© 2018 IJSRST | Volume 4 | Issue 10 | Print ISSN : 2395-6011 | Online ISSN : 2395-602X
Themed Section: Science and Technology

Energy Momentum Tensor for Photonic System

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ABSTRACT

Different expressions for energy momentum tensor in Cartesian co-ordinates under different conditions have been obtained by taking into consideration the transformation of co-ordinates from a photonic system. In doing so, the fundamental tensor, the Christofell's three index symbol, the Riemann-Christofell curvature tensor , scalar curvature have been calculated which leads to the expression of the energy-momentum tensor. The clue for transforming the main expression to those under different conditions has been mentioned in this work.
Keywords : Transformation matrix, Energy-momentum tensor, Christofell three index symbol, Space-time curvature tensor.

I. INTRODUCTION

We know that a light beam consists of photons. Each photon possesses energy. It, also, possesses linear momentum directed along the beam axis normal to the wave front. Again, each one has a spin angular momentum aligned in and opposite to the direction of propagation. Thus, a photon is a complex system as it possesses particle and wave nature and, also, has spin and linear motions simultaneously [1].

Again, there is oscillation of the impulse of the light wave. It is note worthy that oscillation perpendicular to the direction of propagation gives rise to linear as well as angular momentum as also to polarization of the waves. Thus, a photon has linear velocity, a rotation and an oscillation [2, 3].

Now, it is well known that the energy momentum tensor is of fundamental importance in the study of gravitational physics. This parameter is related to the characteristics of gravity, as seen from Einstein's Field Equation (neglecting cosmological constant)

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\frac{8\pi G}{\mu_0 c^4}T_{\mu\nu}=-K\,T_{\mu\nu} \qquad (1)$$

In this equation the terms on the left represent geometry of space-time while that on the right is energy- momentum tensor which is a measure of the matter energy density [4]. It is to be mentioned that for empty space $R_{\mu\nu}=0$. Hence, in absence of matter $T_{\mu\nu}=0$ [5].

It is to be remembered that different types of expressions for energy-momentum tensor were put forward by different workers. The fact is that any one of the expressions could be used for different works [6]. It is evident that the energy momentum tensor for photonic system would be a bit different from that of others. So, if one can calculate the exact energy momentum tensor in the system of photon then Einstein's equations and Einstein's tensor could be easily obtained. On solving these equations one may obtain the characteristics of photon wave [7].

If we consider a generalized field equation for an arbitrary conservative field where K is the coupling constant appropriate for the concerned field then the energy- momentum tensor in the system of photon could be obtained using fundamental tensor of it.

So, in this work we shall try to find out Christofell's three index symbol from the fundamental tensor and, then, calculate the components of energy-momentum tensor in Cartesian co-ordinate system under different conditions.

Mention must be made to the fact that long expressions are not shown in the text when there is the case of mathematical calculation only. But these are shown in the appendix and hints are given in the text so that one can obtain the full form of the expression if he is interested about it.

II. Energy-Momentum Tensor

From (1) we can write the energy-momentum tensor to be

$$T_{\mu\nu} = -\frac{1}{K}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) \tag{2}$$

Due to the symmetry of energy-momentum tensor and for the case of photon $T_{\mu\mu}$, instead of $T_{\mu\nu}$, will suffice to describe the photon. Hence, the energy-momentum tensor could be expressed in the matrix form as

$$T_{\mu\mu} = \begin{pmatrix} T_{11} & 0 & 0 & 0 \\ 0 & T_{22} & 0 & 0 \\ 0 & 0 & T_{33} & 0 \\ 0 & 0 & 0 & T_{44} \end{pmatrix} \tag{3}$$

The elements of energy-momentum tensor could be written as

$$T_{\mu\mu} = -\frac{1}{K}(R_{\mu\mu} - \frac{1}{2}g_{\mu\mu}R) \tag{4}$$

We may find out the expression of $g_{\mu\mu}$ as in [8, 9], $R_{\mu\mu}$, R for the system of photon under different conditions and can have the values of $T_{\mu\mu}$ from (4) as shown in the following sections.

