

Inversion of Time in the Classical Kinematics of Material Point

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ABSTRACT

As a result of the fact that the time in classical physics manifests itself only through the movement of the bodies, in the kinematics of material point the question of reversing the course of time (from the future to the past) is examined theoretically. The method is defined for inversion of time and its impact on the kinematic quantities and principles describing the motion. Three main types of motion in the kinematics have been discussed compared to reversal the course of time.

- a) Fully reversible motions - these are all constant motions. They can occur in the same way both in right and reverse direction of passage of time but they only indicate that the time is running not determining its direction;
- b) Semi reversible movements - these are all movements, in which the acceleration is an even function in case of inversion of time. They amend their nature temporary only in right and reverse direction of passage of time for a certain period. After that the semi-reversible movements run identically. They also show that the time is running but not determining its direction;
- c) Completely irreversible movements - these are movements in which the tangential acceleration is odd function compared to the inversion of time. These can occur only in one direction of the time course; in the opposite direction they are impossible. The irreversible movements can be used to determine the direction of time.

The reasons for these three types of movements have been examined and the relevant laws have been received: of motion, velocity and acceleration in right and reverse direction of passage of time. Their relationship with some basic quantities and laws of classical physics has been shown as well.

Keywords: time, inversion of time

I. INTRODUCTION

The major nature of time is that it is manifested through the movement of the bodies. Let we imagine for a moment that time flows in the opposite direction - from the future to the past. How this affects the movements in the kinematics?

II. METHODS AND MATERIAL

A. Definition

To reverse the direction of time of one randomly moving material point it means the same to go through the same spatial positions in its movement, but in reverse order,

with the same size of velocity but in the opposite direction [1]. The same definition is used also by the other authors [2-12].

B. Description of the inversion of time

Suppose we examine a material point moving along the axis Ox (Figure 1). During the interval of time $\Delta t = t_2 - t_1$ it passes distance $\Delta x = x_2 - x_1$. Projection of its velocity \vec{v} along Ox in the initial and final point is: $v_{x1} \neq v_{x2}$. According to the definition, the inversion of time means a point to go back the distance $x_2 \rightarrow x_1$ with the same size of velocity but in the opposite direction. To reverse the course of time we shall use the

method described in [1], as in every moment of time t of the movement of point a negative value is attributed:

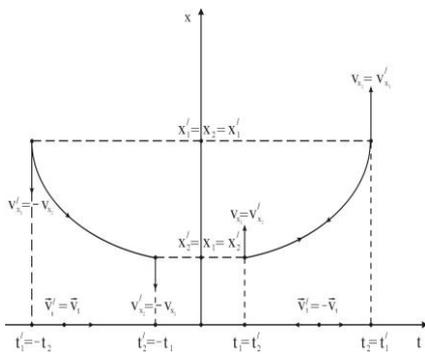


Figure 1: Description of the inversion of time

$$t' = -t$$

This is equivalent to a reflection on the time axis compared to its beginning (Figure 1 – the left part), but without changing the direction of the velocity of time: $\vec{v}'_t = \vec{v}_t$, although formally it now flows from the future to the past. At the same time, the time moments change their signs: $t'_1 = -t_2 < 0$, $t'_2 = -t_1 < 0$ as the final moment becomes initial one and vice versa, and the time intervals preserve their sign.

$$\Delta t' = t'_2 - t'_1 = -t_1 - (-t_2) = t_2 - t_1 = \Delta t > 0$$

The attitude positions preserve their sign, but follow in reverse order:

$$x'_1 = x_2; \quad x'_2 = x_1$$

and spatial intervals change their sign:

$$\Delta x' = x'_2 - x'_1 = x_1 - x_2 = -(x_2 - x_1) = -\Delta x$$

The velocity of the point also changes its direction:

$$v'_x = \frac{\Delta x'}{\Delta t'} = \frac{-\Delta x}{\Delta t} = -v_x$$

thus satisfying the definition for inversion the course of time.

C. Inversion of time in kinematic parameters describing the motion.

We shall examine at how the inversion of time affects the kinematic parameters describing the motion of a material point. The change of time direction we shall reflect as we shall substitute time moments t' with $(-t)$ as the initial moment becomes final one and vice versa, and we shall change the directions of the velocities: $\vec{v}' = -\vec{v}$, according to the mentioned above.

