

# Control of Multi-Link Robots with Link Flexibility

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## ABSTRACT

The flexible robots are favoured in industries and in space applications due to their quick response, low energy consumption, lower overall mass and operation at high speeds compared to conventional industrial rigid link robots. These flexible link robots are inherently flexible and this effects the end-point positioning accuracy of the robot. It is important to model the link kinematics with precision which in turn simplifies the modeling of dynamics and control of flexible link robots. The main objective of this paper is to design two types of controllers (PD and PI) to reduce the error at the tip position of two link Revolute-Revolute type Manipulator and ensure that the end effector will follow the specified vertical path when the two links are flexible and the payload is equal to the links mass.

## I. INTRODUCTION

Robotic manipulators are widely used to help in dangerous, monotonous, and tedious jobs. Most of the existing robotic manipulators are designed and build in a manner to maximize stiffness in an attempt to minimize the vibration of the end-effector to achieve good position accuracy. This high stiffness is achieved by using heavy material and a bulky design.

The advantages of using a lighter weight manipulator as against the rigid link manipulator include: higher manipulation speed, less power consumption, they require less material for their construction, they required smaller actuator. By making the weight of the manipulator to be lighter results in the flexibility of the manipulator that makes the modeling of such a system to become very cumbersome.

Flexible manipulators can find many applications but since the main problem is to control their vibrations, this problem can be solved by improving the dynamic

models and incorporating different control strategies. The study on the control of a flexible arm manipulator started as a part of the space robots research, as a space manipulator should be as light as possible in order to reduce its launching cost and due to the space and weight restriction issues.

The overall flexibility in a robot is due to the flexibility of joints and the flexibility of the links. The flexibility at the joint is due to the lack of rigidity in the drive, deformation of the gear teeth and shaft, and due to the control action. The flexibility of the link due to the deformation in transverse direction and due to shearing and rotary inertia effect.

Link of the robotic system shown in Figure.1 is initially at rest and when actuator is actuated to move the link through an angle  $\theta$  it would move as a whole body to angle  $\theta$  if it were a rigid link robot. However due to its structural flexibility, robot goes to final position but deforms from its steady state position and it vibrates at steady state position and finally settles to

a new steady state position after some time depending upon damping ratio of the flexible system.

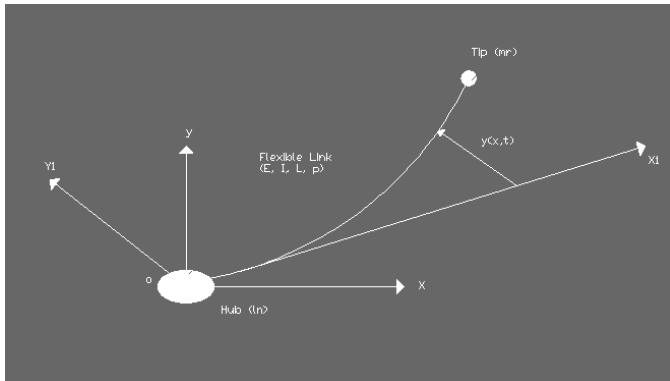


Figure 1. Vibration of Flexible Beam

Considering the flexible link robotic arm as shown in the Figure 1, the flexible link undergoes deformation in motion due to the flexibility of the link. One can observe that a point on this link has a deviation  $y(x,t)$  from the un-deformed position. Therefore the motion of the point, related to  $y(x,t)$ , is not completely determined by joint angle  $\theta$  and it can also be concluded that an infinite number of  $\theta$ 's needed to describe the motion of the entire link. The motion of the end effector is calculated based on Assumed Mode Method (AMM). The links are assumed to behave like Euler-Bernoulli beams.

The position vector along the length of the link depends on the lateral deformation 'y' of the link at that section at a given time. Value of 'y' can be found by Assumed Mode Method (AMM). The problem of flexible link can be solved assuming it as a Euler-Bernoulli's cantilever beam with a payload 'Mp' at the tip of the beam undergoing free vibration, the governing equation to represent the vibration of link can be written as follows

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} + \rho A \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \tag{1}$$

