

Domain Walls Cosmological Model in $f(R,T)$ Theory of Gravity

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ABSTRACT

In this paper, non-static plane symmetric model in the presence of domain walls in the framework of the modified $f(R,T)$ theory of gravitation has been investigated, where R is Ricci scalar and T is the trace of energy momentum tensor. Some physical properties of the model are also obtained and discussed.

Keywords : Non-static plane symmetric space-time, domain walls, $f(R,T)$ gravity.

I. INTRODUCTION

The late time accelerated expansion of the universe has direct evidence of cosmic acceleration comes from high red-shift supernova experiments [1-3]. This is attributed to an unknown exotic component with high negative pressure causes repulsion among galaxies which is popularly known as dark energy (DE). Many candidates of DE have been proposed such as the cosmological constant, quintessence, phantom, quintom as well as the (generalized) Chaplygin gas, and so on. The proposal of extended theories of gravity is one of the attractive approaches to deal with mysterious nature of dark energy. One of these theories is $f(R,T)$ theory of gravity proposed by Harko et al. [4], where the Lagrangian is an arbitrary function of the scalar curvature R and the trace of the energy momentum tensor T . The dependence from the trace T may be induced by exotic imperfect fluids or quantum effects. The cosmological reconstruction in modified $f(R,T)$ gravity has been investigated by Houndjo [5]. Bianchi type-I cosmological model in $f(R,T)$ theory has been constructed by Adhav [6]. Different cosmological models in $f(R,T)$ theory of gravity have been discussed by various relativists [7-36].

Domain walls are the topologically stable objects, which might have been formed during a phase transition in the early universe (Kibble [37]). Domain walls have become more important in recent years from cosmological standpoint in view of a new scenario of galaxy formation proposed by Hill et al. [38]. Vilenkin [39], Ispier and Sikivie [40], Widrow [41], Goetz [42], Mukherji [43], Wang [44], Rahaman et al. [45], Reddy and Subba Rao [46], Adhav et al. [47], Rahaman [48], Rahaman, Mukherji [49], Adhav et al. [50], Reddy et al. [51], Katore et al. [52-53], Kandalkar et al. [54], Venkateswarlu, and Sreenivas [55] are some of the authors who have investigated several aspects of the domain walls in different context.

Motivated by the above discussions, in this paper non-static plane symmetric space-time in presence of domain walls within the framework of $f(R,T)$ theory of gravitation proposed by Harko et al. [4] have been studied. The plan of the paper as follows: Sect. 2 describes $f(R,T)$ gravity formalism in the presence of domain walls. Sect. 3 is devoted to the derivation of field equations and solutions of field equations leading to domain walls model. Sect. 4 contains a detailed

physical discussion of the model. Summary and conclusions are presented in the last section.

II. FIELD EQUATIONS OF $f(R, T)$ GRAVITY WITH DOMAIN WALLS

The field equations of $f(R, T)$ gravity are derived from the Hilbert-Einstein type variation principle. The action for the $f(R, T)$ gravity with domain walls,

$$S = \frac{1}{16\pi} \int \sqrt{-g} f(R, T) d^4x + \int \sqrt{-g} L_m d^4x \quad (2.1)$$

where $f(R, T)$ is an arbitrary function of Ricci scalar (R) , (T) is the trace of stress-energy tensor of the matter (T_{ij}) and L_ϕ is the matter Lagrangian density.

The stress energy tensor of matter T_{ij} is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_\phi)}{\delta g^{ij}} \quad (2.2)$$

Here the dependence of matter Lagrangian is merely on the metric tensor (g_{ij}) is considered rather than on its derivatives and we obtain

$$T_{ij} = g_{ij} L_\phi - \frac{\delta(L_\phi)}{\delta g^{ij}} \quad (2.3)$$

Now varying the action with respect to metric tensor, $f(R, T)$ gravity field equations are obtained as

$$f_R(R, T)R_{ij} - \frac{1}{2} f(R, T)g_{ij} + f_R(R, T)(g_{ij} \nabla^i \nabla_j - \nabla_i \nabla_j) = 8\pi T_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2(T_{ij} + \Theta_{ij})f'(T) + f(T)g_{ij}, \quad (2.4)$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij} L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\delta g^{ij} \delta g^{\alpha\beta}}. \quad (2.5)$$

Here $f_R = \frac{\delta f(R, T)}{\delta R}$, $f_T = \frac{\delta f(R, T)}{\delta T}$,

$\Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ and ∇_i is the covariant derivative.

