

# Some Studies with GEM Equations - A Note

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## ABSTRACT

Considering GEM equations expressions for electrostatic ( $u_e$ ), magnetic ( $u_m$ ) and gravitational ( $u_g$ ) energy densities were found out. The ratios  $\frac{u_e}{u_m}$ ,  $\frac{u_g}{u_e}$ ,  $\frac{u_g}{u_m}$  and their sum have also been obtained. Studies were made to find out the correct form of these ratios i.e.  $\frac{u_g}{u_e}$  or  $\frac{u_e}{u_g}$  and  $\frac{u_g}{u_m}$  or  $\frac{u_m}{u_g}$  and proper arguments are presented in their favour. In similar way the correct form of the wave impedances for electromagnetic ( $Z_{EM}$  or  $Z_{ME}$ ), gravitomagnetic ( $Z_{GH}$  or  $Z_{HG}$ ) and gravitoelectric ( $Z_{GE}$  or  $Z_{EG}$ ) portions of the GEM wave have been derived. Lastly, mention has been made to a mathematical concept which would introduce an error in deducing some expressions. Steps have been provided to show how this error may be avoided and arrive at the speculated result.

**Keywords :** GEM, EM, GE and GH waves

## I. INTRODUCTION

It is well known that energy density and impedance of any type of wave could be found out easily as done in literatures [ 1 ]. Thus expressions for electrostatic ( $u_e$ ), magnetic ( $u_m$ ) and gravitational ( $u_g$ ) energy densities and wave impedances are closely related to the GEM waves. Using GEM equations one could find out the expressions for these parameters as well as different types of ratios of the energy densities. It should be mentioned here that the above parameters depend upon the constants like  $\mu_{G0}, \epsilon_{G0}, \mu_{E0}, \epsilon_{E0}$  etc. Depending upon the values of these constants the energy densities, their ratios and the impedances may be either finite or infinite. In this work trial would be made to find out the correct form of the above parameters so that they are finite. Again, in some cases certain mathematical steps could be used which are mathematically correct apparently

but erroneous so far as physical concept is concerned. This type of error could be avoided if physical reasoning be given importance while writing a mathematical step. In this dissertation such an error would be avoided by providing proper arguments having physical concepts.

## II. THE PROBLEM

We have GEM (Gravitoelectromagnetic) equations proposed in [ 2,3 ] are given by

$$\begin{aligned} (i) \nabla \cdot \mathbf{E} &= 0, (ii) \nabla \cdot \mathbf{G} = 0, (iii) \nabla \cdot \mathbf{H} = 0, \\ (iv) \nabla \times \mathbf{E} &= -\mu_{G0} \frac{\partial \mathbf{H}}{\partial t}, (v) \nabla \times \mathbf{G} = -\mu_{E0} \frac{\partial \mathbf{H}}{\partial t} \text{ and} \\ (vi) \nabla \times \mathbf{H} &= \epsilon_{G0} \frac{\partial \mathbf{E}}{\partial t} - \epsilon_{E0} \frac{\partial \mathbf{G}}{\partial t} \end{aligned} \quad (1)$$

Here  $\mathbf{E}$ ,  $\mathbf{H}$  and  $\mathbf{G}$  are respectively the electric, magnetic and gravitational field vectors. These are connected by electric and magnetic constants  $\epsilon_{G0}$ ,  $\epsilon_{E0}$  and  $\mu_{G0}$ ,  $\mu_{E0}$  which are related to gravitational

and electrical space-time designated by the suffix  $G$  and  $E$  respectively. It may be assumed that in absence of gravitational interaction  $\mu_{G0} = \mu_0$  and  $\varepsilon_{G0} = \varepsilon_0$  and  $\mu_{E0} = 0 = \varepsilon_{E0}$ . It is easily seen that GEM wave in a medium is due to the mutual interaction of electric, magnetic and gravitational fields.

From ( 1 ) we could obtain [ 4,5 ]

$$\begin{aligned} a) \nabla^2 \mathbf{E} &= \mu_{G0} \varepsilon_{G0} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_{G0} \varepsilon_{E0} \frac{\partial^2 \mathbf{G}}{\partial t^2} \\ b) \nabla^2 \mathbf{G} &= \mu_{E0} \varepsilon_{G0} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_{E0} \varepsilon_{E0} \frac{\partial^2 \mathbf{G}}{\partial t^2} \end{aligned} \quad (2)$$

$$\text{and } c) \nabla^2 \mathbf{H} = (\mu_{G0} \varepsilon_{G0} - \mu_{E0} \varepsilon_{E0}) \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

The solution of these are

$$\begin{aligned} a) \mathbf{E} &= \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t) \\ b) \mathbf{G} &= \mathbf{G}_0 \exp(i\mathbf{k}' \cdot \mathbf{r} - i\omega' t) \\ c) \mathbf{H} &= \mathbf{H}_0 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t) \end{aligned} \quad (3)$$

where

$\omega, \omega'$  and  $\mathbf{k}, \mathbf{k}'$  are respectively the frequencies and propagation vectors for the oscillations of EM and gravitational waves respectively.