**Boundary Conditions:** Since, the equation of motion Eq. 4.4 involves a second order derivative with time and a fourth order derivative with 'x', two initial conditions and four boundary conditions are needed

for finding a unique solution for  $y(x,t)$  and they are given in following Equations 4.4 to 4.9

$$y(x,t=0) = y_i \tag{2}$$

$$\frac{\partial y(x,t=0)}{\partial t} = \frac{\partial y(x,t)}{\partial t} \Big|_{t=0} = y'_i \tag{3}$$

$$y(x,t)|_{x=0} = 0; \tag{4}$$

$$\frac{\partial y(x,t)}{\partial x} \Big|_{x=0} = 0; \tag{5}$$

$$EI \frac{\partial^2 y(x,t)}{\partial x^2} \Big|_{x=L} = -J_L \frac{\partial^2}{\partial t^2} \left( \frac{\partial y(x,t)}{\partial x} \Big|_{x=L} \right) \tag{6}$$

$$EI \frac{\partial^3 y(x,t)}{\partial x^3} \Big|_{x=L} = -M_L \frac{\partial^2}{\partial t^2} (y(x,t)|_{x=L}) \tag{7}$$

The solution of the Equation 1 can be expressed as follows.

$$y(x,t) = \sum_{j=1}^n C_{1,j} \sin(\omega_j t) \left\{ (\cos(\beta_j x) - \cosh(\beta_j x)) - \alpha (\sin(\beta_j x) - \sinh(\beta_j x)) \right\}$$

where

$$\alpha = \frac{-\beta_j^3 \cos(\beta_j L) - \beta_j^3 \cosh(\beta_j L) + \frac{M_L}{\rho} \beta_j^4 \sin(\beta_j L) - \frac{M_L}{\rho} \beta_j^4 \sinh(\beta_j L)}{-\beta_j^3 \sin(\beta_j L) - \beta_j^3 \sinh(\beta_j L) - \frac{M_L}{\rho} \beta_j^4 \cos(\beta_j L) + \frac{M_L}{\rho} \beta_j^4 \cosh(\beta_j L)} \tag{9}$$

## II. LITERATURE REVIEW

Hu Zhongling et.al [1] presented a method of Co-simulation technology based on ADAMS and MATLAB for design and research of complex mechanical systems. Results showed that co-simulation is an effective method for the simulation analysis of complex dynamic systems. F. Cheraghpour et.al [2], presented Dynamic modeling and kinematic simulation of Stäubli TX40 robot using MATLAB/ADAMS co-simulation and proposed a precise simulator to develop approaches for experimental simulation in kinematics, dynamics and control analysis. H Liu et.al [3], studied Co-simulation using ADAMS and MATLAB for Active Vibration Control of Flexible beam with Piezoelectric Stack Actuator Co-simulation. The virtual prototype of flexible beam with piezoelectric actuator is created in ADAMS, the controller based on FXLMS algorithm is established in MATLAB. The results and analysis prove that active vibration control for flexible beam

has a great suppression performance. *M.A. Ahmad* [4], studied effects of Vibration and input tracking control of Flexible manipulator using LQR with Non-collocated PID Controller. *Jerzy et. al*[5] Proposed a method for dynamic modeling and adaptive control of a single-link flexible manipulator. *Mahamood et. al* [6], proposed PID Controller design for Two Link Flexible Manipulator.

### III. PROBLEM STATEMENT

The objectives of this paper are to evaluate the effects of link flexibility on the variation in the tip position of a two-link RR type Robotic arm when the end effector (tip) is made to move in a vertical motion and the payload considered is equal to links mass. The difference between the end effector positions of flexible link robot and the rigid links at each time instant is taken as the positional error. Two different control methods (i.e. PD control and PI control) were adopted to reduce the positional error between the Rigid and Flexible links. Based on the results obtained a better control strategy is adopted for future usage. The dimensions and properties considered in this paper are given in Table 1. Software's utilized are MSC Adams and Mat lab.

TABLE 1. LINK PARAMETERS

|          | Length (mm) | Width (mm) | Depth (mm) | Mass (kg) | Density (kg/mm <sup>3</sup> ) |
|----------|-------------|------------|------------|-----------|-------------------------------|
| Link 1   | 300         | 40         | 20         | 2         | 7.8*10 <sup>-6</sup>          |
| Link 2   | 400         | 40         | 20         | 2.6       | 7.8*10 <sup>-6</sup>          |
| End base | 80          | 20         | 20         | 0.5       | 7.8*10 <sup>-6</sup>          |
| Gripper1 | 50          | 10         | 20         | 0.045     | 7.8*10 <sup>-6</sup>          |
| Gripper2 | 50          | 10         | 20         | 0.045     | 7.8*10 <sup>-6</sup>          |
| payload  | -           | -          | -          | 5.1       | 7.8*10 <sup>-6</sup>          |

### IV. RESEARCH METHODOLOGY

The method adopted in this paper is as follows. The dimensions of the links are taken such that they replicate the dimensions of human arm provided in

the Journals. Using MSC Adams software, RR type two-link manipulators (both rigid and flexible) are modelled such that the end effector moves in a specified vertical path with a payload equal to the mass of the links.

