

Contribution of Queuing Theory to Analyse Various Aspects of Queuing Model

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ABSTRACT

The purpose of this paper is to study the important role of Queuing Theory in reducing Waiting time of customer in Queuing model. The various Queuing models were used to study the different aspects and parameters which interpret the lacuna in Queuing models which helps to improve the system. It will reduced the waiting time of customer and service cost of service facility.

Keywords : Queuing Model, Customers Waiting Time, Service Cost.

I. INTRODUCTION

Queuing Theory is the Mathematical study of forming the queue. It comes under the branch of Operations Research. As the population day by day increases, the demand on service facility increases. The result of this is to form a long queue at every service facility centers. Queues are one of the unwanted parts of human's life. Today we are living in the age of busy life. Nobody wants to waste their valuable time to stand in a queue. To overcome this problem of increasing cost of waiting time of customer many of the research carried out in Queuing Theory. The conclusions and results of the Queuing Theory are generally used in the improvement of Queuing models in business, library sciences, banking services, traffic controls, marketing, various reservations, in the sector of information technology, production of industries, hospital management and many more.

Basic Features of Queuing System:

1. Customer Arrivals
2. Service Discipline
3. Service Mechanism
4. Customers Attitude

History behind the Queuing Theory and Contribution:

In Queuing Theory the values which are to be recorded as average arrival time of customer, the inter-arrival time of customers, average service rate of customer and time required for service mechanism to provide the complete service. By using these values the various parameters as given below in the list of formulae's can be calculated and analyze it for the betterment of Queuing model. By using Poisson distribution probability of 'n' customers arriving in time 't' and probability of 'n' customers getting service in time 't' can be calculated. By using Exponential distribution the inter-arrival time and service time distribution of customer can be analyzed. Agner Krarup Erlang in 1909, called the father of Queuing Theory had its beginning in the research on the Waiting Line Theory. Tore Olaus and Erlang were the first developers of Queuing Theory as applicable to the telephone industry. Erlang perform an experiment with fluctuating demand in telephone, traffic, after that er he published a report which addresses the delays in automatic dialing equipment and its cost. In his research he was extended to more general problems and to business applications of the waiting lines. Engset's formulations were not known until

later because of the delay in publishing them and Erlang's model was first used by traffic engineers to develop better systems. Engset's main work was not in Queuing Theory and traffic engineering and his contributions are not as well known. The Danish mathematician, Erlang developed models that accounted for callers that dropped due to frustration from waiting for an operator and those that were patient enough to wait for their call to be connected. Erlang (M/D/1) Queuing model in 1917 and (M/D/K) Queuing model in 1920.

Later when Queuing Theory comes in existence many of the researchers work on this theory to reduce the waiting time of customers and reduced the service cost. This theory helps human being to solve their daily life problems of queuing to some extent .In hospital management queuing problems facing by the patients are resolve, priority queue discipline helps the serious patients to get quick treatment. In banking sector various queue discipline are comes in existence, on priority basis customers satisfaction level increases. In library science Queuing Theory is applied to counter service and circulation of books which reduced the waiting time of customer to wait for the availability of books. Queuing Theory plays an important role in various types of Queuing models for customer's satisfaction.

Characteristics of Queuing System:

There are six items which must be specified for any given Queuing System.

1. Mean arrival time of customer, λ
2. Mean service time of server, μ
3. Customer's behavior in the system
4. System capacity
5. Number of counters for service
6. The service rate is faster than arrival rate.

Different Types of Queuing Models:[16]

i. Poisson-Exponential, Single server-Infinite population model (M/M/1: ∞ /FIFO)

ii. Poisson-Exponential, Single server-Finite population model (M/M/1: N/FIFO)

iii. Poisson-Exponential, Multiple server-Infinite population models (M/M/S: ∞ /FIFO)

iv. Poisson-Exponential, Multiple server-Finite population model (M/M/S: N/FIFO)

v. Poisson Arrivals and Erlang Service Distribution ($M / E_k / 1$)

vi. Poisson Arrivals and General Service Time Distribution ($M / G / 1$)

vii. Poisson Arrivals and Regular Service Time Distribution ($M / D / 1$)

viii. Constant Arrival Rate and Constant Service Rate ($D / D / 1$)

Different Formulae's: [16]

According to the Poisson probability distribution, the probability that n customers will arrive in system during a given interval t is given by

$$P_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}; n = 0, 1, 2, \dots$$

Probability of n customers served in time t is given by

$$P_n(t) = \frac{e^{-\mu t} (\mu t)^n}{n!}; n = 0, 1, 2, \dots$$

The continuous probability density function of of exponential distribution for arrival and service time distribution are

$$f(t) = \lambda e^{-\lambda t}, \quad f(t) = \mu e^{-\mu t}$$

Whose cumulative distribution function is

$$F(t) = 1 - e^{-\lambda t}$$

By using

$$P_n(t) \rightarrow P_n, \text{ as } t \rightarrow \infty$$

i.e.

$$P_n = \frac{e^{-\lambda} (\lambda)^n}{n!}; n = 0, 1, 2, \dots$$

These can be solved by the successive substitution technique to yield

1. Probability of having exactly n customers in the system $P_n = (\rho)^n P_0$, for any value of n

where $\rho = \frac{\lambda}{\mu}$ is utilization factor, $P_0 = \left(1 - \frac{\lambda}{\mu}\right)$ is

the probability of no units in the system ,

2. Percentage of idle workstation = $(1 - \rho)100\%$

3. Expected number of units in the system

$$L_s = \sum_{n=0}^{\infty} nP_n = \frac{\lambda}{\mu - \lambda}$$

4. Expected number of units in the queue waiting for

$$\text{service } L_q = \sum_{n=1}^{\infty} (n-1)P_n = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

5. Expected waiting time a unit spends in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu - \lambda}$$

6. Expected waiting time in system (time in queue plus service time) the queue

$$W_s = W_q + \frac{1}{\mu} = \frac{1}{\mu - \lambda}$$

Little's Theorem: $L = \lambda T$ It describes the relationship between throughput rates. By using this theorem expected number of customers in the system can be determined. Here λ is average arrival rate of customer and T is the average service time for a customer.

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