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Hamiltonian Approach for Finding Shortest Path

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ABSTRACT

Hamiltonian problem is important branch of graph theory in mathematics and computer science. Need of finding shortest path is increased. The aim of the work is to find the shortest Hamiltonian path from a weighted complete graph. It is easy to understand and very useful in real life such as shortest route on road from home to hospital, social network, pizza delivery, mail delivery, gas delivery, tour travel and school bus travel etc. Due to exact timing delivery, the popularity of enterprise is increased and boom the clients, offerings and earnings. The objective of the work is to find shortest path using Hamiltonian technique from a weighted graph.

Keywords : Hamiltonian Circuit, Shortest Path, Time, Cost

I. INTRODUCTION

In day to day life, Hamiltonian graph plays a vital role in graph theory which has popular attention. Graph theory has been used not only in mathematics but also in computer science, electrical engineering and some other scientific areas. In a graph Hamiltonian circuit is a circuit that visits every vertex once with no repeated vertex and the circuit starts and ends at same vertex[6]. Hamiltonian problem can be used to solve pipeline crossing, road intersection, and communication network vertex/cities [2], time scheduling and NP-complete problem.The problem of the proposed work is to find the minimum cost or shortest path Hamiltonian circuit and certain criteria must be satisfied for existence of a Hamiltonian circuit in a graph.

General method to solve the Hamiltonian problem is

- ✓ List all possible Hamiltonian circuits with each vertex visiting once.
- ✓ Find the cost or weight of each circuit.
- ✓ Find the circuit with least cost or weight.

Applications of Shortest Hamiltonian circuit:

- ✓ In Google Maps, GPS finds the shortest route from one origin to another destination. Hamiltonian path can also efficiently determine the shortest path.
- ✓ Social networks connect friends and viral videos. Hamiltonian path can also connect friends where friends are vertices connections are edges.
- ✓ Colleges and schools can also use Hamiltonian technique to pick up the students from their station to school/college. Here station or stops are vertex routes are edges.

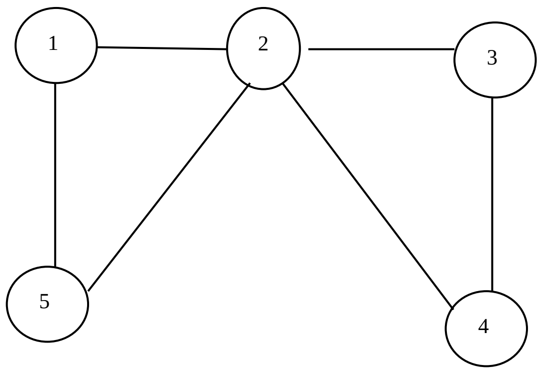

Figure 1: Not Hamiltonian cycle.

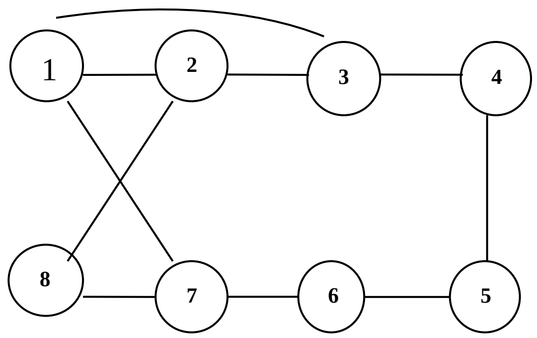

Figure 2 : Hamiltonian Cycle

The remaining of this work is organized as follows; section II deals with the literature review, section III deals with the proposed work, and section IV deals with the results & discussions section V deals with the conclusion of the work.

II. LITEACTURE REVIEW

Dipak et al [3] discussed the backtracking approach to solve Hamiltonian cycle and Travelling Salesman Problem. Aim of this work is to identify the minimum path of Hamiltonian circuit. Directed and Undirected weighted graphs are represented in form of matrix. The weights are non negative as they represent distance between two cities. Using brute force method N! permutation of the vertex/cities are generated and tested to identify Hamiltonian circuit. In weighted graph each vertex/city is examined and every possible combination is identified. The length of the Hamiltonian cycle is computed and compared with the previously calculated minimum Hamiltonian cycle length. Here the optimal solution is obtained using efficient backtracking algorithm.

Dajin Wang et al [4] discussed the embedding of Hamiltonian cycle in the Crossed Cube. Aim of this work is to design the network topology in efficient manner using the Hamiltonian cycle. This is attained by crossing the straight links of a hypercube and proposed an algorithm for Hamiltonian facilitating permutation. Due to the loss of link-topology regularity in crossed cube, producing Hamiltonian cycles in a crossed cube is more complex process than in a regular hypercube. The proposed algorithm works on link permutation of the given Hamiltonian cycle. The candidate networks balancing regularity, competence, suitability and performance criteria for choosing an interconnection network is proposed.