Section II.A. Using Cartesian co-ordinates

Considering the characteristics of photon the fundamental tensor $\bar{g}_{\mu\nu}$ in Cartesian co-ordinates would be given by [10]

$$\bar{g}_{\mu\nu} = \begin{pmatrix} Sin^2\bar{\omega}\bar{t} & 0 & 0 & \frac{1}{2}(\bar{x}\,\bar{\omega}\,Sin\,2\bar{\omega}\bar{t} - 2\bar{y}\,\omega\,Sin\,\bar{\omega}\bar{t}) \\ 0 & 1 & 0 & \bar{x}\,\omega\,Sin\,\bar{\omega}\bar{t} \\ 0 & 0 & 1 & 0 \\ \frac{1}{2}(\bar{x}\,\bar{\omega}\,Sin\,2\bar{\omega}\bar{t} - 2\bar{y}\,\omega\,Sin\,\bar{\omega}\bar{t}) & \bar{x}\,\omega\,Sin\,\bar{\omega}\bar{t} & 0 & \begin{matrix}\bar{x}^2\omega^2 Sin^2\bar{\omega}\bar{t} + \bar{x}^2\bar{\omega}^2 Cos^2\bar{\omega}\bar{t} + \\ 2\bar{x}\,\bar{y}\,\omega\bar{\omega}\,Cos\,\bar{\omega}\bar{t} + \bar{y}^2\omega^2 - c^2\end{matrix} \end{pmatrix} \tag{5}$$

From (5) the determinant of $\bar{g}_{\mu\nu}$ becomes

$$\Delta = -c^2\, Sin^2\,\bar{\omega}\bar{t} \tag{6}$$

Also, we shall have

$$g^{11} = \frac{\bar{x}^2\bar{\omega}^2 Cos^2\bar{\omega}\bar{t} - 2\bar{x}\,\bar{y}\,\omega\bar{\omega}\,Cos\,\bar{\omega}\bar{t} + \bar{y}^2\omega^2 - c^2}{-c^2\, Sin^2\,\bar{\omega}\bar{t}},$$
$$g^{22} = -\frac{\bar{x}^2\omega^2 Sin^2\bar{\omega}\bar{t} - c^2}{c^2},\; g^{33} = 1,\; g^{44} = -\frac{1}{c^2} \tag{7}$$

Using (5) and (7) we can calculate the necessary Christofell's three index symbols to be

$$\Gamma^1_{44} = -\frac{1}{2}g^{11}\partial_{\bar{x}}(g_{44}),\; \Gamma^2_{44} = -\frac{1}{2}g^{22}\partial_{\bar{y}}(g_{44}),\; \Gamma^4_{11} = -\frac{1}{2}g^{44}\partial_{\bar{t}}(g_{11}),$$
$$\Gamma^4_{14} = \frac{1}{2}g^{44}\partial_{\bar{x}}(g_{44}) = \Gamma^4_{41},\Gamma^4_{24} = \frac{1}{2}g^{44}\partial_{\bar{y}}(g_{44}) = \Gamma^4_{42},\; \Gamma^4_{44} = \frac{1}{2}g^{44}\partial_{\bar{t}}(g_{44}) \tag{8}$$

Other Christofell symbols are not required for our purpose. Here $\partial_{\bar{x}}$ represent differentiation of a function with respect to $\bar{x}$ and so on. Using (8) we shall obtain the Riemann-Christofell curvature tensor from $R_{\mu\mu} = \partial_{\bar{x}^\mu}(\Gamma^\beta_{\mu\beta}) - \partial_{\bar{x}^\beta}(\Gamma^\beta_{\mu\mu}) + \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\alpha\mu} - \Gamma^\alpha_{\mu\mu}\Gamma^\beta_{\alpha\beta}$ to be

$$
\begin{aligned}
R_{11} &= \partial_{\bar{x}}(\Gamma^4_{14}) - \partial_{\bar{t}}(\Gamma^4_{11}) + (\Gamma^4_{14})^2 - \Gamma^4_{11}\Gamma^4_{44} \\
R_{22} &= \partial_{\bar{y}}(\Gamma^4_{24}) + (\Gamma^4_{24})^2 \\
R_{33} &= 0 \\
R_{44} &= -\partial_{\bar{x}}(\Gamma^1_{44}) - \partial_{\bar{y}}(\Gamma^2_{44}) + \Gamma^1_{44}\Gamma^4_{41} + \Gamma^2_{44}\Gamma^4_{42}
\end{aligned}
\tag{9}
$$

So, the scalar curvature R $[R = g^{\mu\mu}R_{\mu\mu}]$ comes out to be

$$
\begin{aligned}
R &= g^{11}[\partial_{\bar{x}}(\Gamma^4_{14}) - \partial_{\bar{t}}(\Gamma^4_{11}) + (\Gamma^4_{14})^2 - \Gamma^4_{11}\Gamma^4_{44}] + g^{22}[\partial_{\bar{y}}(\Gamma^4_{24}) + (\Gamma^4_{24})^2] \\
&- g^{44}[\partial_{\bar{x}}(\Gamma^1_{44}) - \partial_{\bar{y}}(\Gamma^2_{44}) - \Gamma^1_{44}\Gamma^4_{41} - \Gamma^2_{44}\Gamma^4_{42}]
\end{aligned}
\tag{10}
$$

Using (9) and (10) we might have the elements of energy-momentum tensor from (4) by substituting required values from APPENDIX.