The problem of the perfect fluids delineated by associate degree energy density, pressure p and four velocities u^i is difficult since there's no distinctive definition of the matter Lagrangian. However, here it

is assumed that the strain energy tensor of the matter is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \quad (2.6)$$

and the matter Lagrangian may be taken as $L_m = -p$ and that we have

$$u^i \nabla_j u_i = 0, \quad u^i u_i = 1. \quad (2.7)$$

Then with the employment of equation (2.5), for the variation of stress-energy of perfect fluid the expression is

$$\theta_{ij} = -2T_{ij} - p g_{ij}. \quad (2.8)$$

Generally, the field equations also depend, through the tensor, on the physical nature of the Θ_{ij} matter field. Hence in the case of $f(R, T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to different matter contributions for gravity. However, Harko et al. [4] gave three classes of these models:

$$f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) + f_3(T) \end{cases}. \quad (2.9)$$

Here the first case is considered, i.e., where is an arbitrary function of the trace of stress-energy tensor. Using these relation $f(R, T)$ gravity field equations (2.4) reduced to

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2(T_{ij} + \Theta_{ij})f'(T) + f(T)g_{ij}, \quad (2.10)$$

where a prime denotes differentiation with respect to the argument.

Generally, the field equations additionally rely through the tensor θ_{ij} , on the physics nature of the matter field. Hence within the case of $f(R, T)$ gravity reckoning on the character of the matter supply, we tend to acquire many theoretical models reminiscent of every alternative of $f(R, T)$. Victimization equation (2.8), in equation (2.10) become

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}. \tag{2.11}$$

III. METRIC AND FIELD EQUATIONS

The universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But during the early stages of evolution, it is unlikely that it could have had such a smoothed out picture. Plane symmetric models are important because they constitute plane wave solutions of the Universe. Venkateswarlu et. al. [56] discussed the non-static plane symmetric cosmological solutions in the context of cosmic strings in a new scalar-tensor theory of gravitation proposed by Sen & Dunn [57]. V. U. M. Rao & D. Neelima [58] have presented a non-static plane symmetric cosmological models filled with perfect fluid in the framework of $f(R, T)$ gravity proposed by Harko et al.[4] and also in general relativity . Hence plane symmetry space-time is considered which provides an opportunity for the study of inhomogeneity. A Riemannian space-time described by the line element is given by

$$ds^2 = e^{2h} (dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2), \tag{3.1}$$

where r , θ , z are the usual cylindrical polar coordinates and h & s are functions of t alone. It is well known that this line element is plane symmetric. The energy–momentum tensor of the domain walls is taken as

$$T_{ij} = \rho(g_{ij} + w_i w_j) + p g_{ij}, \tag{3.2}$$

where ρ is the energy density, p stands for pressure, and w^i is the four-velocity vector satisfying $w_i w^i = -1$. Then, it yields

$$T_1^1 = T_2^2 = T_3^3 = \rho, T_4^4 = -p, T = 3\rho - p. \tag{3.3}$$

Zel'dovich [59] pointed out the stress energy of the domain walls composed of the surface energy density and string tension into spatial directions with

magnitude of tension equal to the surface energy density, this is interesting because there are several indication that tension acts repulsive source of gravity in general relativity. 5-D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain walls have been studied by Yilmaz [60-61] . Adhav et al. [62-63] have discussed string cloud and domain walls with quark matter in n-dimensional Kaluza-Klein cosmological model in general relativity and strange quark matter attached to string cloud in Bianchi type-III space time in general relativity. Katore et al. [64] have studied domain walls and strange quark matter in Einstein–Rosen space-time with the cosmological constant and heat flow. Halife Caglar and Sezgin Aygun[65] researched a higher dimensional at Friedmann-Robertson-Walker (FRW) universe in Barber's second theory when strange quark matter (SQM) and normal matter (NM) are attached to the string cloud and domain walls.

