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Contra – Bc – Continuous Functions in Topological Spaces

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ABSTRACT

In this paper by means of $\mathrm{Bc-open}$ sets, we introduce and investigate certain ramifications of contra continuous and allied functions, namely, $\mathrm{contra-Bc-continuous}$, $\mathrm{almost-Bc-continuous}$, almost $\mathrm{weakly-Bc-continuous}$ and almost $\mathrm{contra-Bc-continuous}$ functions along with their several properties, characterizations and natural relationships. Further, we introduce new types of graphs, called $\mathrm{Bc-closed}$, $\mathrm{contra-Bc-closed}$ and strongly $\mathrm{contra-Bc-closed}$ graphs via $\mathrm{Bc-open}$ sets. Several characterizations and properties of such notions are investigated.

Keywords : Almost $\mathrm{contra-Bc-continuous}$, $\mathrm{contra-Bc-closed}$ Graph, $\mathrm{contra-Bc-continuous}$, $\mathrm{Bc-connected}$, $\mathrm{Bc-open}$.

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I. INTRODUCTION

In recent literature, we find many topologists have focused their research in the direction of investigating different types of generalized continuity. One of the outcomes of their research leads to the initiation of different orientations of contra continuous functions. The notion of contra continuity was first investigated by Dontchev [7]. Subsequently, Jafari and Noiri [15, 16] exhibited $\mathrm{contra-\alpha-continuous}$ and $\mathrm{contra-pre-continuous}$ functions. Contra $\delta\mathrm{-precontinuous}$ [10] was studied by Ekici and Noiri. $\mathrm{contra-e-continuous}$ functions [12] was studied by Gosh and Basu. A good number of researchers have also initiated different types of contra continuous – like functions, some of which are found in the papers [6, 8, 11, 18, 23]. In [1] Al – Abdullah and Abed introduced a new class of sets in a topological space, known as $\mathrm{Bc-open}$ sets. In this paper, we employ this notion of $\mathrm{Bc-open}$ sets to introduce and investigate contra continuous functions, called $\mathrm{contra-Bc-continuous}$ functions. We study fundamental properties of $\mathrm{contra-Bc-continuous}$ functions and use such functions to characterize $\mathrm{Bc-connectedness}$. We introduce and study the properties of almost $\mathrm{contra-Bc-continuous}$ functions as well. We also introduce and study the notions of $\mathrm{Bc-closed}$, $\mathrm{contra-Bc-closed}$ and slightly $\mathrm{contra-closed}$ graphs.

II. PRELIMINARIES

In this paper, spaces X and Y always represent topological spaces (X, τ) and (Y, σ) respectively on which no separation axioms are assumed unless otherwise stated. For a subset A of a space X $\mathrm{Cl}(A)$

and $\text{Int}(A)$ denote the closure and the interior of A respectively.

Definition 2.1. Let (X, τ) be a topological space and $A \subseteq X$. Then A is called $b-\text{open}$ set in X if $A \subseteq \text{Cl}[\text{Int}(A)] \cup \text{Int}[\text{Cl}(A)]$. The family of all $b-\text{open}$ subsets of a topological space (X, τ) is denoted by $BO(X, \tau)$ or (Briefly $BO(X)$).

Lemma 2.2. Arbitrary union of $b-\text{open}$ sets in a topological space is $b-\text{open}$ set.

Proof. Let (X, τ) be a topological space and let $\{A_\alpha : \alpha \in \Delta\}$ be a family of $b-\text{open}$ sets in X. Let $A = \bigcup_{\alpha\in\Delta} A_\alpha$. Then for each $\alpha \in \Delta$, $A_\alpha \subseteq A$. Then it follows immediately that

$\text{Cl}[\text{Int}(A_\alpha)] \subseteq \text{Cl}[\text{Int}(A)]$

and $\text{Int}[\text{Cl}(A_\alpha)] \subseteq \text{Int}[\text{Cl}(A)]$. Further by hypothesis, $A_\alpha \subseteq \text{Cl}[\text{Int}(A_\alpha)] \cup \text{Int}[\text{Cl}(A_\alpha)]$. Hence it implies immediately that $A_\alpha \subseteq \text{Cl}[\text{Int}(A)] \cup \text{Int}[\text{Cl}(A)]$, for each $\alpha \in \Delta$. Thus $A = \bigcup_{\alpha\in\Delta} A_\alpha \subseteq \text{Cl}[\text{Int}(A)] \cup \text{Int}[\text{Cl}(A)]$. Therefore, $A = \bigcup_{\alpha\in\Delta} A_\alpha$ is a $b-\text{open}$ set in X.

Definition 2.3. Let (X, τ) be a topological space and $A \subseteq X$. Then A is called $Bc-\text{open}$ set in X if for each $x \in A \in BO(X, x)$, there exists a closed set F such that $x \in F \subseteq X$. The family of all $Bc-\text{open}$ subsets of a topological space (X, τ) is denoted by $Bc-O(X, \tau)$ or (Briefly $Bc-O(X)$). A is called $Bc-\text{closed}$ if $A^c = X - A$ is $Bc-\text{open}$ set. The family of all $Bc-\text{closed}$ subsets of a topological space (X, τ) is denoted by $Bc-C(X, \tau)$ or (Briefly $Bc-C(X)$).

Remark 2.4. It is clear from the definition that every $Bc-\text{open}$ set is $b-\text{open}$, but the converse is not true in general as shown by the following example.

