

# 3×3 Game Problem Using Linear Programming Problem

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## ABSTRACT

In this paper, a discussion has been done about the 3×3 game problems by using LPP, which does not satisfy the saddle point conditions and calculated the optimal strategy and value of the game. Considering the far wide applications of game theory it becomes essential to establish the most effective method of optimization of games. **Keywords :** Game Problem, Optimal Strategies, Linear Programming Problem, Value of the Game.

## I. INTRODUCTION

The branch of mathematics concerned with the study of mathematical models of strategic interaction between rational decision makers. In Linear programming problem, the objective is either maximization or minimization of the objective function. Each player is trying to maximize his profit and minimize his loss.

### BASIC CONCEPTS:

#### Linear programming problem:

It is a method to achieve the best outcome (such as Maximum profit or lowest cost in a mathematical model), whose requirements are represented by a linear relationships.

#### Optimal Strategies:

Strategies for all players which fully specifies all actions in a game. A strategy profile must include one and only one strategy for every player.

#### Value of the game:

The saddle point payoff is called the value of the game.

#### Mixed strategy:

In case when the game is unstable and no saddle point is reached, there is no pure strategy for player.

#### PROBLEM:

Consider a 3×3 game problem which does not have a saddle point.

	PLAYER B		
PLAYER A	3	-4	2
	1	-3	-7
	-2	4	7

#### Solution:

Value of the game lies between -2 and 3. Hence it does not satisfy the saddle point. So we are taking  $C=3$  and adding the value in game problem.

	Player B		
Player A	6	-1	5
	4	0	-4
	1	7	10

Converting a game problem into a LPP standard form and it can be written as,

$$\begin{aligned} \text{Max } Z &= Y_1 + Y_2 + Y_3 + 0S_1 + 0S_2 + 0S_3 \\ 6Y_1 - Y_2 + 5Y_3 + S_1 &= 1 \\ 4Y_1 + Y_2 - 4Y_3 + S_2 &= 1 \\ Y_1 + Y_2 + 10Y_3 + S_3 &= 1 \\ Y_1 \geq 0 \quad Y_2 \geq 0 \quad Y_3 \geq 0 \end{aligned}$$

INITIAL ITERATION:

$C_B$	$Y_B$	$X_B$	$Y_1$	$Y_2$	$Y_3$	$S_1$	$S_2$	$S_3$
0	$S_1$	1	6	-1	5	1	0	0
0	$S_2$	1	4	0	-4	0	1	0
0	$S_3$	1	1	7	10	0	0	1
		$Z_j - C_j$	-1	-1	-1	0	0	0

FIRST ITERATION:

$C_B$	$Y_B$	$X_B$	$Y_1$	$Y_2$	$Y_3$	$S_1$	$S_2$	$S_3$
0	$S_1$	8/7	43/7	0	45/7	1	0	1/7
0	$S_2$	1	4	0	-4	0	1	0
1	$Y_2$	1/7	1/7	1	10/7	0	0	1/7
	$Z_j - C_j$		-6/7	0	3/7	0	0	1/7

$S_1 \rightarrow$  leaves,  $Y_1 \rightarrow$  enter

FINAL ITERATION:

$C_B$	$Y_B$	$X_B$	$Y_1$	$Y_2$	$Y_3$	$S_1$	$S_2$	$S_3$
1	$Y_1$	8/43	1	0	45/43	7/43	0	1/43
0	$S_2$	11/43	0	0	-352/43	-28/43	1	-4/43
1	$Y_2$	5/43	0	1	1/43	-1/43	0	6/43

$$Y_1 + Y_2 + Y_3 = \frac{8}{43} + \frac{5}{43} + 0 = \frac{13}{43} = \frac{1}{V}$$

$$V = \frac{43}{13}$$

Considering Player A as  $p_1, p_2, p_3$  and Player B as  $q_1, q_2, q_3$ .

$$\text{Prob of strategy } q_1 = Y_1 \times V = \frac{8}{43} \times \frac{43}{13} = \frac{8}{13}$$

$$q_2 = Y_2 \times V = \frac{5}{43} \times \frac{43}{13} = \frac{5}{13}$$

$$q_3 = Y_3 \times V = 0 \times V = 0$$

Real value of the game

$$V^* = V - 3 = \frac{43}{13} - 3 = \frac{4}{13}$$

Dual variables with respect to player is  $X_1, X_2, X_3$

$$X_1 = \frac{6}{43}, X_2 = 0, X_3 = \frac{7}{43}$$

$$p_1 = X_1 \times V = \frac{6}{43} \times \frac{43}{13} = \frac{6}{13}$$

$$p_2 = X_2 \times V = 0 \times V = 0$$

$$p_3 = X_3 \times V = \frac{7}{43} \times \frac{43}{13} = \frac{7}{13}$$

$$X_1 + X_2 + X_3 = \frac{6}{43} + 0 + \frac{7}{43} = \frac{13}{43}$$

$$V = \frac{43}{13}$$

$$\text{Value of game} = \frac{43}{13} \times 3 = \frac{4}{13}$$

## II. CONCLUSION

In this paper a 3×3 Game problem in the game theory can be solved by Linear programming problem method. It has been observed that the optimal strategies and value of the game for the players are calculated and same when comparing the solution.

For Player A and Player B:

optimal strategies  $\Rightarrow p_1 + p_2 + p_3 = 1$  and  $q_1 + q_2 + q_3 = 1$

Value of the game  $\Rightarrow \frac{4}{13}$

## III. REFERENCES

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