The input Torques were applied at the two revolute joints resulting in the angular rotation of two links resulting in a linear motion of the end effector. The end effector is constrained to move in a specified vertical path.

Figure 2 shows the application of constraint in MSC Adams so that the end effector moves in vertical path. The two angular rotations at the two joints and the motion of the end effector are taken as the outputs which have to be monitored as the input torques are applied at the joints. The time taken for total end effector path is about ten seconds in 1000 steps. The end effector position of the rigid link RR type manipulator, angular rotations at joint 1 and joint 2 joint during these 10 seconds in 1000 steps are taken as the reference values. Then when the two links are replaced by flexible links in RR type manipulator and when same input torques are applied at the two joints resulting in the end effector positions, angular rotations at joints 1 and 2 are measured again for 10 seconds in 1000 steps.

The figure 3 shows the way a flexible links are built in MSC Adams.

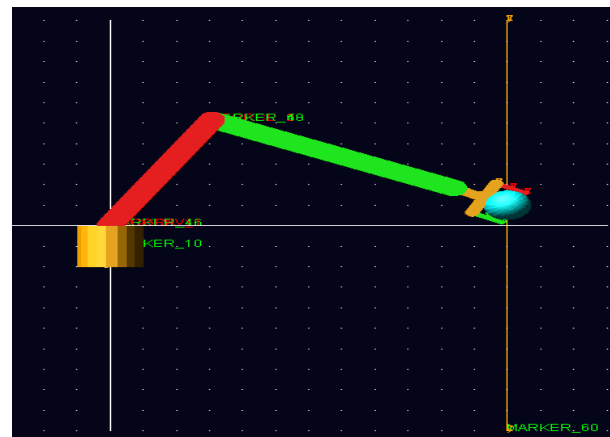


Figure 2. Building model with vertical constraint

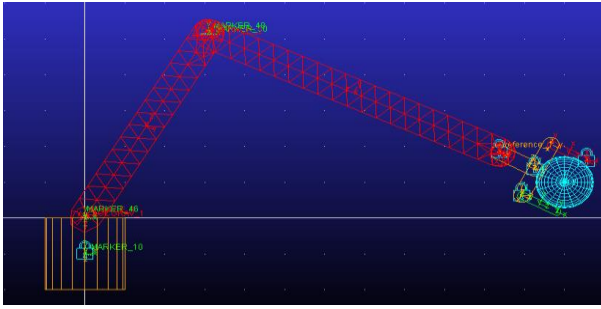


Figure 3. Building flexible model with payload

Positional error is defined as difference between these two tip positions of the end effector and difference in angular rotations at joint 1 and 2 for 1000 steps of the vertical path. From the results obtained given in figure 4 one can conclude that as time and number of steps are increasing the positional error is increasing. Moreover the end effector is also carrying a payload equal to the weight of two links (i.e. 5.1 kg) and this is also influencing the value of positional errors.

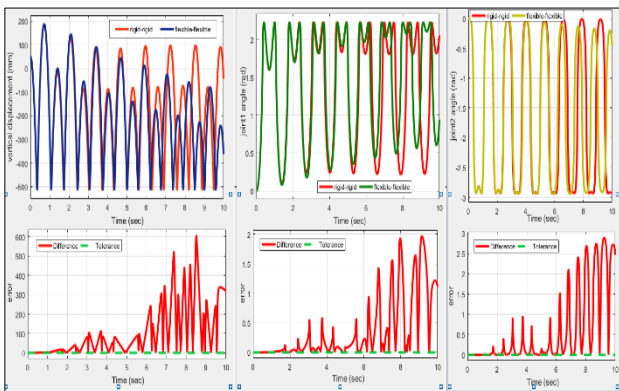


Figure 4. Error at end effector, joint 1 and Joint 2 due to flexibility

To keep this positional errors at end effector, joints 1 and 2 to a minimum value, a control strategy needs to be adopted. From the Literature, two control strategies i.e. Proportional Derivate (PD) and Proportional Integral (PI) methods were selected.

Applying PI and PD control strategy directly in MSC Adams posed problems and were unable to get the required response. Therefore a co-simulation method using MSC Adams and Matlab was envisaged. Both the rigid and flexible links models were developed in

MSC Adams and these models was imported to Matlab. Using Simulink, the Adams model was integrated in Matlab and the block diagrams shown in figure 6 and 8 were developed for both PD & PI control strategies. The positional error in tip position is the output and the angular rotations are the input for the Simulink model.