Example 2.5. Suppose that $X = \{1, 2, 3\}$, and the topology $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$. Then the closed sets are: $X, \phi, \{2, 3\}, \{1,3\}, \{3\}$. Hence $BO(X, \tau) = \{\phi, X, \{1\}, \{2\}, \{1, 2\}, \{1,3\}, \{2,3\}\}$ and $Bc-O(X, \tau) = \{\phi, X, \{1,3\}, \{2,3\}\}$. Then $\{1\}$ is $b-\text{open}$, but $\{1\}$ is not $Bc-\text{open}$.

Remark 2.6. The intersection of two $Bc-\text{open}$ sets is not $Bc-\text{open}$ set in general as shown by the following example.

Example 2.7. Suppose that $X = \{1, 2, 3\}$, and the topology $\tau = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$. Then $\{1,3\}$ and $\{2,3\}$ are $Bc-\text{open}$ sets where as $\{1, 3\} \cap \{2,3\} = \{3\}$ is not $Bc-\text{open}$ set.

Theorem 2.8. Arbitrary union of $Bc-\text{open}$ sets in a topological space is $Bc-\text{open}$ set.

Proof. Let $\{A_\alpha : \alpha \in \Delta\}$ be a family of $Bc-\text{open}$ sets in a topological space (X, τ). Then for each $\alpha \in \Delta$, A_α is a $b-\text{open}$ set in X and by Lemma 2.2, $\bigcup_{\alpha\in\Delta} A_\alpha$ is $b-\text{open}$ set. If $x \in \bigcup_{\alpha\in\Delta} A_\alpha$, then there exists $\gamma \in \Delta$ such that $x \in A_\gamma$. Since $x \in A_\gamma$ is $Bc-\text{open}$ set, there exists a closed set F such that $x \in F \subseteq A_\gamma \subseteq \bigcup_{\gamma\in\Delta} A_\gamma$. Hence $\bigcup_{\gamma\in\Delta} A_\gamma$ is $Bc-\text{open}$ set in X.

Theorem 2.9. A subset A of a topological space (X, τ) is $Bc-\text{open}$ if and only if A is $b-\text{open}$ and it is a union of closed sets.

Proof. Let A be a $Bc-\text{open}$ set. Then A is $b-\text{open}$ and for each $x \in A$, there exists a closed set F_x such that $x \in F_x \subseteq A$. This implies that $A = \bigcup_{x\in A}\{x\} \subseteq \bigcup_{x\in A} F_x \subseteq A$. Thus $A = \bigcup_{x\in A} F_x$ where F_x is closed set for each $x \in A$. The converse is directed from definition of $Bc-\text{open}$ sets.

Definition 2.10. Let (X, τ) be a topological space and $A \subseteq X$. Then $Bc-\text{closure}$ of A in X is the set

$\mathrm{Bc-Cl}(\mathrm{A}) = \bigcap\{\mathrm{K} \subseteq \mathrm{X} : \mathrm{K} \text{ is } \mathrm{Bc-closed} \text{ and } \mathrm{A} \subseteq \mathrm{K}\}$.

Definition 2.11. Let (X, τ) be a topological space and $\mathrm{A} \subseteq \mathrm{X}$. Then $\mathrm{Bc-interior}$ of A in X is the set $\mathrm{Bc-Int}(\mathrm{A}) = \bigcup\{\mathrm{U} \subseteq \mathrm{X} : \mathrm{U} \text{ is } \mathrm{Bc-open} \text{ and } \mathrm{U} \subseteq \mathrm{A}\}$.

Definition 2.12. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the subset $\{(x, f(x)) : x \in X\} \subseteq X \times Y$ is called the graph of f and is denoted by $G(f)$.

III. CONTRA – Bc – CONTINUOUS FUNCTIONS

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called $\mathrm{contra-Bc-continuous}$ if $f^{-1}(V)$ is $\mathrm{Bc-closed}$ in X for every open set V of Y.

Definition 3.2. Let (X, τ) be topological space and $\mathrm{A} \subseteq \mathrm{X}$. Then the intersection of all open sets of X containing A is called kernel of A and is denoted by $\mathrm{Ker}(\mathrm{A})$.

Lemma 3.3. The following properties hold for subsets A and B of a topological space (X, τ).

(a) $x \in Ker(A)$ if and only if $\mathrm{A} \bigcap \mathrm{F} \neq \phi$ for any closed set F of X containing x.

(b) $\mathrm{A} \subseteq \mathrm{Ker}(\mathrm{A})$ and $\mathrm{A} = \mathrm{Ker}(\mathrm{A})$ if A is open in X.

(c) If $\mathrm{A} \subseteq \mathrm{B}$, then $\mathrm{Ker}(\mathrm{A}) \subseteq \mathrm{Ker}(\mathrm{B})$.

Lemma 3.4. The following properties hold for a subset A of a topological space (X, τ):

(i) $\mathrm{Bc-Int}(\mathrm{A}) = \mathrm{X} - [\mathrm{Bc-Cl}(\mathrm{X}-\mathrm{A})]$;

(ii) $\mathrm{x} \in \mathrm{Bc-Cl}(\mathrm{A})$ if and only if $\mathrm{A} \bigcap \mathrm{U} \neq \phi$ for each $\mathrm{x} \in \mathrm{Bc-O}(\mathrm{X}, \mathrm{x})$;

(iii) A is $\mathrm{Bc-open}$ if and only if $\mathrm{A} = \mathrm{Bc-Int}(\mathrm{A})$;

(iv) A is $\mathrm{Bc-closed}$ if and only if $\mathrm{A} = \mathrm{Bc-Cl}(\mathrm{A})$.