1. Positions and moments:

$$\vec{r}' = \vec{r} \quad - \text{radius vector};$$

$$\vec{\varphi}' = \vec{\varphi} \quad - \text{positional angle at motion in a circle};$$

$$t' = -t \quad - \text{instant of time.}$$

2. Changes in the position and time:

$$\Delta \vec{r}' = \vec{r}'_2 - \vec{r}'_1 = \vec{r}_1 - \vec{r}_2 = -(\vec{r}_2 - \vec{r}_1) = -\Delta \vec{r}$$

- displacement;

$$\Delta s' = \Delta s \quad - \text{distance};$$

$$\Delta \vec{\varphi}' = \vec{\varphi}'_2 - \vec{\varphi}'_1 = \vec{\varphi}_1 - \vec{\varphi}_2 = -(\vec{\varphi}_2 - \vec{\varphi}_1) = -\Delta \vec{\varphi}$$

- angle of rotation

$$\Delta t' = t'_2 - t'_1 = -t_1 - (-t_2) = t_2 - t_1 = \Delta t$$

- time interval.

(The same is relevant to the differential changes)

$$T' = T \quad - \text{period};$$

$$v' = \frac{1}{T'} = \frac{1}{T} = v \quad - \text{frequency};$$

$$\omega' = \frac{\Delta \varphi'}{\Delta t'} = \frac{-\Delta \varphi}{\Delta t} = -\omega \quad - \text{circular frequency}$$

$$\vec{v}' = \frac{d\vec{r}'}{dt'} = \frac{-d\vec{r}}{dt} = -\vec{v}$$

3. Velocities: - odd

$$\vec{\omega}' = \frac{d\vec{\varphi}'}{dt'} = \frac{-d\vec{\varphi}}{dt} = -\vec{\omega}$$

functions compared to inversion of time;

$$\vec{v}' = \vec{\omega}' \times \vec{r}' = -\vec{\omega} \times \vec{r} = -\vec{v} \quad - \text{right-hand screw rule preserves.}$$

4. Velocity changes:

$$\Delta \vec{v}' = \vec{v}'_2 - \vec{v}'_1 = -\vec{v}_1 - (-\vec{v}_2) = \vec{v}_2 - \vec{v}_1 = \Delta \vec{v}$$

$$\Delta \vec{\omega}' = \vec{\omega}'_2 - \vec{\omega}'_1 = -\vec{\omega}_1 - (-\vec{\omega}_2) = \vec{\omega}_2 - \vec{\omega}_1 = \Delta \vec{\omega}$$

(The same is relevant to the differential changes.)

5. Accelerations:

First we shall believe that the acceleration is not proportional to the velocity.

$$\vec{a}' = \frac{d\vec{v}'}{dt'} = \frac{d\vec{v}}{dt} = \vec{a}$$

$$\vec{a}'_\tau = \frac{dv'}{dt'} \vec{e}'_v = \frac{-dv}{dt} (-\vec{e}_v) = \frac{dv}{dt} \vec{e}_v = \vec{a}_\tau$$

(Here the unit vector indicating the direction of the velocity, changes its direction: $\vec{e}'_v = -\vec{e}_v$.)

$$\vec{a}'_n = \omega'^2 \rho' \vec{n}' = (-\omega)^2 \rho \vec{n} = \vec{a}_n$$

(Here the radius of curvature of the trajectory and the normal vector to it does not change in case of inversion of time.)

$$\vec{a} = \vec{a}_\tau + \vec{a}_n = \vec{a}'_\tau + \vec{a}'_n = \vec{a}'$$

$$\vec{\alpha}' = \frac{d\vec{\omega}'}{dt'} = \frac{d\vec{\omega}}{dt} = \vec{\alpha}$$

$$\vec{a}'_\tau = \vec{\alpha}' \times \vec{r}' = \vec{\alpha} \times \vec{r} = \vec{a}_\tau$$

All accelerations are even functions in case of inversion of time.

The time derivatives of higher order can be considered similarly as well.

6. Law of motion, velocity and acceleration.

According to the first part of the definition of the inversion of time - the material point to pass through the same spatial positions, but in reverse order - it follows that the law of motion has to be even function in case of the inversion of time:

$$\vec{r}'(t') = \vec{r}(-t)$$

According to the second part of the definition of the inversion of time - the velocity of the material point to preserve its size, but to alter its direction backwards - it follows that the law of velocity has to be odd function in case of the inversion of time:

$$\vec{v}'(t') = -\vec{v}(-t)$$

The above two laws require in case of the inversion of time the law of acceleration to be even function as well:

$$\vec{a}'(t') = \vec{a}(-t)$$

But this condition is only necessary, but not sufficient, because the acceleration is determined not by law of movement but by the force acting on the point, and it can be not an even function (see below).