Wadee Alhalabi et al [1] discussed about handshaking theorem to solve Hamilton cycle of a complete graph. Purpose of this work is to find the optimal Hamiltonian cycle. The result of Peterson graph is a n-factor graph which is not Hamiltonian. The proposed n-factors graph detects the extra edges and deletes it. If is necessary the edges are added to the n-factor graph. Using java applet Peterson graph is developed to add more constraints to the destroyer and the connector.

Vidhi Sutaria et al [5] discussed about step by step making a Hamiltonian and Eulerian cycles. Aim of this work is to design the framework of telecom topology using Hamiltonian and Eulerian cycles. The Hamiltonian is Dirac's theorem can be used this work. The theorem to find NP complete finished problem with the proof and condition. The optimal algorithm output is an optimal algorithm. Euler's cycles using the Routing problem defined and found ways to delivery of good services (i.e. mail, news paper)

III. PROPOSED SHORTEST PATH METHOD

Aim of this work is to find the shortest path of Hamiltonian circuit. Consider G (V, E) be a weighted complete graph. V (G) is the vertex set and E (G) is the Edge set. Here vertices are represented as cities and edges are represented as weight or cost from one city to another city. In this work six numbers of vertices has been taken for cities and six edges has been taken for path/cost. The graph of the problem is represented in the following diagram. To find the shortest path using Hamiltonian circuit, a matrix is constructed with number of vertices as row and column numbers. The weight of the edge is taken the values inside the matrix connecting one vertex/city to another. Since, the problem is six vertex, a 6x6 matrix is constructed to find the shortest path using Hamiltonian approach.

The matrix is filled with zero values where there is no edge connected. The Hamiltonian path can be generated starting from any vertex. Hence, let the pathe finding starts from vertex 0. The shortest path is built from vertex 0 to all other vertices occurring only one. From the graph a 6x6 matrix is shown below.

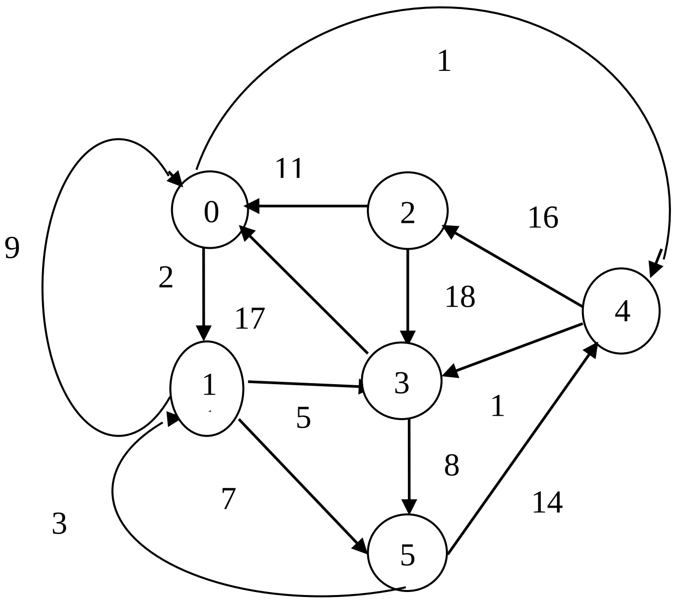

Figure 3: Hamiltonian cycle

The weights of edges between cities are

{{0, 1} = 2, {0, 4} = 12,
{1, 3} = 5, {1, 5} = 7,
{2, 0} = 11, {2, 3} = 18,
{3, 0} = 17, {3, 5} = 8,
{4, 2} = 16, {4, 3 } = 1,
{5, 1} = 3, {5, 4} = 14}

The remaining weights are 0 since their edges are not connected.

TABLE I. DISTANCE MATRIX

ˊ	0	1	2	3	4	5
0	0	2	0	0	12	0
1	9	0	0	5	0	7
2	11	0	0	18	0	0
3	17	0	0	0	0	8
4	0	0	16	1	0	0
5	0	3	0	0	14	0

The principle behind the shortest path is to find sum of weights of circuits satisfying the Hamiltonian criteria. Here the possibilities of circuits are starting from node 0, node 2, node 4 and node 5. Hence, the sum of weights from node 0 to all other nodes is found.