Theorem 3.5. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following conditions are equivalent:

(a) f is $\mathrm{contra-Bc-continuous}$;

(b) for each $\mathrm{x} \in \mathrm{X}$ and each closed subset F of Y containing $f(x)$, there exists $\mathrm{U} \in \mathrm{Bc-O}(\mathrm{X}, \mathrm{x})$ such that $f(U) \subseteq F$;

(c) for each closed subset F of Y, $f^{-1}(F)$ is $\mathrm{Bc-open}$ in X;

(d) $f[Bc-Cl(A)] \subseteq Ker[f(A)]$ for every subset A of X;

(e) $Bc-Cl[f^{-1}(B)] \subseteq f^{-1}[Ker(B)]$ for every subset B of Y.

Proof. $(a) \Rightarrow (b)$: Let $\mathrm{x} \in \mathrm{X}$ and F be any closed set of Y containing $f(x)$. Using (a), we have $f^{-1}(Y-F) = X - f^{-1}(F)$ is $\mathrm{Bc-closed}$ in X and so $f^{-1}(F)$ is $\mathrm{Bc-open}$ in X. Taking $U = f^{-1}(F)$, we get $\mathrm{x} \in \mathrm{U}$ and $f(U) \subseteq \mathrm{F}$.

$(b) \Rightarrow (c)$: Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists a $\mathrm{Bc-open}$ subset U_{x} containing x such that $f(U_x) \subseteq \mathrm{F}$. Therefore, we obtain $f^{-1}(F) = \bigcup\{U_x : x \in f^{-1}(F)\}$ – which is $\mathrm{Bc-open}$ in X.

$(c) \Rightarrow (a)$: Let V be any open set of Y. Then since $(\mathrm{Y}-\mathrm{V})$ is closed in Y, by (c) $f^{-1}(Y-V) = \mathrm{X} - f^{-1}(V)$ is $\mathrm{Bc-open}$ in X. Therefore, $f^{-1}(V)$ is $\mathrm{Bc-closed}$ in X.

$(c) \Rightarrow (d)$: Let A be any subset of X. Suppose that $y \notin Ker[f(A)]$. Then by Lemma 3.3, there exists a closed set F of Y containing y such that $f(A) \bigcap F = \phi$. This implies that $A \bigcap f^{-1}(F) = \phi$ and so $Bc-Cl(A) \bigcap f^{-1}(F) = \phi$. Therefore, we obtain $f\left[Bc-Cl(A)\right]\cap F=\phi$ and $y\notin f\left[Bc-Cl(A)\right]$. Hence, $f\left[Bc-Cl(A)\right]\subseteq Ker\left[f(A)\right]$.

$(d)\Rightarrow(e)$: Let B be any subset of Y. Using (d) and Lemma 3.3 we have $f\left[Bc-Cl\left(f^{-1}(B)\right)\right]\subseteq Ker\left[f\left(f^{-1}(B)\right)\right]$ $\subseteq Ker(B)$. Thus it follows that $Bc-Cl\left[f^{-1}(B)\right]\subseteq f^{-1}\left[Ker(B)\right]$.

$(e)\Rightarrow(a)$: Let V be an open subset of Y. Then from Lemma 3.3 and (e) we have $Bc-Cl\left[f^{-1}(V)\right]\subseteq f^{-1}\left[Ker(V)\right]=f^{-1}(V)$ and hence $Bc-Cl\left[f^{-1}(V)\right]=f^{-1}(V)$. This shows that $f^{-1}(V)$ is Bc – closed in X.

The following lemma can be verified easily.

Lemma 3.6. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is Bc – continuous if and only if for each $x\in X$ and for each open set V of Y containing $f(x)$, there exists $U\in Bc-O(X,x)$ such that $f(U)\subseteq V$.

Theorem 3.7. Suppose that a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is contra – Bc – continuous and Y is regular. Then f is Bc – continuous.

Proof. Let $x\in X$ and V be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set G in Y containing $f(x)$ such that $Cl(G)\subseteq V$. Again, since f is $contra-Bc-continuous$, so by Theorem 3.5, there exists $U\in Bc-O(X,x)$ such that $f(U)\subseteq Cl(G)$. Then $f(U)\subseteq Cl(G)\subseteq V$. Hence f is $Bc-continuous$.

Definition 3.8. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called $almost-Bc-continuous$ if for each $x\in X$ and each open set V of Y containing $f(x)$, there exists $U\in Bc-O(X,x)$ such that $f(U)\subseteq Bc-Int\left[Cl(V)\right]$.

$Almost-Bc-continuous$ function can be equivalently defined as in the following proposition.

Proposition 3.9. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function. Then the following statements are equivalent:

(a) f is $almost-Bc-continuous$.

(b) For each $x\in X$ and each regular open set V of Y containing $f(x)$, there exists $U\in Bc-O(X,x)$ such that $f(U)\subseteq V$.

(c) $f^{-1}(V)$ is $Bc-open$ in X for every regular open set V of Y.

Definition 3.10. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be $pre-Bc-open$ if image of each $Bc-open$ set of X is a $Bc-open$ set of Y.

Denition 3.11. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be $Bc-irresolute$ if preimage of a $Bc-open$ subset of Y is a $Bc-open$ subset of X.

Theorem 3.12. Suppose that a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is $pre-Bc-open$ and contra – Bc – continuous. Then f is almost – Bc – continuous.

Proof. Let $x\in X$ and V be an open set containing $f(x)$. Since f is contra – Bc – continuous, then by Theorem 3.5, there exists $U\in Bc-O(X,x)$ such that $f(U)\subseteq Cl(V)$. Again, since f is pre – Bc – open, $f(U)$ is Bc – open in Y. Therefore, $f(U)=Bc-Int\left[f(U)\right]$ and hence $f(U)\subseteq Bc-Int\left[Cl\left(f(U)\right)\right]\subseteq Bc-Int\left[Cl(V)\right]$. So f is almost – Bc – continuous.