Section II. A.1. Using Cartesian co-ordinates with $\bar{\omega}\bar{t} = \pi/2$.

In this case

$$
(\bar{g}_{\mu\nu})_{\bar{\omega}\bar{t}=\pi/2} = \begin{pmatrix} 1 & 0 & 0 & -\omega\bar{y} \\ 0 & 1 & 0 & \omega\bar{x} \\ 0 & 0 & 1 & 0 \\ -\omega\bar{y} & \omega\bar{x} & 0 & \omega^2(\bar{x}^2+\bar{y}^2)-c^2 \end{pmatrix}
\tag{11}
$$

The determinant of the above matrix becomes $\Delta_1 = -c^2$. This would lead one to

$$
g^{11} = -(\frac{\omega^2 y^2}{c^2} - 1),\ g^{22} = -(\frac{\omega^2 x^2}{c^2} - 1),\ g^{33} = 1,\ g^{44} = -\frac{1}{c^2}
\tag{12}
$$

With the use of (12) we obtain

$$
\begin{aligned}
&\Gamma^1_{44} = \frac{\omega^2\,\bar{x}}{c^2}(\omega^2\,\bar{y}^2 - c^2),\ \Gamma^2_{44} = \frac{\omega^2\,\bar{y}}{c^2}(\omega^2\,\bar{x}^2 - c^2),\ \Gamma^4_{11} = 0, \\
&\Gamma^4_{14} = -\frac{\omega^2\,\bar{x}}{c^2} = \Gamma^4_{41},\ \Gamma^4_{24} = -\frac{\omega^2\,\bar{y}}{c^2} = \Gamma^4_{42}
\end{aligned}
\tag{13}
$$

All other Christofell symbols are zero and are not required in the present case. Using (13) we obtain the space-time curvature tensors to be

$$
R_{11} = \frac{\omega^2}{c^2}(\frac{\omega^2\,\bar{x}^2}{c^2} - 1),\ R_{22} = \frac{\omega^2}{c^2}(\frac{\omega^2\,\bar{y}^2}{c^2} - 1),\ R_{33} = 0,\ R_{44} = -\frac{2\,\omega^6\,\bar{x}^2\,\bar{y}^2}{c^4} + 2\,\omega^2
\tag{14}
$$

The scalar curvature tensor R comes out to be

$$
R = 2\frac{\omega^4}{c^4}(\bar{x}^2 + \bar{y}^2) - 4\frac{\omega^2}{c^2}
\tag{15}
$$

Now, we shall have the elements of the energy-momentum tensor as shown below

$$T_{11}=\frac{1}{K}\frac{\omega^2}{c^2}[\frac{\omega^2\,\bar{y}^2}{c^2}-1]$$
$$T_{22}=\frac{1}{K}\frac{\omega^2}{c^2}[\frac{\omega^2\,\bar{x}^2}{c^2}-1]$$
$$T_{33}=\frac{1}{K}\frac{\omega^2}{c^2}[\frac{\omega^2}{c^2}(\bar{x}^2+\bar{y}^2)-2] \tag{16}$$
$$T_{44}=-\frac{1}{K}\frac{\omega^4}{c^2}[4\frac{\omega^2\,\bar{x}^2\,\bar{y}^2}{c^2}+\frac{\omega^2}{c^2}(\bar{x}^4+\bar{y}^4)-3(\bar{x}^2+\bar{y}^2)]$$

We may put these in the form of (3) if we like so. It is to be pointed out that all these relations could be obtained easily by substituting $\varpi\,\bar{t}=\pi/2$ in the corresponding relations of section II. A.

Section II. A. 2. Using Cartesian co-ordinates with $\omega = 0$.

