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Odd Vertex Even Mean Labeling of H-Graph

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ABSTRACT

A graph with m vertices and n edges is said to have an odd vertex even mean labeling if there exist an injective function f:V(G)→{1,3,5…2s-1} such that the induced map f*:E(G)→{2,4,…2s-2,2s} define by $f^*(u'v')=\frac{f(u')+f(v')}{2}$ is a bijection. A graph that admits an odd vertex even mean labeling is called an odd vertex even mean graph. Here we study the odd vertex even mean behavior of H-graph.

Keywords : Odd Vertex Even Mean Labeling, Odd Vertex Even Mean Graph.

I. INTRODUCTION

In this paper we define the set of vertices and the set of edges of a graph G will be denoted by V(G) and E(G) respectively and m= $|V(G)|$, n=$|E(G)|$. For general graph theoretic notations we follow F.Harary[6]. A graph labeling is a mapping that carries a set of elements into a set of numbers.

The concept of mean labeling was introduced and studied by somasundaram and ponjar[7]. A graph G is said to be a mean graph if there exist an injective function f:V(G)→{1,2,…s} such that the induced map f*:E(G)→{1,2,…s} defined by $f^*(u'v')=\left\lceil\frac{f(u')+f(v')}{2}\right\rceil$ is bijection.

The concept of even mean labeling was introduced and studied by gayathri and gopi[4]. A graph G is said to be even mean if there exist an injective function f:V(G)→{1,2,…s} such that the induced map f*:E(G)→{2,4…2s} defined by $f^*(u'v')=\left\lceil\frac{f(u')+f(v')}{2}\right\rceil$ is a bijection.

Manikam and marudai [8] have introduced the concept of odd mean labeling of a graph. A graph G is said to be odd mean if there exist an injective function f:V(G)→{1,…2s-1} defined by $f^*(u'v')=\left\lceil\frac{f(u')+f(v')}{2}\right\rceil$ is a bijection.

A graph G is said to have an odd vertex even mean labeling if there exist an injective function f:V(G)→{1,3…2s-1} such that the induced map f*:E(G)→{2,…2s-2,2s} defined by $f^*(u'v')=\left\lceil\frac{f(u')+f(v')}{2}\right\rceil$ is a bijection. A graph that admits an odd vertex even mean labeling is called odd vertex even mean graph.

In this paper, we proved that the odd vertex even mean labeling of H-graphs.

1.1 Definition:

The H-graph of a path P_m is the graph obtained from two copies of Pm with vertices $v'_1,v'_2,\dots v'_m$ and $u'_1,u'_2,\dots u'_m$ by joining the vertices $V_{\frac{m+1}{2}}$ and $U_{\frac{m+1}{2}}$ by an edges if m is even the vertices $V_{\frac{m}{2}+1}$ and $U_{\frac{m}{2}}$ if m is odd.

1.2 Theorem:

The H-graph of a path P_m in ($m \geq 3$) is a odd vertex even mean graph.

Proof:

Let $\{v'_k, 1 \leq k \leq m,\ u'_k,\ 1 \leq k \leq m\}$ be the vertices and $\{e, e_k,\ 1 \leq k \leq m-1,\ e'_k,\ 1 \leq k \leq m-1\}$ be the edges which are denoted as in figure 1.

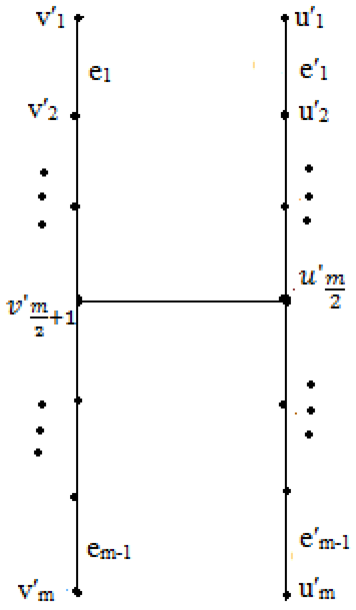

Figure 1. Ordinary labeling of H-graph of path P_m

Define $f: v' \rightarrow \{1,3,5 \ldots .2s-1\}$

For $1 \leq k \leq m$

$f(v'_k) = 2k-1$

$f(u'_k) = 2m+2k-1$

Then the induced edge labels are:

For $1 \leq k \leq m-1$

$f^*(e_k) = 2(i-1)$

$f^*(e'_k) = 2m+2(k-1)$

$f^*(e) = 2(n-1)$

Therefore $f^*(E) = \{2,4, \ldots 2s\}$. So f is a odd vertex even mean labeling and hence the H-graph of a path P_m ($m \geq 3$) is a odd vertex even mean graph.