Theorem 3.13. Let $\{(X_\lambda,\tau_\lambda):\lambda\in\Lambda\}$ be any family of topological spaces. If a function $f:X\longrightarrow\prod_{\lambda\in\Lambda}X_\lambda$ is contra – Bc – continuous, then

$\pi_\lambda o f : X \longrightarrow X_\lambda$ is contra – Bc – continuous, for each $\lambda \in \Lambda$, where π_λ is the projection of $\prod_{\lambda\in\Lambda} X_\lambda$ onto X_λ.

Proof. For a fixed $\lambda \in \Lambda$, let V_λ be any open subset of X_λ. Since π_λ is continuous, $\pi_\lambda^{-1}(V_\lambda)$ is open in $\prod_{\lambda\in\Lambda} X_\lambda$. Since f is *contra – Bc – continuous*, $f^{-1}\left[\pi_\lambda^{-1}(V_\lambda)\right] = (\pi_\lambda o f)^{-1}(V_\lambda)$ is Bc – closed in X. Therefore, $\pi_\lambda o f$ is contra – Bc – continuous for each $\lambda \in \Lambda$,

Definition 3.14. Let (X, τ) be a topological space. Then the Bc – frontier of a subset A of X, denoted by $Bc - Fr(A)$, is defined as $Bc - Fr(A) = \left[Bc - Cl(A)\right] \cap \left[Bc - Cl(X - A)\right] = \left[Bc - Cl(A)\right] - \left[Bc - Int(A)\right]$.

Theorem 3.15. The set of all points x of X at which $f : (X, \tau) \to (Y, \sigma)$ is not contra – Bc – continuous is identical with the union of Bc – frontier of the inverse images of closed sets of Y containing $f(x)$.

Proof. ***Necessity:*** Let f be not contra – Bc – continuous at a point $x \in X$. Then by Theorem 3.5, there exists a closed set F of Y containing $f(x)$ such that $f(U) \cap (Y - F) \neq \phi$ for every $U \in Bc - O(X, x)$, which implies that $U \cap f^{-1}(Y - F) \neq \phi$. Therefore, $x \in Bc - Cl\left[f^{-1}(Y - F)\right] = Bc - Cl\left[X - f^{-1}(F)\right]$. Again, since $x \in f^{-1}(F)$, we get $x \in Bc - Cl\left[f^{-1}(F)\right]$ and so it follows that $x \in Bc - Fr\left[f^{-1}(F)\right]$.

Sufficiency: Suppose that $x \in \left(Bc - Fr\left[f^{-1}(F)\right]\right)$ for some closed set F of Y containing $f(x)$ and f is contra – Bc – continuous at x. Then there exists $U \in Bc - O(X, x)$ such that $f(U) \subseteq F$. Therefore $x \in U \subseteq f^{-1}(F)$ and hence it follows that $x \in Bc - Int\left[f^{-1}(F)\right] \subseteq X - \left(Bc - Fr\left[f^{-1}(F)\right]\right)$. But this is a contradiction. So f is not contra – Bc – continuous at x.

Definition 3.16. A function $f : (X, \tau) \to (Y, \sigma)$ is called almost weakly – Bc – continuous if, for each $x \in X$ and for each open set V of Y containing $f(x)$, there exists $U \in Bc - O(X, x)$ such that $f(U) \subseteq Cl(V)$.

Theorem 3.17. Suppose that a function $f : (X, \tau) \to (Y, \sigma)$ is contra – Bc – continuous. Then f is almost weakly – Bc – continuous.

Proof. For any open set V of Y, $Cl(V)$ is closed in Y. Since f is contra – Bc – continuous, $f^{-1}\left[Cl(V)\right]$ is Bc – open set in X. We take $U = f^{-1}\left[Cl(V)\right]$, then $f(U) \subseteq Cl(V)$. Hence f is almost weakly – Bc – continuous.

Theorem 3.18. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \mu)$ be any two functions. Then the following properties hold:

(i) If f is contra – Bc – continuous function and g is a continuous function, then gof is contra – Bc – continuous.

(ii) If f is Bc – irresolute and g is contra – Bc – continuous, then gof is contra – Bc – continuous.

Proof. (i) For x ∈ X, let W be any closed set of Z containing $(gof)(x)$. Since g is continuous, $V = g^{-1}(W)$ is closed in Y. Also, since f is contra – Bc – continuous, there exists $U \in Bc - O(X, x)$ such that $f(U) \subseteq V$. Therefore $(gof)(U) \subseteq g\left[f(U)\right] \subseteq g(V) \subseteq W$ and so it implies that $(gof)(U) \subseteq W$. Hence, gof is contra – Bc – continuous.

(ii) For x ∈ X, let W be any closed set of Z containing $(gof)(x)$. Since g is contra – Bc – continuous, there exists $V \in Bc-O(Y, f(x))$ such that $g(V) \subseteq W$. Again, since f is *Bc – irresolute*, there exists $U \in Bc-O(X, x)$ such that $f(U) \subseteq V$. This shows that $(gof)(U) \subseteq W$. Hence, *gof* is contra – Bc – continuous.

Theorem 3.19. Let $f:(X, \tau) \to (Y, \sigma)$ be surjective *Bc – irresolute* and *pre – Bc – open* function and $g:(Y, \sigma) \to (Z, \eta)$ be any function. Then $gof:(X, \tau) \to (Z, \eta)$ is contra – Bc – continuous if and only if g is contra – Bc – continuous.