This case is that of a plane polarized photon beam. In this case we shall have

$$(\bar{g}_{\mu\nu})_{\omega=0}=\begin{pmatrix} Sin^2\,\varpi\,\bar{t} & 0 & 0 & \varpi\,\bar{x}\,Sin\,\varpi\,\bar{t}\,Cos\,\varpi\,\bar{t} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \varpi\,\bar{x}\,Sin\,\varpi\,\bar{t}\,Cos\,\varpi\,\bar{t} & 0 & 0 & \omega^2\,\bar{x}^2\,Cos^2\,\varpi\,\bar{t}-c^2 \end{pmatrix} \tag{17}$$

This will lead to

$$\Delta_2=-c^2\,Sin^2\,\varpi\,\bar{t} \tag{18}$$

Now, we shall have the following

$$g^{11}=-\frac{\varpi^2\,\bar{x}^2\,Cos^2\,\varpi\,\bar{t}-c^2}{c^2\,Sin^2\,\varpi\,\bar{t}},\ g^{22}=1,\ \ g^{33}=1,\ g^{44}=-\frac{1}{c^2} \tag{19}$$

The three index Christofell symbols could be easily obtained as

$$\Gamma^1_{44}=-\frac{\varpi^2\,\bar{x}^2\,c^2\,Cos^2\,\varpi\,\bar{t}-\varpi^4\,\bar{x}^3\,Cos^4\,\varpi\,\bar{t}}{c^2\,Sin^2\,\varpi\,\bar{t}},\ \Gamma^2_{44}=0,\ \Gamma^4_{11}=\frac{\varpi\,Sin\,2\,\varpi\,\bar{t}}{2\,c^2},$$
$$\Gamma^4_{14}=-\frac{\varpi^2\,\bar{x}^2\,Cos^2\,\varpi\,\bar{t}}{c^2}=\Gamma^4_{41},\ \Gamma^4_{24}=0=\Gamma^4_{42},\ \Gamma^4_{44}=\frac{\varpi^3\,\bar{x}^2\,Sin\,2\,\varpi\,\bar{t}}{2c^2} \tag{20}$$

With the help of (17) and (20) we obtain the Riemann-Christofell curvature tensor to be

$$R_{11}=\frac{\varpi^2}{c^2}[\frac{\varpi^2\,\bar{x}^2\,Cos^4\,\varpi\,\bar{t}}{c^2}-Cos^2\,\varpi\,\bar{t}-Cos\,2\,\varpi\,\bar{t}-\frac{\varpi^2\,\bar{x}^2\,Sin^2\,2\,\varpi\,\bar{t}}{4}],\ R_{22}=0,$$
$$R_{33}=0,\quad R_{44}=\varpi^2\,Cot^2\,\varpi\,\bar{t}[1-\frac{4\,\varpi^2\,\bar{x}^2\,Cos^2\,\varpi\,\bar{t}}{c^2}+\frac{\varpi^4\,\bar{x}^4\,Cos^4\,\varpi\,\bar{t}}{c^4}] \tag{21}$$

Hence, the scalar curvature tensor R becomes

$$R=\frac{6\,\bar{x}^2\,\varpi^4\,Cot^2\,\varpi\,\bar{t}\,Cos^2\,\varpi\,\bar{t}}{c^2}-\frac{2\,\bar{x}^4\,\varpi^6}{c^6}Cot^2\,\varpi\,\bar{t}\,Cos^4\,\varpi\,\bar{t}+\frac{\bar{x}^2\varpi^4\,Cot^2\,\varpi\,\bar{t}\,Cos\,2\,\varpi\,\bar{t}}{c^4}$$
$$+\frac{\bar{x}^4\,\varpi^6\,Cot^2\,\varpi\,\bar{t}\,Sin^2\,2\,\varpi\,\bar{t}}{4\,c^4}-\varpi^2\,Cot^2\,\varpi\,\bar{t}-\frac{\varpi^2\,Cos\,2\,\varpi\,\bar{t}}{c^2\,Sin^2\,\varpi\,\bar{t}}-\frac{\bar{x}^2\,\varpi^4\,Cos^2\,\varpi\,\bar{t}}{2\,c^2}$$
$$-\frac{\varpi^2\,Cot^2\,\varpi\,\bar{t}}{c^2} \tag{22}$$

We can easily find out the elements of energy-momentum tensor by using (17), (21) and (22) in (4). Of course, all the expressions could be easily obtained from those in section A by substituting $\omega = 0$.

Section II. A. 3. Using Cartesian co-ordinates with both $\bar{\omega}\bar{t} = \pi/2$ and $\omega = 0$ simultaneously.