7. Equations of motion:

We shall examine how the inversion of time affects the fundamental equation of motion in classical mechanics - the second law of Newton [11]:

$$\begin{aligned} \vec{F} &= \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt} = \\ &= -\frac{dm'}{dt'}(-\vec{v}') + m'\frac{d\vec{v}'}{dt'} = \\ &= \frac{dm'}{dt'}\vec{v}' + m'\frac{d\vec{v}'}{dt'} = \frac{d(m'\vec{v}')}{dt'} = \frac{d\vec{p}'}{dt'} = \vec{F}' \end{aligned}$$

(Here $\vec{p} = m\vec{v}$ - momentum of the body; $m = m'$ - body mass, which does not change in case of inversion of time; $\frac{dm}{dt} = -\frac{dm'}{dt'}$ - rate of change of mass in reactive movements, which is odd function compared to the inversion of time.)

It is seen that the equation of motion (the second principle of dynamics) preserves in case of the inversion of time i.e. it is reversible over time. But and here this is only necessary but not sufficient condition.

D. Examples of movements inverted in time.

The question placed - can we use a particular motion in order to determine the direction of course of time - from the past to the future or vice versa? At first sight, the answer is negative, because the equations of motion are symmetric toward the inversion of time - movements themselves can run in forward or reverse direction. But, as we shall see below, this is not exactly so.

1. Movements fully reversible in time:

Rest

When the material point is at rest compared to the frame of reference, the kinematic variables are: $\vec{r} = \text{const}$; $\Delta\vec{r} = \mathbf{0}$; $\Delta s = 0$; $\vec{a} = \mathbf{0}$. Only the time interval Δt is uncertain quantity. Reversing the direction of time does not affect all these quantities - they will not change. This is because the rest is a condition but not motion. We can put the question - whether in this case the time is running at all (Δt grows indefinitely) and if it is running - in what direction? Kinematics cannot answer the question in this case: the time may not run ($\Delta t = 0$, $t = \text{const}$), and we cannot determine its direction as well (if it is running) - from the past to the future, or vice versa. The reason for this is that we define the time through the movement, but in this case it is absent.

Constant straight-line motion (Fig. 2)

Let design the vector laws for acceleration, velocity and movement along the axis Ox of the selected coordinate frame of reference:

$$a_x(t) = 0$$

$$v_x(t) = v_{0x} = \text{const}$$

$$x(t) = x_0 + v_x(t - t_0), \quad t \in (-\infty, +\infty)$$

and to replace in them v'_x with $-v_x$, v'_{0x} with $-v_{0x}$ and moments of time t with: $t' = -t$, $t'_0 = -t_0$, which

corresponds to the reversal the course of time. We shall receive:

$$\begin{aligned}
 a'_x(t') &= a_x(-t) = 0 \\
 v'_x(t') &= -v_x(-t) = -v_{0x} = -const \\
 x'(t') &= x'_0 + v'_x(t' - t'_0) = x_0 - v_x(-t + t_0) = \\
 &= x_0 + v_x(t - t_0) = x(t) \\
 t' &\in (-\infty, +\infty)
 \end{aligned}$$

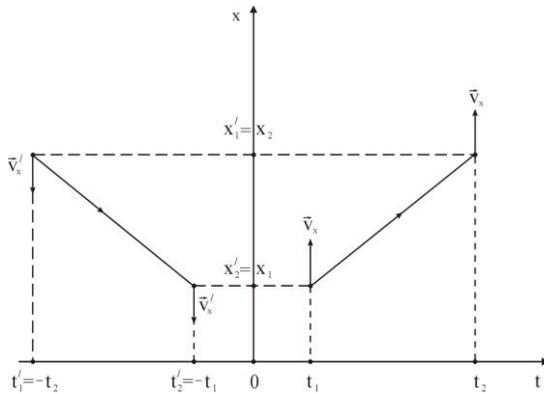


Figure 2 : Inversion of time at Constant straight-line motion

The resulting laws for acceleration, velocity and movement show that the point moves constantly in a straight line in the opposite direction of Ox, as it passes through the same spatial positions with the same size of velocity, but in reverse order. (This is also true when $t_0 = 0$).

It turns out that the law of velocity is odd, and the law for the acceleration and movement - even function toward the reversal of time t.

It is apparent that the inverted movement is completely analogical to the initial, but in reverse direction. Such a movement we shall call **fully reversible in time**. Moreover, the direction of movement shall be determined by the selection of coordinate frame of reference. Please note that if we invert the spatial axis Ox compared to its starting position accurate within an additive constant the movement does not differ from the initial one. What is more, if we consider two identical material points, moving constantly in a straight line with equal velocities against each other and if we turn the direction of time, we shall receive again two material points, moving against each other with the same velocities. The only difference is that two points have exchanged their places.

That is why we can draw a fantastic at first sight conclusion that we not able a priori to define for which bodies moving constantly in straight-line the time flows in one direction or in the other direction, i. e. the constant straight-line movements shows that the time is running out, but not determine its direction.