For the particular choice of the function $f(T) = \mu T$ (Harko et al. [4]), where μ is a constant.

The field equations (2.11) for the metric (3.1) using (3.2) and (3.3) can be written as

$$e^{-2h} \left(2h_{44} + h_4^2 + \frac{2h_4 s_4}{s} + \frac{s_{44}}{s} \right) = -(8\pi + 5\lambda)\rho - p\lambda, \tag{3.4}$$

$$e^{-2h} (2h_{44} + h_4^2) = -(8\pi + 5\lambda)\rho - p\lambda, \tag{3.5}$$

$$e^{-2h} \left(\frac{2h_4 s_4}{s} + 3h_4^2 \right) = (8\pi + \lambda)p - 3\lambda\rho. \tag{3.6}$$

Equations (3.4)-(3.6) are a set of three independent equations with four unknowns h , s , p and ρ .

Using equations (3.4) and (3.6) we get,

$$2h_4 s_4 + s_{44} = 0 \tag{3.7}$$

From equations (3.7), we obtain the metric coefficients as

$$e^h = (at + b)^m \tag{3.8}$$

$$s = \kappa (at + b)^{1-2m}, \quad m \neq \frac{1}{2}, \quad (3.9)$$

where $\kappa = \frac{\psi}{(1-2m)a}$ and a, b, κ, ψ & m are arbitrary constants.

The metric (3.1) can be written as

$$ds^2 = (at + b)^{2m} \left\{ dt^2 - dr^2 - r^2 d\theta^2 - \left[\kappa^4 (at + b)^{2-4m} \right] dx^2 \right\} \quad (3.10)$$

The model has point type singularity for $0 < m < \frac{1}{2}$ and has cigar type singularity for $m < 0$ & $m > \frac{1}{2}$. Thus the universe (3.10) possesses initial singularity of the point type at $t = t_0 = \frac{-b}{a}$.

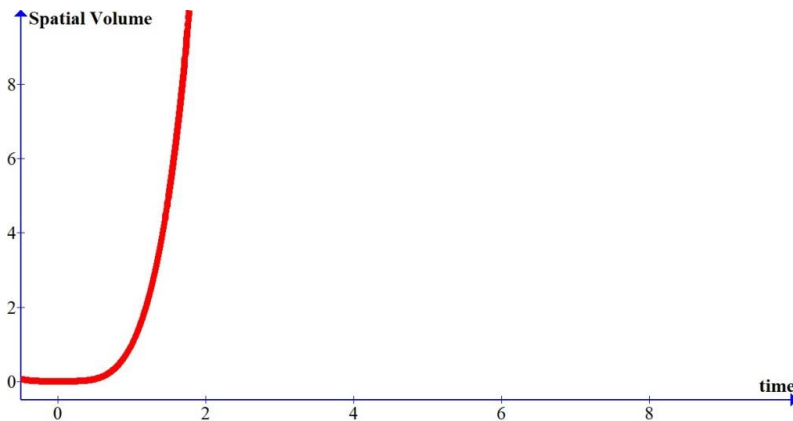


Figure 1. Spatial Volume vs time.

It is seen that at $t = \frac{-b}{a}$, the spatial volume vanishes and it increases with time becomes infinite for large values of time, which shows consistency with the concept of an expanding universe. Thus inflation is possible for large value of time. Also volume becomes zero at the instant $t = 0$ as shown in figure 1, there is a Big-Bang at $t = 0$ which resembles with the investigations of Katore et. al. [52].

The Hubble parameter is

$$H = \frac{1}{3} \frac{V_4}{V} = \frac{a(1+m)}{3(at+b)} \quad (4.2)$$

IV. PHYSICAL DISCUSSION

Equation (3.10) represents non-static plane symmetric domain walls cosmological model in $f(R,T)$ theory of gravity. The physical and geometrical parameters which are significant in the discussion of cosmology are discussed.