Proof. The "if" part is easy to prove. To prove "only if" part, let $gof:(X, \tau) \to (Z, \eta)$ be contra – Bc – continuous and let F be a closed subset of Z. Then $(gof)^{-1}(F)$ is a Bc – open subset of X i.e. $f^{-1}[g^{-1}(F)]$ is Bc – open in X. Since f is *pre – Bc – open*, $f[f^{-1}(g^{-1}(F))]$ is a Bc – open subset of Y and so $g^{-1}(F)$ is Bc – open in Y. Hence, g is contra – Bc – continuous.

Definition 3.20. A topological space (X, τ) is said to be Bc – normal if each pair of *non – empty* disjoint closed sets can be separated by disjoint Bc – open sets.

Definition 3.21. A topological space (X, τ) is said to be ultranormal if each pair of *non – empty* disjoint closed sets can be separated by disjoint clopen sets.

Theorem 3.22. Suppose that $f:(X, \tau) \to (Y, \sigma)$ is a contra – Bc – continuous, closed injection and Y is ultranormal. Then X is Bc – normal.

Proof. Let A and B be disjoint closed subsets of X. Since f is closed injection, $f(A)$ and $f(B)$ are disjoint closed subsets of Y. Again, since Y is ultranormal, $f(A)$ and $f(B)$ are separated by disjoint clopen sets P and Q (say) respectively. Therefore, $f(A) \subseteq P$ and $f(B) \subseteq Q$ i.e., $A \subseteq f^{-1}(P)$ and $B \subseteq f^{-1}(Q)$, where $f^{-1}(P)$ and $f^{-1}(Q)$ are disjoint Bc – open sets of X (since f is contra – Bc – continuous). This shows that X is Bc – normal.

Definition 3.23. A topological space (X, τ) is called Bc – connected provided that X is not the union of two disjoint nonempty Bc – open sets of X.

Theorem 3.24. Suppose that $f:(X, \tau) \to (Y, \sigma)$ is contra – Bc – continuous surjection, where X is Bc – connected and Y is any topological space, then Y is not a discrete space.

Proof. If possible, suppose that Y is a discrete space. Let P be a proper nonempty open and closed subset of Y. Then $f^{-1}(P)$ is a proper nonempty Bc – open and Bc – closed subset of X, which contradicts to the fact that X is Bc – connected. Hence the theorem follows.

Theorem 3.25. Suppose that $f:(X, \tau) \to (Y, \sigma)$ is contra – Bc – continuous surjection and X is Bc – connected. Then Y is connected.

Proof. If possible, suppose that Y is not connected. Then there exist nonempty disjoint open sets P and Q such that $Y = P \cup Q$. So P and Q are clopen sets of Y. Since f is contra – Bc – continuous function, $f^{-1}(P)$ and $f^{-1}(Q)$ are Bc – open sets of X. Also $f^{-1}(P)$ and $f^{-1}(Q)$ are nonempty disjoint Bc – open sets of X and $X = f^{-1}(P) \cup f^{-1}(Q)$, which contradicts to the fact that X is Bc – connected. Hence Y is connected.

Theorem 3.26. A topological space (X, τ) is Bc – connected if and only if every contra – Bc – continuous function from X into any T_1 – *space* (Y, σ) is constant.

Proof. Let X be Bc – connected. Now, since Y is a T_1 – *space*, $\Omega = \{f^{-1}(y) : y \in Y\}$ is disjoint

$Bc-open$ partition of X. If $|\Omega| \geq 2$ (where $|\Omega|$ denotes the cardinality of Ω), then X is the union of two nonempty disjoint $Bc-open$ sets. Since X is $Bc-connected$, we get $|\Omega| = 1$. Hence, f is constant.

Conversely, suppose that X is not $Bc-connected$ and every $contra-Bc-continuous$ function from X into any $T_1-space$ Y is constant. Since X is not $Bc-connected$, there exists a non-empty proper $Bc-open$ as well as $Bc-closed$ set V (say) in X. We consider the space $Y = \{0,1\}$ with the discrete topology σ. The function $f:(X, \tau) \to (Y, \sigma)$ defined by $f(V) = \{0\}$ and $f(X-V) = \{1\}$ is obviously $contra-Bc-continuous$ and which is $non-constant$. This leads to a contradiction. Hence X is $Bc-connected$.

Definition 3.27. A topological space (X, τ) is said to be $Bc-T_2$ if for each pair of distinct points x, y in X there exist $U \in Bc-O(X, x)$ and $V \in Bc-O(X, y)$ such that $U \cap V = \phi$.

Theorem 3.28. Let (X, τ) and (Y, σ) be two topological spaces and suppose that for each pair of distinct points x and y in X there exists a function $f:(X, \tau) \to (Y, \sigma)$ such that $f(x) \neq f(y)$ where Y is an Urysohn space and f is $contra-Bc-continuous$ function at x and y. Then X is $Bc-T_2$.

Proof. Let $x, y \in X$ and $x \neq y$. Then by assumption, there exists a function $f:(X, \tau) \to (Y, \sigma)$, such that $f(x) \neq f(y)$ where Y is Urysohn and f is $contra-Bc-continuous$ at x and y. Now, since Y is Urysohn, there exist open sets U and V of Y containing $f(x)$ and $f(y)$ respectively, such that $Cl(U) \cap Cl(V) = \phi$. Also, f being $contra-Bc-continuous$ at x and y there exist $Bc-open$ sets P and Q containing x and y respectively such that $f(P) \subseteq Cl(U)$ and $f(Q) \subseteq Cl(V)$. Then $f(P) \cap f(Q) = \phi$ and so $P \cap Q = \phi$. Therefore X is $Bc-T_2$.

Corollary 3.29. If $f:(X, \tau) \to (Y, \sigma)$ is $contra-Bc-continuous$ injection where Y is an Urysohn space, then X is $Bc-T_2$.