The constants ( $K_p$ ,  $K_i$  and  $K_d$ ) in PI and PD controllers are chosen in a trial and error basis, as the inputs are varied and the outputs are monitored till those values are reached for which the Proportional error become minimum.

### V. RESULTS AND DISCUSSION

When there was no control strategy to control the effect of flexibility, the maximum error in the tip position is equal to 600 mm and an average RMS error of 106 mm. the maximum angular position error at joint 1 and 2 is about 2 radians and 2.8 radians respectively. Figure 4 shows these results and one can conclude, these errors are considerably large implying the flexibility has significant effect on the tip positional accuracy.

To reduce these, two kinds of controllers are used to find the error at the tip position and ensure that the end effector will follow the specific vertical path.

#### A. Applying Proportional-Integral (PI) Controller at both Joints:

To reduce the error in vertical displacement and joint angles a PI controller for the two link RR manipulator with specific constant value (given in table 2) which ensure the reduction of the error is developed. The Figure 5 shows the Simulink model, so developed in MATLAB. Figure 6 shows the effect of PI controller.

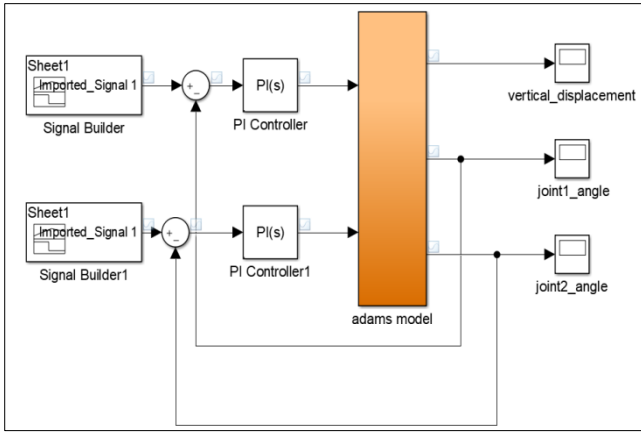


Figure 5. Simulink model for PI controller

The next two figures (Fig. 7-Fig. 8) shows the MATLAB Simulink configuration of PD controller and the errors in the tip position and joints angles.

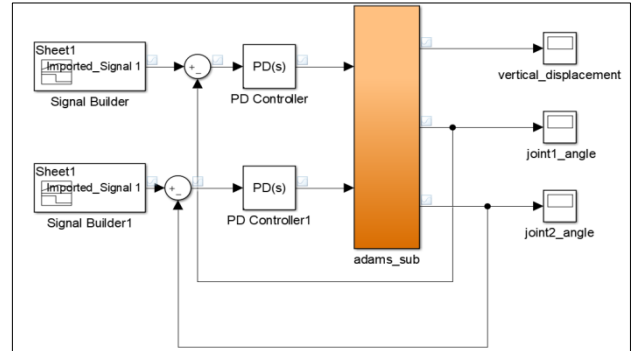


Figure 7. Simulink Model of PD controller

Table 2 :PI Controller Constants

|         | P constant | I constant |
|---------|------------|------------|
| Joint 1 | 4794.8811  | 2845.9666  |
| Joint 2 | 200        | 100        |

When PI control strategy was applied, the maximum error in the tip position reduced to 180 mm and Average RMS error of 34 mm. the maximum angular position error at joint 1 and 2 is about 0.76 radians and 1.2 radians respectively. Figure 6 shows these results and one can conclude, these errors are considerably reduced implying that PI control is satisfactorily countering the effects of flexibility.

Table 3. Pd Controller Constant

|         | P constant | D constant | Filter coefficient |
|---------|------------|------------|--------------------|
| Joint 1 | 4794.8811  | 2500       | 1                  |
| Joint 2 | 300        | 300        | 1                  |

When PD control strategy was applied, the maximum error in the tip position reduced to 165 mm and average RMS error of 30 mm. the maximum angular position error at joint 1 and 2 reduced to 0.78 radians and 1.22 radians respectively. Figure 6 shows these results and one can conclude, these errors are considerably reduced implying that PD control is satisfactorily countering the effects of flexibility