Now, we have the ratios of EM(electromagnetic), GE(gravitoelectric) and GM(gravitomagnetic) energy densities respectively to be

$$\begin{aligned} a) \frac{u_e}{u_m} &= \frac{\mu_{G0}}{\mu_{G0} - \varepsilon_{E0}} \\ b) \frac{u_G}{u_e} &= \frac{\mu_{E0}}{\mu_{G0}} \end{aligned} \quad (4)$$

$$\text{and } c) \frac{u_G}{u_m} = \frac{\mu_{E0}}{\mu_{G0} - \varepsilon_{E0}}$$

where, similar to [ 5 ], we have

$$\text{Electrostatic energy density } u_e = -\frac{1}{2} \varepsilon_{G0} (\mathbf{E})^2$$

$$\text{Magnetic energy density } u_m = -\frac{1}{2} (\mu_{G0} - \varepsilon_{E0}) (\mathbf{H})^2 \quad (5)$$

$$\text{and Gravitational energy density } u_G = -\frac{1}{2} \varepsilon_{E0} (\mathbf{G})^2$$

In order that the relations may be viable they should be written as in ( 4 ) so that the ratios  $\frac{u_G}{u_e}$  and  $\frac{u_G}{u_m}$  become zero in absence of the effect of gravity. Also, the total ratio

$$X \text{ (say)} = \frac{u_e}{u_m} + \frac{u_G}{u_e} + \frac{u_G}{u_m} = \frac{\mu_{G0}}{\mu_{G0} - \varepsilon_{E0}} + \frac{\mu_{E0}}{\mu_{G0}} + \frac{\mu_{E0}}{\mu_{G0} - \varepsilon_{E0}} \quad (6)$$

would have a finite value in presence of  $\mathbf{G}$  and unity in absence of it. Otherwise, if we write the ratios as  $\frac{u_e}{u_G}$  and  $\frac{u_m}{u_G}$  then these ratios as well as  $X$  would become infinity in absence of the effect of gravity which is impossible.

Now, we may consider GEM waves as a combination of EM, GE and GH waves. Hence, GEM waves should have three impedances combined together. These may be written as  $Z_{EM}, Z_{GE}$  and  $Z_{GH}$ . But, the question is what type of combination it is? Whether the three  $Z$ 's would be combined in series or in parallel? Let us try to study it.

Using ( 3 ) we get respectively from (iv), (v) and (vi) of ( 1 )

$$\begin{aligned} (a) \mathbf{k} \times \mathbf{E} &= \mu_{G0} \omega \mathbf{H}, \quad (b) \mathbf{k}' \times \mathbf{G} = \mu_{E0} \omega \mathbf{H} \text{ and} \\ (c) \mathbf{k} \times \mathbf{H} &= -\varepsilon_{G0} \omega \mathbf{E} + \varepsilon_{E0} \omega' \mathbf{G} \end{aligned} \quad (7)$$

We can easily find out the impedances of EM, GM and GE waves from ( 7 ). These are shown below.

$$Z_{EM} = \left| \frac{\mathbf{E}}{\mathbf{H}} \right| = \mu_{G0} c_1 \quad (8)$$

$$Z_{GH} = \left| \frac{\mathbf{G}}{\mathbf{H}} \right| = \mu_{E0} c_2 \quad (9)$$

$$Z_{GE} = \left| \frac{\mathbf{G}}{\mathbf{E}} \right| = \frac{\mu_{E0} c_2}{\mu_{G0} c_1} \quad (10)$$

$$\text{where } c_1 = \frac{1}{\sqrt{\mu_{G0} \varepsilon_{G0}}} \text{ and } c_2 = \frac{1}{\sqrt{\mu_{E0} \varepsilon_{E0}}} \quad (11)$$

It is clearly seen that in absence of gravity only  $Z_{EM}$  exists while other two impedances would be zero

( since  $\mu_{E0} = 0 = c_2$  ). Again, if we take series combination of them then, also, the resultant impedances would be finite in presence or in absence of gravity. In the later case it is, again,  $Z_{EM}$  . This is justified.

But, if we write  $Z_{GH}$  and  $Z_{GE}$  in the inverse form then  $Z_{HG}(=\frac{1}{\mu_{E0}c_2})$  and  $Z_{EG}(=\frac{\mu_{G0}c_1}{\mu_{E0}c_2})$  would become infinity in absence of gravitational interaction. Let us take the series combination of these impedances. Then, the resultant

$$Z_S = Z_{EM} + Z_{EG} + Z_{HG} = \mu_{G0}c_1 + \frac{\mu_{G0}c_1}{\mu_{E0}c_2} + \frac{1}{\mu_{E0}c_2}$$

would become infinity in absence of gravitational interaction. This means that in absence of gravity the energy of EM wave will not propagate at all. This is not true for all media. Hence, the impedances should be written in the forms as in ( 8 ), ( 9 ) and ( 10 ).