Another important note: the process of constant straight-line movement can be considered reversible, as there is no reason for this movement.

There are many other movements fully reversible in time, such as constant movement in a circle, ellipse, spread of a flat sine wave in environment non-swelling energy, etc., which will not be considered here by us.

General case of fully reversible movements.

We shall prove the following theorem: any unspecified constant motion is completely reversible in time. Indeed, for any unspecified constant movement (in a curve or in straight-line) we have:

$$\begin{aligned}
 \vec{a}'_r(t') &= \vec{a}_r(-t) = 0 \\
 \vec{a}'_n(t') &= a'_n \vec{n}'(t') = (-\omega)^2 \rho' \vec{n}'(t') = \omega^2 \rho \vec{n}(-t) = \\
 &= \vec{a}_n(-t); a'_n = a_n = \omega^2 \rho = const \\
 \vec{\alpha}'(t') &= \vec{\alpha}(-t) = 0 \\
 \vec{a}'(t') &= \vec{a}'_r + \vec{a}'_n = \vec{a}_r + \vec{a}_n = \vec{a}(-t)
 \end{aligned}$$

It is apparent that the law of acceleration is an even function compared to the inversion of time, and this, together with the requirements for odd function of the law of velocity and for even function of the law of the movement leads to the fully reversible movement. The reason for this is that the force acting on the point in this case is only normal (perpendicular to velocity) and such force shall not modify the kinetic energy of the moving body.

And here is in effect the claiming that all fully reversible movements (whether they are periodical or not) do not determine the direction of the arrow of time.

2. Movements semi - reversible in time:

Uniformly variable straight-line motion (Figure 3)

We shall consider the particular case of constant accelerated straight-line motion. If we design the vector laws for acceleration, velocity and movement along of the coordinate axis Ox, we shall receive:

$$a_x(t) = a_{0x} = const$$

$$v_x(t) = v_{0x} + a_x(t-t_0)$$

$$x(t) = x_0 + v_{0x}(t-t_0) + \frac{1}{2}a_x(t-t_0)^2$$

The inversion of time leads to the following laws for acceleration, velocity and movement:

$$a'_x(t') = a'_{0x} = a_{0x} = a_{0x} = const = a_x(-t)$$

$$v'_x(t') = v'_{0x} + a'_x(t'-t'_0) = -v_{0x} + a_x(-t+t_0) = -v_{0x} - a_x(t-t_0) = -v_x(t)$$

$$x'(t') = x'_0 + v'_{0x}(t'-t'_0) + \frac{1}{2}a'_x(t'-t'_0)^2 =$$

$$= x_0 - v_{0x}(-t+t_0) + \frac{1}{2}a_x(-t+t_0)^2 =$$

$$= x_0 + v_{0x}(t-t_0) + \frac{1}{2}a_x(t-t_0)^2 = x(t)$$

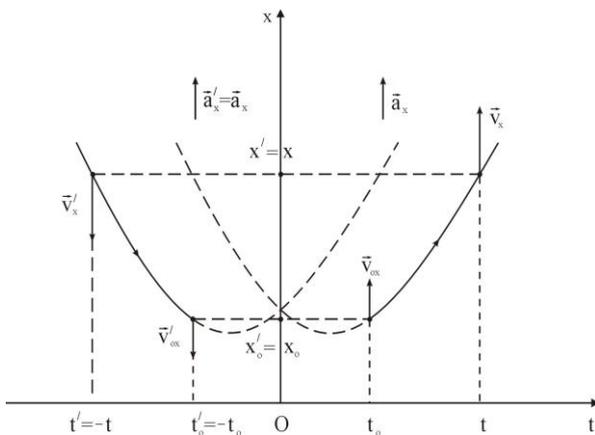


Figure 3. Inversion of time at uniformly accelerated straight-line motion

It is apparent that the law of velocity is an odd function and the laws of acceleration and movement are even functions compared to the inversion of time t . The reversed movement represents a uniformly decelerated motion conversely to Ox, as the point goes through the same spatial positions with the same size of velocity, but in reverse order. Or the nature of the motion will be amended temporarily – from uniformly accelerated motion it will become uniformly decelerated motion. After the interval of time $\Delta t = 2v/a$ inverted movement again becomes uniformly accelerated without any difference from the initial one. We shall name such movement semi - reversible in time because it amends

its nature only temporarily - for a certain time interval, which we shall name a period of "whirlwind of time."