Here, the situation is that of a polarized wave. We shall have

$$\left.\bar{g}_{\mu\nu}\right)_{\bar{\omega}\bar{t}=\pi/2,\ \omega=0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -c^2 \end{pmatrix} \quad (23)$$

This leads to

$$\Delta_3 = -c^2 \quad \text{and} \quad g^{11} = 1 = g^{22} = g^{33}, \quad g^{44} = -\frac{1}{c^2} \quad (24)$$

It could be easily shown that all the three index Christofell symbols as well as all the curvature tensors are zero. Hence, all the elements of energy-momentum tensor are zero.

Now, according to Einstein's law of gravitation the Riemann curvature tensor is zero for empty space. So, the result signifies that there is absence of matter in the space considered and also the space-time is flat. The tidal force is, also, absent in this case [11, 12]. So, the condition $\bar{\omega}\bar{t} = \pi/2$ and $\omega = 0$ means absence of matter in the space-time continuum.

III. CONCLUSION

It is obvious from this work that energy-momentum tensor could be easily obtained from the principle of co-ordinate transformation in photonic system. From the main expression for a particular co-ordinate system, expressions for other cases could be obtained by simple substitution of the relevant condition in the appropriate expression for energy-momentum tensor. Last but not the least, we may conclude that the expression for energy-momentum tensor gives us information about the characteristics of the medium and the state of polarization of the photon wave. This is clearly seen especially from section II. A. 3.

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Appendix

Equation (8) of the text

$$\Gamma^1_{44} = \frac{1}{c^2\, Sin^2\,\bar{\omega}\,\bar{t}}[\bar{x}^3\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t}\;Sin\,\bar{\omega}\,\bar{t} + \bar{x}^3\,\bar{\omega}^4\,Cos^4\,\bar{\omega}\,\bar{t} - \bar{x}^2\,\bar{y}\,\omega\,\bar{\omega}^3\,Cos^3\,\bar{\omega}\,\bar{t}$$

$$-2\,\bar{x}^2\,\omega^3\,\bar{\omega}\,Sin^2\,\bar{\omega}\,\bar{t}\;Cos\,\bar{\omega}\,\bar{t} - 2\,\bar{x}^2\,\bar{y}\,\omega^3\,\bar{\omega}\,Cos^3\,\bar{\omega}\,\bar{t} - 2\,\bar{x}\,\bar{y}^2\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t}$$

$$+\bar{x}\,\bar{y}^2\,\omega^4\,Sin^2\,\bar{\omega}\,\bar{t} + \bar{x}\,\bar{y}^2\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t} - \bar{y}^3\,\omega^3\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t} - \bar{x}\,\omega^2\,c^2\,Sin^2\,\bar{\omega}\,\bar{t}$$

$$-\bar{x}\,\bar{\omega}^2\,c^2\,Cos^2\,\bar{\omega}\,\bar{t} - \bar{y}\,\omega\,\bar{\omega}\,Cos^2\,\bar{\omega}\,\bar{t}]$$

$$\Gamma^2_{44} = \frac{1}{c^2}[-\bar{x}^3\,\omega^3\,\bar{\omega}\,Sin^2\,\bar{\omega}\,\bar{t}\;Cos\,\bar{\omega}\,\bar{t} + \bar{x}^2\,\bar{y}\,\omega^4\,Sin^2\,\bar{\omega}\,\bar{t} + c^2\,\bar{x}\,\omega\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t} - c^2\,\bar{y}\,\omega^2]$$

$$\Gamma^4_{11} = \frac{\bar{\omega}}{2c^2}\,Sin\,2\,\bar{\omega}\,\bar{t}, \qquad \Gamma^4_{14} = -\frac{\bar{x}\,\omega^2\,Sin^2\,\bar{\omega}\,\bar{t} + \bar{x}\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t} - \bar{y}\,\omega\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t}}{c^2} = \Gamma^4_{41},$$

$$\Gamma^4_{24} = \frac{\bar{x}\,\omega\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t} - \bar{y}\,\omega^2}{c^2} = \Gamma^4_{42}, \quad \Gamma^4_{44} = -\frac{\bar{x}^2\,(\omega^2\,\bar{\omega} - \bar{\omega}^3)\,Sin\,2\,\bar{\omega}\,\bar{t} + 2\,\bar{x}\,\bar{y}\,\omega\,\bar{\omega}^2\,Sin\,\bar{\omega}\,\bar{t}}{2\,c^2}$$

The curvature tensors in equation (9) of the text are given below.