H-graph of P_7 is shown in figure 2.

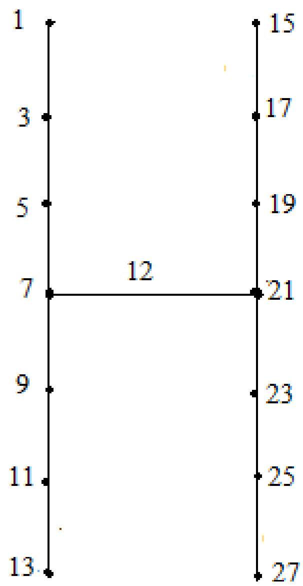

Figure 2. Odd vertex even mean labeling of H-graph of a path P_7

1.3 Definition:

The graph $H \Theta n_l$ is a graph obtained from the H-graph by attaching k pendent vertices at each k^{th} vertex on the two paths on m vertices for $1 \leq k \leq m$.

1.4 Theorem:

The $H \Theta n_l$ is a odd vertex even sum graph.

Proof:

Let $\{v'_k, u'_k,\ 1 \leq k \leq m$ and $v_{kk'}, u_{kk'},\ 1 \leq k \leq m,\ 1 \leq k' \leq n\}$ be the vertices and $\{x_k, y_k,\ 1 \leq k \leq m-1$and $x_{kk'}, y_{kk'},\ 1 \leq k \leq m,\ 1 \leq k' \leq n, y\}$ be the edges which are denoted as in figure1.3

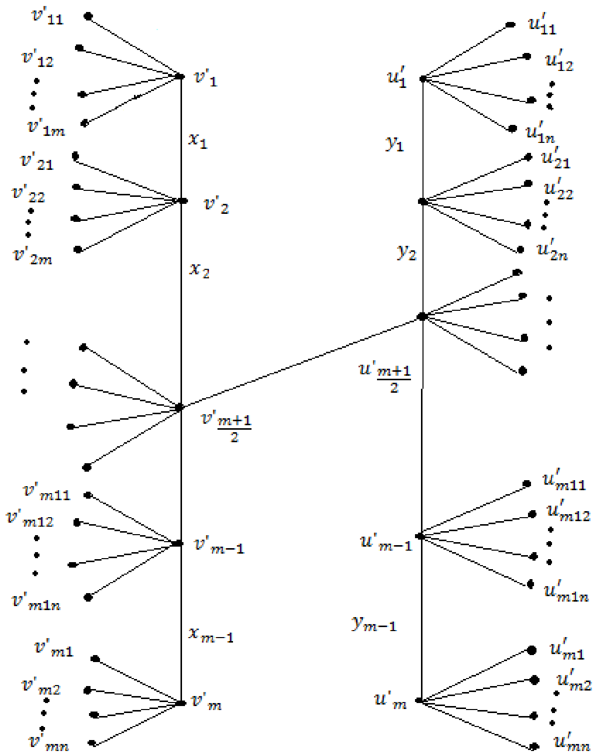

Figure 3. Ordinary labeling of HΘni1

Case 1: m is odd

First we label the vertice's as follows:

Define f:v′→{1,3…2s-1}

For 1≤k≤m

$$f(v_k) = \begin{cases} 2(k-1)(n+1)+1 & k \text{ is odd} \\ 4n+3+2(k-2)(n+1) & k \text{ is even} \end{cases}$$

$$f(u'_k) = \begin{cases} 2(m-1)(n+1)+4n+3+2(i-1)(n+1) & k \text{ is odd} \\ 2(m-1)(n+1)+1+4(n+1)+2(k-2)(n+1) & k \text{ is even} \end{cases}$$

1≤k≤m, 1≤k′≤n

$$f(v'_{kk'}) = \begin{cases} 4k'-1+2(k-1)(n+1) & k \text{ is odd} \\ 4k'+1+2(k-2)(n+1) & k \text{ is even} \end{cases}$$

$$f(u'_{kk'}) = \begin{cases} 2(m-1)(n+1)+4k'+1+2(k-1)(m+1) & k \text{ is odd} \\ 2(m-1)(n+1)+4n+3+4k'+2(k-2)(m+1) & k \text{ is even} \end{cases}$$

case 2: m is even

for 1≤k≤m

$$f(v'_k) = \begin{cases} 2(k-1)(n+1)+1 & k \text{ is odd} \\ 4n+3+2(k-2)(n+1) & k \text{ is even} \end{cases}$$