Corollary 3.30. If f is $contra-Bc-continuous$ injection of a topological space (X, τ) into an ultra Hausdorff space (Y, σ), then X is $Bc-T_2$.

Proof. Let $x, y \in X$ where $x \neq y$. Then, since f is an injection and Y is ultra Hausdorff, $f(x) \neq f(y)$ and there exist disjoint closed sets U and V containing $f(x)$ and $f(y)$ respectively. Again, since f is $contra-Bc-continuous$, $f^{-1}(U) \in Bc-O(X, x)$ and $f^{-1}(V) \in Bc-O(X, y)$ with $f^{-1}(U) \cap f^{-1}(V) = \phi$. This shows that X is $Bc-T_2$.

IV. ALMOST CONTRA – Bc – CONTINUOUS FUNCTIONS

Definition 4.1. A function $f:(X, \tau) \to (Y, \sigma)$ is called almost $contra-Bc-continuous$ if $f^{-1}(V)$ is $Bc-closed$ for every regular open set V of Y.

Theorem 4.2. Let $f:(X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:

(a) f is almost $contra-Bc-continuous$;

(b) $f^{-1}(F)$ is $Bc-open$ in X for every regular closed set F of Y;

(c) for each $x \in X$ and each regular open set F of Y containing $f(x)$, there exists $U \in Bc-O(X, x)$ such that $f(U) \subseteq F$.

(d) for each $x \in X$ and each regular open set V of Y non-containing $f(x)$, there exists a $Bc-closed$ set K of X non-containing x such that $f^{-1}(V) \subseteq K$.

Proof. (a) $\Leftrightarrow$ (b): Let F be any regular closed set of Y. Then $(Y-F)$ is regular open and therefore $f^{-1}(Y-F) = X - f^{-1}(F) \in \text{Bc-C}(X)$. Hence, $f^{-1}(F) \in \text{Bc-O}(X)$. The converse part is obvious.

(b) $\Rightarrow$ (c): Let F be any regular closed set of Y containing $f(x)$. Then $f^{-1}(F) \in \text{Bc-O}(X)$ and $x \in f^{-1}(F)$. Taking $U = f^{-1}(F)$ we get $f(U) \subseteq F$.

(c) $\Rightarrow$ (b): Let F be any regular closed set of Y and $x \in f^{-1}(F)$. Then, there exists $U_x \in \text{Bc-O}(X, x)$ such that $f(U_x) \subseteq F$ and so $U_x \subseteq f^{-1}(F)$. Also, we have $f^{-1}(F) = \bigcup_{x \in f^{-1}(F)} U_x$. Hence $f^{-1}(F) \in \text{Bc-O}(X)$.

(c) $\Rightarrow$ (d): Let V be any regular open set of Y non-containing *f*(*x*). Then $(Y-V)$ is regular closed set of Y containing *f*(*x*). Hence by (c), there exists $U \in \text{Bc-O}(X, x)$ such that $f(U) \subseteq (Y-V)$. Hence, we obtain $U \subseteq f^{-1}(Y-V) \subseteq X - f^{-1}(V)$ and so $f^{-1}(V) \subseteq (X-U)$. Now, since $U \in \text{Bc-O}(X)$, $(X-U)$ is Bc – closed set of X not containing *x*. The converse part is obvious.

Theorem 4.3. Let $f:(X, \tau) \to (Y, \sigma)$ be almost contra – Bc – continuous. Then f is almost weakly – Bc – continuous.

Proof. For $x \in X$, let H be any open set of Y containing $f(x)$. Then $\text{Cl}(H)$ is a regular closed set of Y containing $f(x)$. Then by Theorem 4.2, there exists $G \in Bc-O(X, x)$ such that $f(G) \subseteq Cl(H)$. So f is almost weakly – Bc – continuous.

Theorem 4.4. Let $f:(X, \tau) \to (Y, \sigma)$ be an almost contra – Bc – continuous injection and Y is weakly Hausdorff. Then X is $\text{Bc}-T_1$.

Proof. Since Y is weakly Hausdorff, for distinct points *x*, *y* of Y, there exist regular closed sets U and V such that $f(x) \in U$, $f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$. Now, f being almost contra – Bc – continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are Bc – open subsets of X such that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $y \in f^{-1}(V)$, $x \notin f^{-1}(V)$. This shows that X is $\text{Bc}-T_1$.

Corollary 4.5. If $f:(X, \tau) \to (Y, \sigma)$ is a contra – Bc – continuous injection and Y is weakly Hausdorff, then X is $\text{Bc}-T_1$.

Theorem 4.6. Let $f:(X, \tau) \to (Y, \sigma)$ be an almost contra – Bc – continuous surjection and X be $Bc-connected$. Then Y is connected.

Proof. If possible, suppose that Y is not connected. Then there exist disjoint non-empty open sets U and V of Y such that $Y = U \bigcup V$. Since U and V are clopen sets in Y, they are regular open sets of Y. Again, since f is almost contra – Bc – continuous surjection, $f^{-1}(U)$ and $f^{-1}(V)$ are Bc – open sets of X and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not $Bc-connected$. But this is a contradiction. Hence Y is connected.

Definition 4.7. A topological space (X, τ) is said to be $Bc-compact$ if every $Bc-open$ cover of X has a finite subcover.

Definition 4.8. A topological space (X, τ) is said to be countably $Bc-compact$ if every countable cover of X by $Bc-open$ sets has a finite subcover.

Definition 4.9. A topological space (X, τ) is said to be $Bc-Lindeloff$ if every $Bc-open$ cover of X has a countable subcover.