The spatial volume is

$$V = r s e^{4h} = \frac{r\psi(at+b)^{1+2m}}{(1-2m)a} \quad (4.1)$$

The scalar expansion is

$$\theta = 3H = \frac{a(1+m)}{(at+b)} \quad (4.3)$$

The expansion scalar and Hubble parameter decrease with the increase in time i.e. the Hubble parameter H , Expansion Scalar θ vanish as time approaches infinity while they diverge at $t = 0$.

The average anisotropy parameter is

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2 = \frac{7}{3} \quad (4.4)$$

Since the average anisotropic parameter $A_m \neq 0$, the model is anisotropic throughout the evolution of the Universe.

The shear scalar is

$$\sigma^2 = \frac{3}{2} A_m H^2 = \frac{7a^2(1+m)^2}{18(at+b)^2}. \quad (4.5)$$

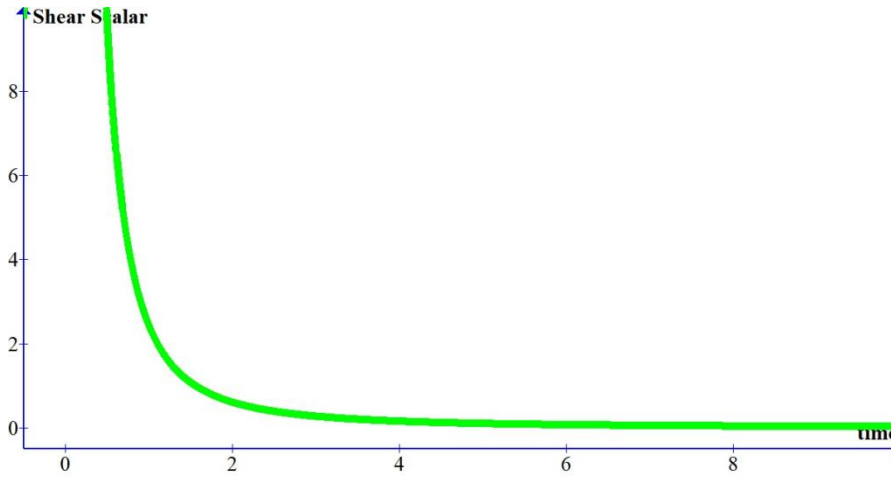


Figure 2. Shear Scalar vs time.

The shear scalar tends to infinity as $t \rightarrow 0$ shown in figure 4, whereas when $t \rightarrow \infty$, shear scalar tends to zero which resembles with the results of Katore and Shaikh [66]. Since $\frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy throughout the evolution of the universe.

The deceleration parameter

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 = \frac{2-m}{1+m}, m \neq -1. \quad (4.6)$$

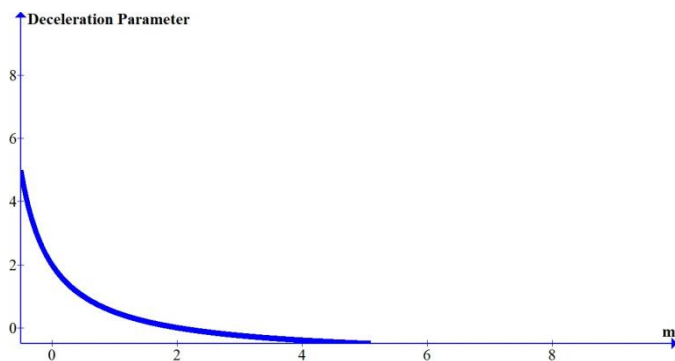


Figure No. 3 : Deceleration Parameter vs m.

The sign of q indicates whether the model inflates or not. A positive sign of q indicates decelerating model

whereas negative sign of q indicates inflation. The above results reveal that the deceleration parameter q in this case, depends on the values of m , i.e., the universe accelerates or decelerates according as q is negative or positive respectively. The deceleration parameter $q > 0$ i.e deceleration parameter q is positive for $-1 < m < 2$ and $q < 0$ i.e. inflationary accelerating universe for $-\infty < m < -1$ & $m > 2$. Here the model sometimes decelerate in the standard way and later accelerate which is in accordance with the present day scenario as shown in figure 3. However, in spite of the fact that the universe, in this case, decelerates in the standard way it will accelerate in finite time due to cosmic recollapse where the universe in turns inflates decelerates and then accelerates[67]. Thus the theoretical model is consistent with the results of recent observations.