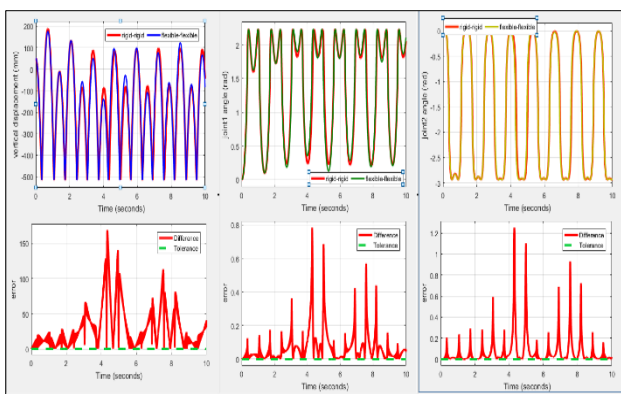


Figure 6. The error in tip position, joint 1 & joint 2 angles with PI controller

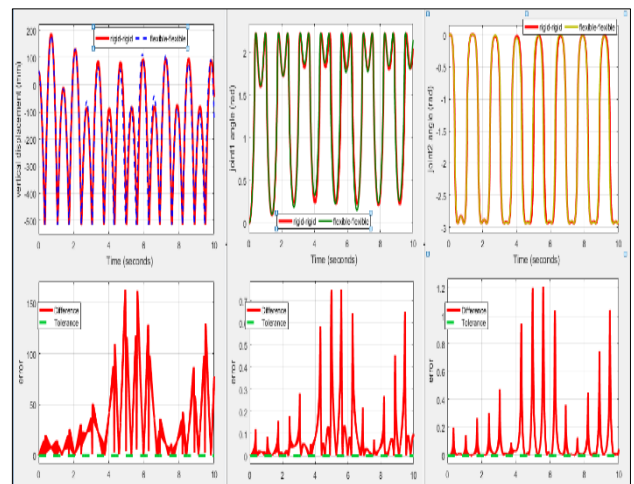
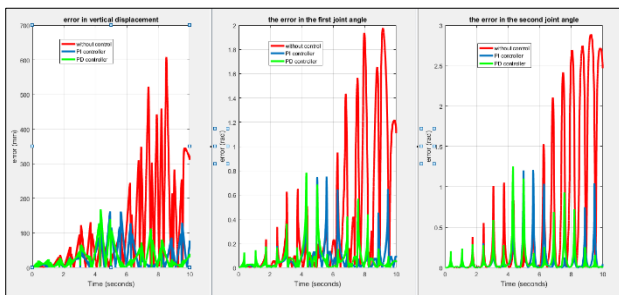


Figure 8. The errors at tip position, joint1 and joint2 with PD control

**B. Applying Proportional-Derivative (PD) Controller at the Joints:**

**C. The Effect of the Controllers on the Input and Output Response:**

The figure 9 shows the effect of PD and PI control strategies to control and reduce the position error at the tip position and joints angles with respect to a system with no control. When PI control is adopted the maximum positional error reduced by 70% and 72.5% reduction using PD control. The angular error reduced by 62% at joint 1 and 40% at joint 2 using PI control. When PD control is used, the error at joint 1 reduced by 60 % and 39 % at joint 2. It is obvious from the figure 9 that the PD controller is performing better than PI control for the given conditions and therefore it should be chosen to reduce error and follow the desired path.



**Figure 9.** Effects of PI and PD control strategies

**VI. CONCLUSIONS**

A two-link rigid and flexible manipulator has been successfully co-simulated in Adams software and MATLAB. From the model created in ADAMS it is found that the flexibility of link significantly affects the system behavior. The tip position, first joint angle and second joint angle values are compared by using MATLAB and two types of controllers were (i.e. PD & PI) applied. The conclusions inferred are:

- The link flexibility has considerable effect on tip position, joint angles.
- For the robotic system without control the averaged RMS error in the tip position is equal to 106.2 mm.

- The averaged rms error in the first joint angle in the case of both links are flexible is equal to 0.37 radians.
- The averaged rms error in the second joint angle in the case of both links are flexible is equal to 0.59 radians.
- The percentage deviation of end effectors position in vertical motion varied between 6.061%.
- The percentage deviation of first joint angle in vertical motion varied between 0.016%.
- The percentage deviation of second joint angle in vertical motion varied from 0.0288%.

1) With PI controller: - the average error in the tip position when both links are flexible is equal to 33.49 mm.

- The average rms error in the first joint angle in the case of both links are flexible is equal to 0.08 rad.
- The average rms error in the second joint angle in the case of both links are flexible is equal to 0.09 rad.

2) With PD controller: - the average error in the tip position when both links are flexible is equal to 30.21 mm.

- The average rms error in the first joint angle in the case of both links are flexible is equal to 0.07 rad.
- The average rms error in the second joint angle in the case of both links are flexible is equal to 0.07 rad.

3) the performance of PD controller is better than PI controller for the given conditions

**VII. REFERENCES**

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