Here the fact is essential that the reversal of time flow does not change the direction of acceleration. We shall note something more: each uniformly decelerated straight-line motion becomes uniformly accelerated in the opposite direction after a certain interval of time after the velocity passes through 0 and reverses its direction. The opposite is impossible – any uniformly accelerated straight-line motion will always remain the same as the velocity will increase to infinity. Even in case of reverse the time flow the uniformly accelerated motion becomes uniformly decelerated in the opposite direction only for a fixed time interval, only until the velocity passes through 0, then the motion is again uniformly accelerated in straight line direction as the velocity increases to infinity.

Note: Formally, we can consider that in case of uniformly decelerated straight-line motion when the velocity becomes 0 and reverses its direction (respectively its move), the time reverse occurs certainly!

The same consideration also applies when a non-inverting movement is uniformly decelerated straight-line motion.

Can such a movement to be used to determine the direction of the "arrow" of time? Unfortunately it is not possible. Here it only shows, as well, that the time is running out, but its direction is not determined.

Many other examples could be given of semi reversible movements - harmonic oscillation of the material point, parabolic motion, hyperbolic motion, elliptic motion in the field of gravitational force, etc., which will not be considered here by us.

General case of semi - reversible movements

All results received above in the case of considered cases of movement in kinematics are mainly due to fact that the laws of motion and acceleration are even functions compared to the inversion of time, which in reflection (mirror image) in the course of time do not alter their shape. Law of velocity is odd function compared to t and in case of reversal in the course of time the sign of the velocity changes. The requirement

the Law for acceleration to be even function is significant as it does not follow from the definition of inversion of time as at the other two laws. We shall prove the following theorem: if tangential acceleration at random movement of a material point is even function compared to the inversion of time, the movement is completely or semi - reversible in time.

We have

$$\vec{a}(t) = \vec{a}_\tau(t) + \vec{a}_n(t)$$

For normal acceleration we have already proven that it is always an even function at the inversion of time:

$$\vec{a}'_n(t') = \vec{a}_n(-t)$$

Then, from the condition that: $\vec{a}'_\tau(t') = \vec{a}_\tau(-t)$ is an even function, it follows that the full acceleration is an even function: $\vec{a}'(t') = \vec{a}(-t)$.

Then the movement meets the conditions of reversibility in time and is completely or semi - reversible.

The requirement $\vec{a}_\tau(t)$ to be an even function is fulfilled when the acceleration depends only on the position in space and time, but does not depend on the velocity of the body:

$$\vec{a} = \vec{a}(\vec{r}(t), t)$$

This is true for homogeneous stationary field of the acceleration (respectively the force) - completely reversible movements - or for central stationary field - semi - reversible movements. Non-stationary fields require further consideration.

In all these cases the conclusion is in force that completely or semi - reversible movements cannot be used to determine the direction of the course of time.

3. Irreversible movements:

What will happen to the reversibility of movements when acceleration is odd function compared to the inversion of time? We shall consider that the acceleration of the material point is proportional to a power of velocity of the body:

$$\vec{a} = \pm k |\vec{v}|^n \vec{e}_v$$

where k is a coefficient of proportionality, and n = 0, 1, 2, 3, ... is an integer.

The mark (-) corresponds to the acceleration obtained under the action of so-called dissipative forces of friction and resistance at movement of the body in a given environment.

Examples of such forces:

TABLE 1

n = 0	-	Forces of dry friction at sliding of the body on some surface [11];
n = 1	-	the resistance forces in case of movement of the body with minor velocity into a viscous fluid - so-called resistance as a result of friction [11];
n = 2	-	vortex resistance on the body, moving at high velocity in a fluid [11];
n = 3	-	resistance due to shock wave at movement of the body with a supersonic velocity in a fluid [11], etc.

All of these forces tend to reduce the kinetic energy of the moving body, which is dissipated to the environment [11]. The work of these forces is negative.

The sign (+) corresponds to the accelerations, obtained by the action of so-called cumulative forces that act on the body, when the environment is moving along with it. These forces increase the kinetic energy of the body at the expense of energy from the environment and the work they do is positive.

In both cases, the acceleration of the body is parallel to the velocity and the movement is delayed or accelerating.

Movement with dry friction at sliding (Figure 4)

Lets a body moves in a straight line along the axis Ox (glides loosely) on a support. Its movement will be decelerated under the action of the friction force. According to the first law of Coulomb for a dry friction at sliding [11], the resistance force acting on the body is:

$$\vec{F}_T = -fN \frac{\vec{v}}{v} = -fN \vec{e}_v = const$$

where f is the coefficient of friction at sliding, N - the pressure of the body on the support, and \vec{e}_v is the unit vector having always the direction of the velocity. From the second principle of the dynamics for the acceleration of the body we shall receive:

$$\vec{a} = -k|\vec{v}|^0 \vec{e}_v = -\frac{fN}{m} \vec{e}_v = const$$

where m is the mass of the body. What is important here is that the direction of acceleration is always opposite to the direction of the velocity. From the projection of the above law of acceleration along the axis Ox and integration, we get the law of the velocity.