$$f(u'_k) = \begin{cases} 4n+2(m-2)(n+1)+5+2(n+1)(k-1 & k \text{ is odd} \\ 8n+2(m-2)(n+1)+7+2(n+1)(k-2) & k \text{ is even} \end{cases}$$

for 1≤k≤m, 1≤k′≤n

$$f(v'_{kk'}) = \begin{cases} 4k'+1-2+2(k-1)(n+1) & k \text{ is odd} \\ 4k'+1+2(k-2)(n+1) & k \text{ is even} \end{cases}$$

$$f(u'_{kk'}) = \begin{cases} 4n+3+2(m-2)(n+1)+4k'+2(k-1)(n+1) & k \text{ is odd} \\ 4n+2(m-2)(n+1)+4k'+5+2(k-2)(n+1) & k \text{ is even} \end{cases}$$

Then the induced edges labels are:

For $1 \le k \le m-1$

$f^*(x_k) = 2n+2+2(k-1)(n+1)$

$f^*(y_k) = 4n+4+2(n+1)(m-1)+2(k-1)(n+1)$

For $1 \le k \le m$, $1 \le k' \le n$

$f^*(x_{kk'}) = 2k'+2(n+1)(k-1)$

$f^*(y_{kk'}) = 2n+2+2(n+1)(m-1)+2k'+2(k-1)(n+1)$

$f^*(y) = 2n+2+2(n+1)(m-1)$

Therefore, $f^*(E) = \{2,4\ldots 2s\}$. So, f is a odd vertex even mean labeling and hence, the $H\Theta n_l$, is an odd vertex even mean graph $H\Theta 3_l$, and $H\Theta 4_l$, is shows in the figure 1.4 and figure 1.5 respectively.

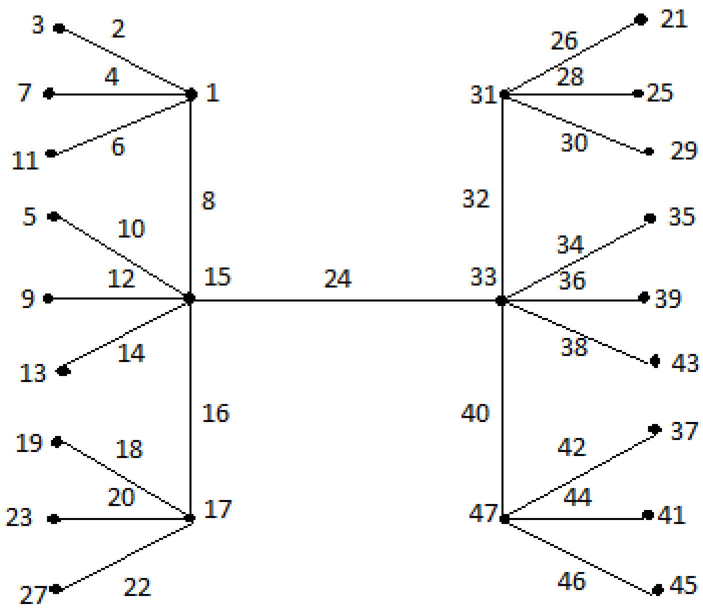

Figure 4. $H\Theta 3i_1$

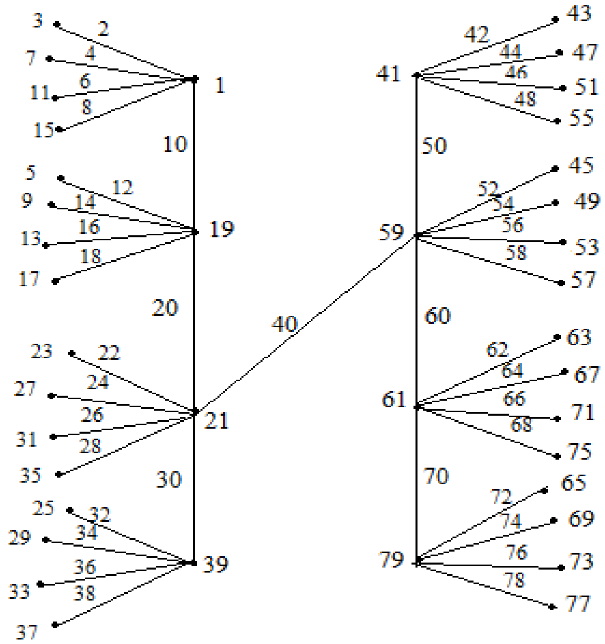

Figure-5: $H\Theta 4i_1$

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