$$R_{11} = \frac{1}{c^2}[-\omega^2\,Sin^2\,\bar{\omega}\,\bar{t} - \bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t} - \bar{\omega}^2\,Cos\,2\,\bar{\omega}\,\bar{t} + \frac{\bar{x}^2\,\omega^4\,Sin^4\,\bar{\omega}\,\bar{t}}{c^2} + \frac{\bar{x}^2\,\bar{\omega}^4\,Cos^4\,\bar{\omega}\,\bar{t}}{c^2}$$

$$+\frac{\bar{y}^2\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t}}{c^2} + \frac{\bar{x}^2\,\omega^2\,\bar{\omega}^2\,Sin^2\,2\,\bar{\omega}\,\bar{t}}{2\,c^2} - \frac{\bar{x}\,\bar{y}\,\omega^3\,\bar{\omega}\,Sin\,\bar{\omega}\,\bar{t}\;Sin\,2\,\bar{\omega}\,\bar{t}}{c^2} -$$

$$\frac{2\,\bar{x}\,\bar{y}\,\omega\,\bar{\omega}^3\,Cos^3\,\bar{\omega}\,\bar{t}}{c^2} + \frac{1}{4}\bar{x}^2\,\omega^2\,\bar{\omega}^2\,Sin^2\,2\,\bar{\omega}\,\bar{t} - \frac{1}{4}\bar{x}^2\,\omega^4\,Sin^2\,2\,\bar{\omega}\,\bar{t} + \bar{x}\,\bar{y}\,\omega\,\bar{\omega}^3\,Sin^2\,\bar{\omega}\,\bar{t}\;Cos\,\bar{\omega}\,\bar{t}]$$

$$R_{22} = -\frac{\omega^2}{c^2} + \frac{1}{c^4}[\bar{x}^2\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t} + \bar{y}^2\,\omega^4 - 2\,\bar{x}\,\bar{y}\,\omega^3\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t}], \qquad R_{33} = 0,$$

$$R_{44} = -\frac{1}{c^2\,Sin^2\,\bar{\omega}\,\bar{t}}[3\,\bar{x}^2\,\omega^2\,\bar{\omega}^2 Cos^2\,\bar{\omega}\,\bar{t}\;Sin^2\,\bar{\omega}\,\bar{t} + 3\,\bar{x}^2\,\bar{\omega}^4\,Cos^4\,\bar{\omega}\,\bar{t} - 2\,\bar{x}\,\bar{y}\,\omega\,\bar{\omega}^3\,Cos^3\,\bar{\omega}\,\bar{t}$$

$$-4\,\bar{x}\,\bar{y}\,\omega^3\,\bar{\omega}\;Cos\,\bar{\omega}\,\bar{t} + 3\,\bar{y}^2\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t} + \bar{y}^2\,\omega^4\,Sin^2\,\bar{\omega}\,\bar{t} - \omega^2\,c^2 + \{\bar{x}^3\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t}\;Sin^2\,\bar{\omega}\,\bar{t}$$

$$+\bar{x}^3\,\bar{\omega}^4\,Cos^4\,\bar{\omega}\,\bar{t} - \bar{x}^2\,\bar{y}\,\omega\,\bar{\omega}^3\,Cos^3\,\bar{\omega}\,\bar{t} - 2\,\bar{x}^2\,\bar{y}\,\omega^3\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t} + 3\,\bar{x}\,\bar{y}^2\,\omega^2\,\bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t}$$

$$+\bar{x}\,\bar{y}^2\,\omega^4\,Sin^2\,\bar{\omega}\,\bar{t} - \bar{y}^3\,\omega^3\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t} - \bar{x}\,c^2(\omega^2\,Sin^2\,\bar{\omega}\,\bar{t} + \bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t}) - \bar{y}\,\omega\,\bar{\omega}\,c^2\,Cos\,\bar{\omega}\,\bar{t}\}$$

$$\frac{1}{c^2}\{\bar{x}\,(\omega^2\,Sin^2\,\bar{\omega}\,\bar{t} + \bar{\omega}^2\,Cos^2\,\bar{\omega}\,\bar{t}) - \bar{y}\,\omega\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t}\}] + \frac{1}{2c^4}[-2\,\bar{x}\,\omega\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t} - 2\,\bar{y}\,\omega^2]$$

$$[\bar{x}^2\,\omega^2\,Sin^2\,\bar{\omega}\,\bar{t} - c^2][\bar{x}\,\omega\,\bar{\omega}\,Cos\,\bar{\omega}\,\bar{t} - \bar{y}\,\omega^2].$$