Case (i)

{(0,1),(1,3),(3,5),(5,4),(4,2),(2,0)}
=(0→1→3→5→4→2→0)
=2+5+8+14+16+11 =56

Case (ii)

{(0,1),(1,5),(5,4),(4,2),(2,3),(3,0)}
=(0→1→5→4→2→3→0)
=2+7+14+16+18+17 =74

Case (iii)

{(0,4),(4,2),(2,3),(3,5),(5,1),(1,0)}
=(0→4→2→3→5→1→0)
=12+16+18+8+3+9 =66

Sum of weights from node 2 to all other nodes

{(2,3),(3,5),(5,1),(1,0),(0,4)(4,4)},
=(2→3→5→1→0→4→2)
=18+8+3+19+112+16 =66

Sum of weights from node 4 to all other nodes

{(4,2),(2,3),(3,5),(5,1),(1,0),(0,4)},
=(4→2→3→5→1→0→4)
=16+18+8+3+9+12 =66

Sum of weights from node 5 to all other nodes

Case (i)

{(5,1),(1,0),(0,4),(4,2),(2,3),(3,5)},

=(5→1→0→4→2→3→5)

=3+9+12+16+18+8 =66

Case(ii)

{(5,4),(4,2),(2,3),(3,0),(0,1),(1,5)}

= (5→4→2→3→0→1→5)

=14+16+18+17+2+3 =70

3.1 Hamiltonian Path Algorithm

```
Algorithm Hamiltonian (k)
{
  Repeat
  {
     Next Value (k)
      If  (x[k]=0)  then return)
      else
          Hamiltonian (k+1);
   }
       Until (false)
 }
Algorithm Next Value (k)
{
   Repeat
   {
      X[k]=x[k+1];
      If ( x[k]=0) then return
          Sum=0
      for j=1 to k-1 do
      {
        sum=sum[k]) then break;
        If (x[j] =x[k]) then break;
       }
       Return (x[k].........x[k-1][xk])
    }
    Until (false)
  }
```

IV. RESULTS AND DISCUSSIONS

It is seen that there are seven cases raised from the six vertex problem.. Various paths from starting from different nodes are found. The shortest paths are defined below. It is noted that three of them are starting from 0 to remaining all other nodes, one path from starting node 2 to all other nodes, one path from node 4 to remaining other nodes and two possible paths from starting node 5 to all other nodes.

TABLE II

HAMILTONIAN PATHS

Starting Node	Path	Sum Weight
0	**0→1→3→5→4→2→0**	56
0	0→1→5→4→2→3→0	74
0	0→4→2→3→5→1→0	66
2	2→3→5→1→0→4→2	66
4	4→2→3→5→1→0→4	66
5	5→1→0→4→2→3→5	66
5	(5→4→2→3→0→1→5	70

Out of these seven paths, it found that the path stating from 0 to all other nodes is shortest and the route of path is denoted as

0→1→3→5→4→2→0

V. CONCLUSION

This work discusses the shortest path to travel one place to other place using the Hamiltonian cycle theory. A problem of six vertices is considered for discussion. The weights of edges starting from one node and ending at the same node are calculated and listed. There are some path which are not meeting the criteria of Hamiltonian circuit. Hence such paths are omitted. It is observed that out of the seven possibilities, only one path is optimum giving minimum total weight. Hence such a path is considered as the shortest path. The advantages of this approach preserves cost and time of problems depending on distance or costs of transportation.

VI. REFERENCES

[1]. Wadee Alhalabi , Omar Kitannah , Amira Alharbi, Zain Balfakin and Akila Sarirete., "Efficient solution for finding Hamiltonian cycles in undirected graph", SpringPlus(2016), 5:11:92.

[2]. M Radhika ., "A Comparative Study Between Eulerian And Hamiltonian Graphs", IJETST-Vol.03,Issue,05,Page 435-439,May,ISSN 2348-9480,2016.

[3]. Dipak Patel,Nishant Doshi,Shakthi Patel,Dishant Soni., "Hamiltonian cycle and TSP:A backtracking approach",InternationalJournal on Computerscience and Engineering(IJCSE),Vol.3,No.4,Apr 2011.

[4]. Dajin Wang., "On Embedding Hamiltonian Cycles in Crossed Cubes", IEEE Transaction on Parallel and Distributed Systems", Vol.19, No.3, March 2008,

[5]. Vidhi Sutaria., "Hamiltonian and Eulerian Cycles", International Journal of Trend in Research and Development,Vol.3,ISSN:2394-9333.

[6]. Ref:Courses.Lumenlearning Visited on 19:5:18

[7]. ANagoor Gani and S.R.Latha., "A new algorithm to find fuzzy Hamiltonian cycle in a fuzzy network using adjacency matrix and minimum vertex degree",SpringPlus(2016).

[8]. MSohel Rahman,M.Kaykobadn and Jesun Sahariar Firoz., "New Sufficient Conditions for Hamiltonian Paths",Hindawi Publishing Corporation Scientific World Journal,Vol 2014.

[9]. Mushtaq Ahmad Stah., "A study on hypo Hamiltonian graphs",African journal of Mathematics and Computer Science Research,Vol.6(2),pp.16-19,February 2013.

[10]. Dr.G.Nirmala,M.Vijaya., "Hamiltonian fuzzy Cycles on k 2n+1 Fuzzy Graphs",Internatonal journal of scientific and Research Publications,Volume 2,Issue 11,November 2012.