$$v_x(t) = v_{0x} - \frac{fN}{m} e_v t$$

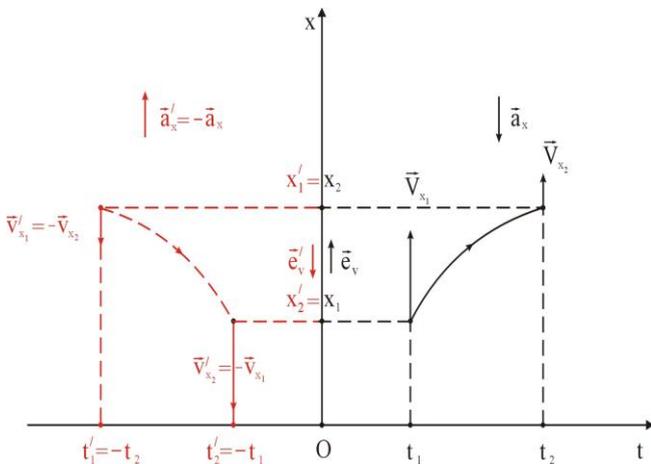


Figure 4: Inversion of time in case of movement with dry friction at sliding

The movement is uniformly decelerated. Through re-integrating in t , we receive the Law of movement as well:

$$x(t) = x_0 + v_{0x}t - \frac{1}{2} \frac{fN}{m} e_v t^2$$

Let us now reverse to the direction of time:

$$a'(t') = -\frac{fN'}{m'} e'_v = -\frac{fN}{m} (-e_v) = -a(-t)$$

which is an odd function of time;

$$\begin{aligned} v'(t') &= v'_{0x} - \frac{fN'}{m'} e'_v t' = -v_{0x} - \frac{fN}{m} (-e_v)(-t) = \\ &= -v_{0x} - \frac{fN}{m} e_v t \end{aligned}$$

which is neither even nor odd function. In the particular case when $v_0 = 0$, the law of velocity appears even function, but such movement (decelerated motion with zero initial speed) is impossible;

$$\begin{aligned} x'(t') &= x'_0 + v'_{0x} t' - \frac{1}{2} \frac{fN'}{m'} e'_v t'^2 = \\ &= x_0 + (-v_{0x})(-t) - \frac{1}{2} \frac{fN}{m} (-e_v)(-t)^2 = \\ &= x_0 + v_{0x} t + \frac{1}{2} \frac{fN}{m} e_v t^2 \end{aligned}$$

which also is neither even nor odd function. The law becomes an odd function only in the particular case when $x_0 = 0$ and $v_0 = 0$, but such movement is impossible.

Inverted movement should be uniformly accelerated in the opposite direction of the Ox with acceleration \vec{a}' , opposite of the velocity \vec{v}' ! But such movement is impossible!

Or the general conclusion at considered movement is that it is absolutely irreversible in time. This is illustrated in Figure 4. It is seen that the body cannot go through the same spatial positions with the same size of the velocity, but in reverse order. The reason for this is that the acceleration changes its direction in case of inversion of time - it is always reverse to the velocity, and in this case it is an odd function of time.

Flow of body by fluid with a moderate velocity (Figure 5)

Consider a body initially at rest, flown by fluid evenly moving along it with small velocity $\vec{v}_f = const$. The fluid will act on the body with cumulative force:

$$\vec{F} = r(\vec{v}_f - \vec{v}(t))$$

where $\vec{v}(t)$ is the velocity of the body in a subsequent moment of time, and r is a coefficient of proportionality. The body will start to move accelerating in the direction of the force while its velocity becomes equal to that of the fluid. Accordingly, the acceleration of the body will be:

$$\vec{a}(t) = k|\vec{v}_f - \vec{v}(t)|^1 \vec{e}_v = \frac{r}{m} |\vec{v}_f - \vec{v}(t)|^1 \vec{e}_v$$

From the projection of the vector law along the axis Ox by integrating we receive the law on velocity:

$$v_x(t) = v_{fx} - v_{fx} e^{-\frac{r}{m}t}$$

By reintegrating we receive the law on movement:

$$x(t) = x_0 - \frac{m}{r} v_{fx} + v_{fx} t + \frac{m}{r} v_{fx} e^{-\frac{r}{m}t}$$

Let us now reverse to the direction of time. The above laws become as follows:

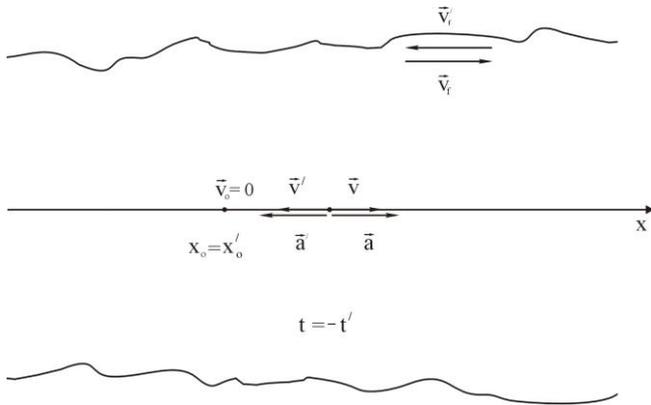


Figure 4: Inversion of time in the case when the body flowing by fluid

$$\begin{aligned} \vec{a}'(t') &= \frac{r'}{m'} |\vec{v}'_f - \vec{v}'_v(t')| \vec{e}_v = \frac{r}{m} |-\vec{v}_f - (-\vec{v}(-t))| (-\vec{e}_v) = \\ &= -\frac{r}{m} |\vec{v}(-t) - \vec{v}_f| \vec{e}_v = -\frac{r}{m} |\vec{v}_f - \vec{v}(-t)| \vec{e}_v = -\vec{a}(-t) \end{aligned}$$

which represents an odd function compared to the inversion of time;

$$\begin{aligned} v'_x(t') &= v'_{fx} - v'_{fx} e^{-\frac{r'}{m'}t'} = \\ &= -v_{fx} - (-v_{fx}) e^{-\frac{r}{m}(-t)} = -v_{fx} + v_{fx} e^{\frac{r}{m}t} \end{aligned}$$

which is neither even nor odd function (it is odd only in the initial moment ($t = 0$));

$$\begin{aligned} x'(t') &= x'_0 - \frac{m'}{r'} v'_{fx} + v'_{fx} t' + \frac{m'}{r'} v'_{fx} e^{-\frac{r'}{m'}t'} = \\ &= x_0 - \frac{m}{r} (-v_{fx}) + (-v_{fx})(-t) + \frac{m}{r} (-v_{fx}) e^{-\frac{r}{m}(-t)} = \\ &= x_0 + \frac{m}{r} v_{fx} + v_{fx} t - \frac{m}{r} v_{fx} e^{\frac{r}{m}t} \end{aligned}$$

which also is neither even nor odd function compared to the inversion of time (it becomes even only at $v_{fx} = 0$ at the moment of time $t = 0$).

Inverted movement should be decelerated in the opposite direction to the axis Ox with acceleration in the same direction as the velocity, but such movement is impossible. Or this movement is also fully irreversible over time.

Other examples can be given of fully irreversible movements with law of the motion:

$$x(t) = \sqrt[n]{At}$$

where $n = 2, 3, \dots$, and A is a constant, connecting with the measure units: $A = [m^p/s]$, but because of the unknown physical character of the forces leading to such movements, they will be not considered here by us.

General case of completely irreversible movements

The main conclusion that can be made from these examples is that, when the acceleration is an odd function compared to the inversion of time (\vec{a} has always had a direction parallel to the direction of the velocity), then the respective movement is fully irreversible over time.

All that remains in force when the movement is curvilinear as well, as in contrast to the normal acceleration (which is always even function compared to the inversion of time), the tangential acceleration remains odd function.

The reason for the irreversibility of such movements is that they become under the influence of dissipative or cumulative forces which always dissipate the energy of the moving object in the environment or vice versa. The source of these forces is any point of the environment in which the body moves, in contrast to all other forces, the source of which is localized in space and does not change in the case of inversion of time. Moreover, the presence of such forces at presence of fully reversible or semi-reversible movements radically alter their nature - they become completely irreversible. For example, to consider the constant straight line motion (which otherwise is completely reversible) with dry friction at sliding. Such movement accomplishes through the action of force \vec{F}_0 and force of friction \vec{F}_T , whose amount is always: $\vec{F}_0 + \vec{F}_T = 0$. Acceleration of the point in this movement is: $\vec{a}(t) = \vec{a}_0(t) + \vec{a}_T(t) = 0$. Inversion in time changes acceleration as follows:

$\vec{a}'(t') = \vec{a}'_0(t') + \vec{a}'_T(t') = \vec{a}_0(-t) - \vec{a}_T(-t) \neq 0$, which leads to irregular motion - movement becomes irreversible in time.