Theorem 4.10. Let $f:(X,\tau)\to(Y,\sigma)$ be an almost contra – Bc – continuous surjection. Then the following statements hold:

(a) If X is $Bc-compact$, then Y is $S-closed$.

(b) If X is $Bc-Lindeloff$, then Y is $S-Lindeloff$.

(c) If X is *countably* $Bc-compact$, then Y is *countably* $S-closed$.

Proof. (a): Let $\{V_\alpha:\alpha\in I\}$ be any regular closed cover of Y. Since f is almost contra – Bc – continuous, then $\{f^{-1}(V_\alpha):\alpha\in I\}$ is a $Bc-open$ cover of X. Again, since X is $Bc-compact$, there exist a finite subset I_0 of I such that $X=\bigcup\{f^{-1}(V_\alpha):\alpha\in I_0\}$ and hence $Y=\{V_\alpha:\alpha\in I_0\}$. Therefore, Y is $S-closed$.

The proofs of (b) and (c) are being similar to (a): omitted.

Definition 4.11. A topological space (X,τ) is said to be $Bc-closed$ compact if every $Bc-closed$ cover of X has a finite subcover.

Definition 4.12. A topological space (X,τ) is said to be countably $Bc-closed$ if every countable cover of X by $Bc-closed$ sets has a finite subcover.

Definition 4.13. A topological space (X,τ) is said to be $Bc-closed$ *Lindeloff* if every $Bc-closed$ cover of X has a countable subcover.

Theorem 4.14. Let $f:(X,\tau)\to(Y,\sigma)$ be an almost contra – Bc – continuous surjection. Then the following statements hold:

(a) If X is $Bc-closed$ compact, then Y is nearly compact.

(b) If X is $Bc-closed$ *Lindeloff*, then Y is *nearly Lindeloff*.

(c) If X is *countably* $Bc-closed$ *compact*, then Y is *nearly countable compact*.

Proof. (a): Let $\{V_\alpha:\alpha\in I\}$ be any regular open cover of Y. Since f is almost contra – Bc – continuous, then $\{f^{-1}(V_\alpha):\alpha\in I\}$ is a $Bc-closed$ cover of X. Again, since X is $Bc-closed$ *compact*, there exists a finite subset I_0 of I such that $X=\bigcup\{f^{-1}(V_\alpha):\alpha\in I_0\}$ and hence $Y=\{V_\alpha:\alpha\in I_0\}$. Therefore, Y is *nearly compact*.

The proofs of (b) and (c) are being similar to (a): omitted.

V. CLOSED GRAPHS VIA Bc – OPEN SETS

Definition 5.1. Let $f:(X,\tau)\to(Y,\sigma)$ be a function. Then the graph $G(f)=\{(x,f(x)):x\in X\}$ of f is said to be $Bc-closed$ (resp. $contra-Bc-closed$) if for each $(x,y)\in(X\times Y)-G(f)$, there exist a $U\in Bc-O(X,x)$ and an open set (resp. a closed set) V in Y containing y such that $(U\times V)\cap G(f)=\phi$.

Lemma 5.2. A graph $G(f)$ of a function $f:(X,\tau)\to(Y,\sigma)$ is $Bc-closed$ (resp. *contra* $Bc-closed$) in $X\times Y$ if and only if for each $(x,y)\in(X\times Y)-G(f)$, there exist $U\in Bc-O(X,x)$ and an open set (resp. a closed set) V in Y containing y such that $f(U)\cap V=\phi$.

Proof. We shall prove that $f(U)\cap V=\phi \Leftrightarrow (U\times V)\cap G(f)=\phi$. Let $(U\times V)\cap G(f)\neq\phi$. Then there exists $(x,y)\in(U\times V)$ and $(x,y)\in G(f)$. This implies that $x\in U$, $y\in V$ and $y=f(x)\in V$. Therefore, $f(U)\cap V\neq\phi$. Hence the result follows.

Theorem 5.3. Suppose that $f:(X,\tau)\to(Y,\sigma)$ is *contra* $Bc-continuous$ and Y is Urysohn. Then $G(f)$ is *contra* $Bc-closed$ in $X\times Y$.

Proof. Let $(x, y) \in (X \times Y) - G(f)$. It follows that $f(x) \neq y$. Since Y is Urysohn, there exist open sets V and W in Y such that $f(x) \in V$, $y \in W$ and $Cl(V) \cap Cl(W) = \phi$. Now, since f is contra $Bc-continuous$, there exists a $U \in Bc-O(X, x)$ such that $f(U) \subseteq Cl(V)$ which implies that $f(U) \cap Cl(W) = \phi$. Hence by Lemma 5.2, $G(f)$ is contra $Bc-closed$ in $X \times Y$.

Theorem 5.4. Let $f:(X, \tau) \to (Y, \sigma)$ be a function and $g: X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra $Bc-continuous$, then f is contra $Bc-continuous$.

Proof. Let G be an open set in Y, then $X \times G$ is an open set in $X \times Y$. Since g is contra $Bc-continuous$, it implies that $f^{-1}(G) = g^{-1}(X \times G)$ is a $Bc-closed$ set of X. Therefore, f is contra $Bc-continuous$.

Theorem 5.5. Let $f:(X, \tau) \to (Y, \sigma)$ have a contra $Bc-closed$ graph. If f is injective, then X is $Bc-T_1$.

Proof. Let x_1 and x_2 be any two distinct points of X. Then, we have $(x_1, f(x_2)) \in (X \times Y) - G(f)$. Then, there exists a $Bc-open$ set U in X containing x_1 and $F \in C(Y, f(x_2))$ such that $f(U) \cap F = \phi$. Hence $U \cap f^{-1}(F) = \phi$. Therefore, we have $x_2 \notin U$. This implies that X is $Bc-T_1$.

