821	1.5295	0.064	0.936
0.25	65876.8657	2.0725	0.0192	0.9808
0.3	79480.7214	2.3652	0.0091	0.9909
0.35	89197.7612	2.5422	0.0055	0.9945
0.4	96785.541	2.659	0.004	0.996
0.45	102153.814	2.74125	0.0031	0.9969
0.5	106688.433	2.80197	0.0026	0.9974
0.55	110398.575	2.8485	0.0023	0.9977
0.6	113490.361	2.8852	0.002	0.998

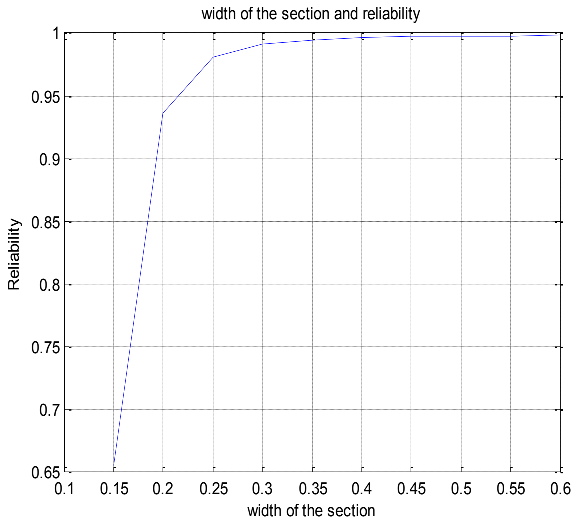

Figure 7. Reliability when width of the section is increase

IV. CONCLUSION

One of the best ways of representing the capacity, serviceability and durability of the structure in the theory of reliability is the reliability index β .Most of the engineering constructions consist of a system with connected components and its elements. When thinking about the reliability of the system, it is important to note that whether failure of the individual elements cause a failure of the entire structure.

By using the Hasofer-Lind Method, the reliability of the structural design (A Rectangular Reinforced Concrete Section) was calculated for its capacity, durability and serviceability. Failure occurs due to the heavy bending moment of the rectangular concrete section. It is observed that if cross sectional area increased then the failure rate decreased. It is also observed that when the value of the other design parameters like specified yield strength, specified compressive strength of concrete, distance between reinforcement and the extreme compressive fiber and width of the section increased then the system reliability increased.

Several factors must be considered in the structure to improve system reliability; those are security, usability, strength and flammability of construction.

The structure should be designed and implemented in such a manner that during its life cycle, with a certain degree of reliability, it will not collapse due to an explosion, shock, loads or human error.

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