From equations (3.4)-(3.5) and (3.8)-(3.9), we get the energy density and pressure as

$$p = \rho = \frac{a^2(2m - m^2)}{(8\pi + 2\mu)(at + b)^{2-2m}}. \quad (4.7)$$

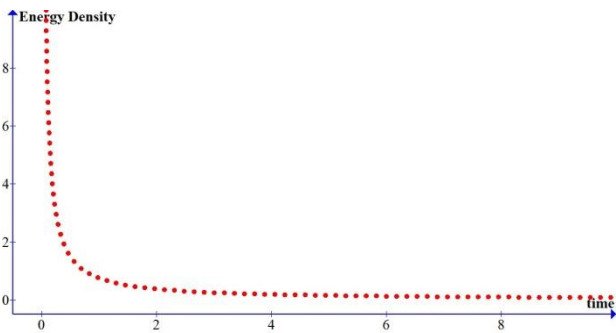


Figure 4. Energy Density vs time.

The equation of state $\rho = p$ represents self-gravitating or stiff domain walls. From (4.7) it can be concluded that only self-gravitating domain walls universe exists for non-static plane symmetric metric in $f(R, T)$ gravity. For $m = 2$, we can see that matter pressure and energy density will vanish. It is observed that the energy density of matter is decreasing function of time as shown in figure 4. Adhav et. al. [68] have shown that plane symmetric domain walls model which represent the self-gravitating or stiff fluid do not survive in the scale covariant theory of gravitation formulated by Canuto et al. [69]. Venkateswarlu, and Sreenivas [55] observed that the energy density of the domain walls $\rho \rightarrow 0$ as $t \rightarrow 0$ and $\rho \rightarrow \infty$ as $t \rightarrow \infty$, which shows the existence of Big-bang singularity in the model when $p = \rho$, which is analogous to stiff matter in general relativity.

Stability of the model

We discuss the stability of the model using the function $c_s^2 = \frac{dp}{d\rho}$. The stability of the model occurs when the function c_s^2 is positive. It is clear that the function is positive, therefore the model is stable.

V. CONCLUSIONS

In this paper, non-static plane symmetric cosmological models filled with domain walls in the framework of $f(R, T)$ gravity proposed by Harko et al. [4]. It is

observe that at $t = \frac{-b}{a}$, the spatial volume vanishes and it increases with time becomes infinite for large values of time, which shows consistency with the concept of an expanding universe. The Hubble parameters H , Expansion Scalar θ vanish as time approaches infinity while they diverge at $t = 0$. The mean anisotropy parameter is uniform throughout whole expansion of the universe. The behavior of the physical parameters resemble with the results obtained by Reddy et. al. [70] in the presence of thick domain walls in the scalar-tensor theory formulated by Brans and Dicke [71] for Kaluza-Klein Space-time. It can be seen that only self-gravitating domain walls universe exists for non-static plane symmetric metric in $f(R, T)$ gravity which differs with the investigations of Reddy and Naidu [72] which stated that stiff or self-gravitating domain walls do not survive in scale covariant theory of gravitation.

The deceleration parameter $q > 0$ i.e deceleration parameter q is positive for $-1 < m < 2$ and $q < 0$ i.e. inflationary accelerating universe for $-\infty < m < -1$ & $m > 2$. Recent observations of type Ia supernovae [73–77] reveal that the present-day universe is in an accelerating phase and the deceleration parameter lies somewhere in the range $-1 < q \leq 0$ (Fig. 3). It follows that obtained model of the universe is consistent with the recent observations. It is clear that the function c_s^2 is positive, therefore the model is stable. Since this model remains anisotropic throughout, it will be useful to discuss the structure formation at the early stages of evolution of the universe in modified theory of gravity.

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