Let us study the effect in case the impedances in ( 8 ), ( 9 ) and ( 10 ) are combined in parallel. Using these equations the resultant impedance  $Z_p$  would be

$$Z_p = \frac{\mu_{G0}\mu_{E0}c_1c_2}{\mu_{E0}c_2 + \mu_{G0}^2c_1^2 + \mu_{G0}c_1} \quad (12)$$

This  $Z_p$  has a finite value. Now, in absence of gravitational interaction  $\mu_{E0} = 0 = c_2$ . Hence,  $Z_p$  becomes zero leading to the fact that the GEM wave would travel with infinite velocity in any type of medium. This is, again, impossible.

Thus, in case of any resultant wave formed by superposition of several waves ( GEM wave = EM wave + GE wave + GM wave ), the resultant impedance should be found out by combining the individual impedances in series. This is a well known fact valid for any type of wave. So, it is also valid for GEM waves.

Again, using ( 1- vi ) we have in ( 7- c )

$$\mathbf{k} \times \mathbf{H} = -\varepsilon_{G0}\omega\mathbf{E} + \varepsilon_{E0}\omega'\mathbf{G} \quad (13)$$

Substituting  $\mathbf{G}$  from ( 10 ) it will lead to

$$\left| \frac{\mathbf{E}}{\mathbf{H}} \right| = \frac{\mu_{G0}\omega}{-\varepsilon_{G0}\mu_{G0}\omega c_1 + \varepsilon_{E0}\mu_{E0}\omega'c_2} \quad (14)$$

$$\text{as } k = \frac{\omega}{c_1}$$

Now, equating ( 8 ) and ( 14 ) we get

$$\frac{c_1}{c_2} = 2 \frac{\omega}{\omega'} \quad (15)$$

Again, using ( 3 ) we get from ( 2 a ) and ( 2 c ) respectively

$$k^2\mathbf{E} = \mu_{G0}\varepsilon_{G0}\omega^2\mathbf{E} - \mu_{G0}\varepsilon_{E0}\omega'^2\mathbf{G} \quad (16)$$

$$\text{and } k^2 = (\mu_{G0}\varepsilon_{G0} - \mu_{E0}\varepsilon_{E0})\omega^2 \quad (17)$$

$$\text{Using ( 17 ) we get from} \quad (16)$$

$$\frac{\omega}{\omega'} = \sqrt{\frac{c_2}{c_1}} \quad (18)$$

Also, applying ( 18 ) we obtain from ( 15 )

$$\frac{c_1}{c_2} = 2^{\frac{2}{3}} \quad (19)$$

This gives the relation between the velocities of EM and gravitational part of the GEM wave.

Now, from ( 18 ) we have  $\frac{c_2}{c_1} = \frac{\omega^2}{\omega'^2}$ . Hence, from ( 15 )

$$\frac{\omega}{\omega'} = 2^{-\left(\frac{1}{3}\right)} \quad (20)$$

It is seen that ( 19 ) gives the relation between the velocities of EM and gravitational waves while ( 20 ) is that between their frequencies. We have to note that the results obtained in ( 19 ) and ( 20 ) seem to be fallacious and unusual. Since, in ( 8 ) and ( 14 ) values of the parameters describing the characteristics of the medium and the wave are unknown to us then how can we arrive at the equations like ( 19 ) or ( 20 ) with numerical values on the right hand side?

### III. SOLUTION OF THE PROBLEM

The possible explanations to solve the fallacy may be as follows:-

i) To obtain ( 15 ) we have equated ( 8 ) and ( 14 ). The entity  $\left| \frac{\mathbf{E}}{\mathbf{H}} \right|$  in ( 8 ) seems to be independent of  $\mathbf{G}$ . But, that in ( 14 ) depends on  $\mathbf{G}$ . So, while deducing ( 15 ) we have equated the same entities but with different characteristics. This is improper.

ii) Relation (14) is not proper as in absence of  $\mathbf{G}$  it will lead to  $\left| \frac{\mathbf{E}}{\mathbf{H}} \right| = -\frac{1}{\epsilon_{G0}c_1} = -\sqrt{\frac{\mu_{G0}}{\epsilon_{G0}}} = -\sqrt{\frac{\mu_0}{\epsilon_0}}$ . But the modulus of an entity cannot be negative. Also, this does not tally with ( 8 ).

Again,  $\mathbf{E}$  and  $\mathbf{G}$  are oppositely directed as seen from ( 1 – vi ) and ( 16 ). Hence, ( 14 ) may be written as

$$\left| \frac{\mathbf{E}}{\mathbf{H}} \right| = \frac{\mu_{G0}\omega}{\epsilon_{G0}\mu_{G0}\omega c_1 - \epsilon_{E0}\mu_{E0}\omega' c_2} \quad (21) \text{ which when}$$

equated to ( 8 ) gives

$$\omega' = 0 \quad (22)$$

This is justified. Since, we are dealing only with  $\mathbf{E}$  and  $\mathbf{H}$  when  $\mathbf{G}$  is not included, then the fact is that the impedance of the EM part of GEM wave has been dealt with. As a result, no gravitational wave exists and frequency  $\omega'$  of gravitational part of the wave becomes zero. Thus, by slight modification of the mathematical understanding the above self-contradictory relations ( 19 ) and ( 20 ) could be explained and the fallacy may be resolved.

#### IV. REFERENCES

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