We shall note the following fact: the irreversible movements can still be carried out at the inversion of time, but this requires the presence of additional force \vec{F}_{ad} at inverted movement to compensate the irreversibility of the movement. For example, in the above case the inverted movement must be done under the action of forces:

$$\vec{F}'_{ad} + \vec{F}'_0 + \vec{F}'_T = \vec{F}_{ad} + \vec{F}_0 - \vec{F}_T = \vec{F}_{ad} - 2\vec{F}_T = 0$$

i.e. it cannot take place without external influence.

The question arises: can irreversible movements completely to be used to determine the direction of the arrow of time? Yes, because of their irreversibility they can take place only in a certain direction of the course of time. The question is what is this direction - from the past to the future or vice versa? Classical kinematics cannot answer this question. A priori here should be postulated that completely irreversible movements occur always in the direction of the arrow of time "past → future."

III. RESULTS AND DISCUSSION

We shall summarize the conclusions made here.

1. The time and movement in kinematics are inextricably linked. The very flow of time can only be described by the movement of the bodies. The rest does not allow determining whether time flows at all.
2. Reversal of course of time in kinematics means moving material point to go in reverse order through the same spatial positions with the velocity with the same size but opposite direction.
Upon inversion of time the spatial positions of the body, covered distance and acceleration (when it does not depend on speed) preserve but direction of motion and velocity change.
3. Equations in kinematics (law of motion, velocity, acceleration, etc.) are reversible when changing direction of time in case of absence of dissipative or cumulative forces. Similar results are obtained also in the papers [6-10].

4. There are three types of movements concerning the inversion of time:
 - a) Fully reversible movements - these are all constant movements. They display that the time is running out, but not determine its direction.
 - b) Semi reversible movements – they are all movements, in which the acceleration is an even function at the inversion of time. They also indicate that time is running out, but not determine its direction.
 - c) Completely irreversible movements - these are movements in which the tangential acceleration is odd function compared to the inversion of time. They can be used to determine the direction of the arrow of time.
5. Reversible and irreversible processes introduced in the thermodynamics ([11], [2], [7], [8], [9], [10]), connecting to the concept of entropy and its growth in a closed thermodynamic system. As can be shown, these processes and the entropy are associated with reversibility and irreversibility of the movement in the kinematics.
6. None movement in kinematics does not allow individual moments of time to be measured directly, but only to be determined time intervals (finite or infinitesimal). This is because the time continuum is assumed to be continuous, i.e. "quantum-time" does not exist. The same applies to the space - we can determine only the lengths (the difference between the positions of two points in it), but we are not able to determine directly the positions themselves of the points in the space. The spatial continuum is continuous and "quantum-space" does not exist as well. The above mentioned is reflected in the main measuring units of time and distance in the system SI - second and meter that are interval values, but not absolute ones.

IV. REFERENCES

- [1]. Детлаф А., Б. Яворский. *Курс физики*. Высшая школа, Москва, 1989. (Detlaf A., Iavorski B. *Kurs Fiziki*. Vissha Shkola, Moskva, 1989.)
- [2]. North J., *Time in Thermodynamics*, The Oxford Handbook of Philosophy of Time, Oxford University Press, p. 312-350, 2011.
- [3]. Zelevinsky V., *Discrete Symmetries*, Quantum Physics, v.1, p.187, 2011.
- [4]. Roberts J.A.G., Quispel G.R.W., *Chaos and Time-reversal symmetry. Order and Chaos in Reversible*

- Dynamical Systems*, Physics Reports, v.216, № 2,3, p.63-171, 1992.
- [5]. Sargsian M., *Lecture2: Classical Space-Time Symmetries*, Advanced Quantum Mechanics, 6645 (6), 2014.
- [6]. Domingos J. M., *Time Reversal in Classical and Quantum Mechanics*, International Journal of Theoretical Physics, v.18, № 3, p.213-230, 1979.
- [7]. Savitt S.F., *Is Classical Mechanics Time Reversal Invariant?*, Brit. J. Phil. Sci., v.45, p.907-913, 1994.
- [8]. Hollinger H., Zenzen M., *Time Reversal Invariance and Irreversibility*, P. REIDER Publishing Company, Tokyo, 1985.
- [9]. Snieder R., *Time-Reversal Invariance and the Relation Between Wave Chaos and Classical Chaos*, Topics Appl. Phys., v.84, p.1-16, 2002.
- [10]. Haddad W.M., Chellaboina V.S., Nersesov S.G., *Time-reversal symmetry, Poincare recurrence, irreversibility, and the entropic arrow of time: From mechanics to system thermodynamics*, Nonlinear analysis: Real World Applications, v.9, p250-271, 2008.
- [11]. Надолйски М., З. Пейков, *Учебник по физика*, УАСГ, София, 2011. (Nadoliiski M., Peykov Z. *Physics Textbook*. UACG, Sofia, 2011.)