Definition 5.6. The graph $G(f)$ of a function $f:(X, \tau) \to (Y, \sigma)$ is said to be strongly contra $Bc-closed$ if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in Bc-O(X, x)$ and regular closed set V in Y containing y such that $(U \times V) \cap G(f) = \phi$.

Lemma 5.7. The graph $G(f)$ of a function $f:(X, \tau) \to (Y, \sigma)$ is strongly contra $Bc-closed$ in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in Bc-O(X, x)$ and regular closed set V in Y containing y such that $f(U) \cap V = \phi$.

Theorem 5.8. Let $f:(X, \tau) \to (Y, \sigma)$ be an almost $weakly-Bc-continuous$ and Y is Urysohn. Then $G(f)$ is strongly contra $Bc-closed$ in $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is Urysohn, there exist open sets G, H in Y such that $f(x) \in G$, $y \in H$ and $Cl(G) \cap Cl(H) = \phi$. Now, since f is almost $weakly-Bc-continuous$, there exists $U \in Bc-O(X, x)$ such that $f(U) \subseteq Cl(G)$. This implies that $f(U) \cap Cl(H) = f(U) \cap Cl[Int(H)] = \phi$, where $Cl[Int(H)]$ is regular closed in Y. Hence by above Lemma 5.7, $G(f)$ is strongly contra $Bc-closed$ in $X \times Y$.

Theorem 5.9. Let $f:(X, \tau) \to (Y, \sigma)$ be an almost $Bc-continuous$ and Y is T_2. Then $G(f)$ is strongly contra $Bc-closed$ in $X \times Y$.

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is T_2, there exist open sets G and H containing y and $f(x)$, respectively, such that $G \cap H = \phi$; which is equivalent to $Cl(G) \cap Int[Cl(H)] = \phi$. Again, since f is almost $Bc-continuous$ and $Int[Cl(H)]$ is regular open, by proposition 3.9, there exists $W \in Bc-O(X, x)$ such that $f(W) \subseteq Int[Cl(H)]$. This implies that $f(W) \cap Cl(G) = \phi$ and by Lemma 5.7, $G(f)$ is strongly contra $Bc-closed$ in $X \times Y$.

Definition 5.10. A filter base $\Im$ on a topological space (X, τ) is said to be $Bc-convergent$ to a point x in X if for any $U \in Bc-O(X, \tau)$ containing x, there exists an $F \in \Im$ such that $F \subseteq U$.

Theorem 5.11. Prove that every function $\psi:(X, \tau)\to(Y, \sigma)$, where (Y, σ) is compact with $Bc-closed$ graph is $Bc-continuous$.

Proof. Let ψ be not $Bc-continuous$ at $x \in X$. Then there exists an open set S in Y containing $\psi(x)$ such that $\psi(T) \not\subset S$ for every $T \in Bc-O(X, x)$. It is obvious to verify that $\wp = \{T \subseteq X : T \in Bc-O(X, x)\}$ is a filterbase on X that $Bc-converges$ to x. Now we shall show that $\Upsilon_{\wp} = \{\psi(T)\cap(Y-S) : T \in Bc-O(X, x)\}$ is a filterbase on Y. Here for every $T \in Bc-O(X, x)$, $\psi(T) \not\subset S$, i.e. $\psi(T)\cap(Y-S) \neq \phi$. So $\phi \notin \Upsilon_{\wp}$. Let $G, H \in \Upsilon_{\wp}$. Then there are $T_1, T_2 \in \wp$ such that $G = \psi(T_1)\cap(Y-S)$ and $H = \psi(T_2)\cap(Y-S)$. Since $\wp$ is a filterbase, there exists a $T_3 \in \wp$ such that $T_3 \subseteq T_1 \cap T_2$ and so $W = \psi(T_3)\cap(Y-S) \in \Upsilon_{\wp}$ with $W \subseteq G\cap H$. It is clear that $G \in \Upsilon_{\wp}$ and $G \subseteq H$ imply $H \in \Upsilon_{\wp}$. Hence $\Upsilon_{\wp}$ is a filterbase on Y. Since $Y-S$ is closed in compact space Y, S is itself compact. So, $\Upsilon_{\wp}$ must adheres at some point $y \in Y-S$. Here $y \neq \psi(x)$ ensures that $(x, y) \notin G(\psi)$. Thus Lemma 5.2 gives us a $U \in Bc-O(X, x)$ and an open set V in Y containing y such that $\psi(U)\cap V = \phi$, *i.e.* $[\psi(U)\cap(Y-S)]\cap V = \phi$. But this is a contradiction.

Theorem 5.12. Suppose that an open surjection $\psi:(X, \tau)\to(Y, \sigma)$ possesses a $Bc-closed$ graph. Then Y is T_2.

Proof. Let $p_1, p_2 \in Y$ with $p_1 \neq p_2$. Since ψ is a surjection, there exists an $x_1 \in X$ such that $\psi(x_1) = p_1$ and $\psi(x_1) \neq p_2$. Therefore $(x_1, p_2) \notin G(\psi)$ and so by Lemma 5.2, there exist $U_1 \in Bc-O(X, x_1)$ and open set V_1 in Y containing p_2 such that $\psi(U_1)\cap V_1 = \phi$. Since ψ is $Bc-open$, $\psi(U_1)$ and V_1 are disjoint open sets containing p_1 and p_2 respectively. So Y is T_2.

Corollary 5.13. If a function $\psi:(X, \tau)\to(Y, \sigma)$ is a surjection and possesses a $Bc-closed$ graph, then Y is T_1.

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