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Study of Fluid Flow Using Matrix Analysis

D. R. Patil[1], A. P. Bhadane[2], M. S. Punia[3]

[1]Research Scholar, JJT University, ZunZun, Rajasthan, and Assistant Professor, Sangamner College, Sangamner, Maharashtra, India

[2]Associate Professor and Head, Dept. of Mathematics Panchavati College, Nashik, Maharashtra, India

[3]Associate Professor and Dean, Faculty of Science, JJT University, ZunZun, Rajasthan, India

ABSTRACT

Fluids are either gases or liquids. Mathematical methods in Chemistry are one of the core subjects in applied mathematics. Fluid dynamics is one of important branch of Mathematical Chemistry deals with study of motion of fluids i.e. liquid, gases and vapors. Fluid flows as a continuum. It is considered as a single entity whiles it in motion. Electricity, heat transfer, light prorogate are various forms of energy are some of the important examples of fluid flow. The all equations of motions of fluid are very important to study and it is important to note that all these equations are expressed in mathematical tools such as differential equations, nonlinear equations partial differential equations. In present paper, we are converting the Nevier-Stokes equations into matrix form. The motion of fluid is studied using properties of matrix such as determinant, rank, eigen value properties are used to study path of the motion of fluid.

Keywords : Fluid Flow, Chemistry, Navier-Stokes Equations, Matrix Analysis

I. INTRODUCTION

Most of these fluid flow equations are nonlinear in nature. They are called Navier-Stokes equations as they are firstly derived by French mathematician Claude-Louis Navier (1822) and then by Englishman G. G. Stokes (1845) independently.

II. BASIC TERMINOLOGIES

In general fluid motion takes place in 3-d space. Also it involves more than one independent variable. It is important to study multivariable calculus in order to understand the equations. First consider the space R^3. It is set of all ordered 3- tuples.

i.e. $R^3 = \{(x, y, z) / x, y, z \varepsilon R\}$. It's dimention is 3. It is real vector space under component wise addition and scalar multiplication. In general $R^n = \{(x_1, x_2 ,\dots x_n) / x_1, x_2 \dots x_n \varepsilon R\}$. is n-dimentional real vector space under component wise addition and scalar multiplication. R^n is a metric space under Euclidean metric. As study is on motion of fluid in R^3, let us define some of the important terminologies in fluid dynamics in order to formulate the Navier-Stokes equations.

Fluid[1]**:** A fluid is a liquid, gas or vapor. A liquid has no shape. It will fill the container which contains it. **Ideal and rear flow**[1]**:** when there is no friction between adjacent moving particles, the flow is called an ideal flow, internal forces at section are always normal to the section. **Velocity**[1]**:** Velocity of a particle is a time derivative of displacement of the particle in a certain direction. **Density:** Mass of particle divided by it's volume is density and is denoted by ρ. In the CGS system the dimensions of ρ are g/cm^3. C**ompressible fluid:** A fluid whose pressure changes it's volume is called compressible fluid. **Acceleration**[2] : The time rate of change of velocity is called acceleration. The velocity involves both directions and magnitude. So change in either magnitude or direction of the velocity produces acceleration. **Viscosity**[3]**:** The viscosity of a fluid is a measure of it's resistance to shear or angular deformation. Consider two parallel plates large enough that edge conditions may be neglected and placed a small distance apart and assume the space between is filled with the fluid shown in

Absolute Viscosity: In the metric system, the units of absolute viscosity is the poise (1 poise = 1 dyne-sec / cm^2). **Kinematic Viscosity**[2]**:** kinematic viscosity is the ratio of viscosity to the density. That is kinematic viscosity $\nu = \frac{\mu}{\rho}$. To derive equations of motion of liquid, let us first understand some of the basic physical laws. By Newton's 2nd law[2] of motion $\sum f_B + \sum f_S = m.\ a$. where f_B and f_S are body force and surface force as total external force while right hand side represents the inertial force.

Derivation of Navier-Stokes equations:

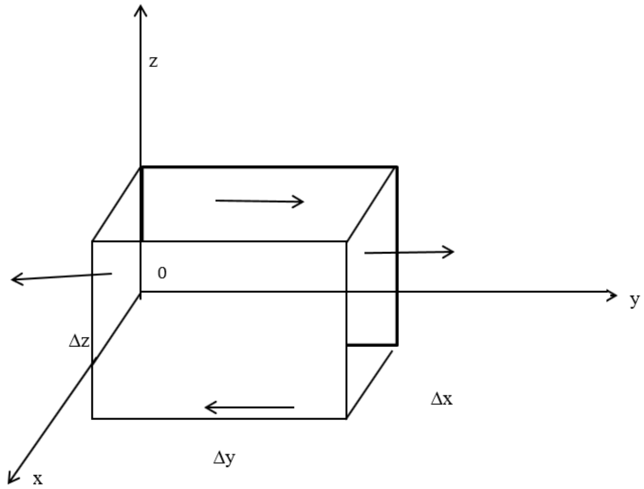

Fig 1. Dynamic of fluid element

Consider the motion of infinitesimal fluid element as shown in Fig. No. 1.3. No by Newtons laws of motion, the net resultant force on fluid element in y-directions with control volume[3] $\Delta v = \Delta x.\Delta y.\Delta z$ is equal to mass into acceleration[4] in y direction. So mathematically,

$$\sum F_{By} + \sum F_{Sy} = m.\ a_y \ldots \text{ is the momentum equation} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Body force along y direction $F_{By} = \rho.B_y.\Delta x.\Delta y.\Delta z$,

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u.\frac{\partial v}{\partial x} + v.\frac{\partial v}{\partial y} + w.\frac{\partial v}{\partial z},$$

$$\begin{aligned}\text{Also } \sum F_{Sy} &= \left(\sigma_{yy} + \frac{\partial \sigma_{yy}}{\partial y}.\frac{\Delta y}{2}\right)\Delta x.\Delta z - \left(\sigma_{yy} - \frac{\partial \sigma_{yy}}{\partial y}.\frac{\Delta y}{2}\right)\Delta x.\Delta z \\ &+ \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}.\frac{\Delta x}{2}\right)\Delta y.\Delta z - \left(\tau_{xy} - \frac{\partial \tau_{xy}}{\partial x}.\frac{\Delta x}{2}\right)\Delta y.\Delta z \\ &+ \left(\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z}.\frac{\Delta z}{2}\right)\Delta x.\Delta y - \left(\tau_{xy} - \frac{\partial \tau_{zy}}{\partial z}.\frac{\Delta z}{2}\right)\Delta x.\Delta y \\ &= 2.\ \frac{\partial \sigma_{yy}}{\partial y}.\frac{\Delta y}{2}.\Delta x.\Delta z + 2.\ \frac{\partial \tau_{xy}}{\partial y}.\frac{\Delta x}{2}.\Delta y.\Delta z + 2.\ \frac{\partial \tau_{zy}}{\partial z}.\frac{\Delta z}{2}.\Delta x.\Delta y \\ &= \left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z}\right).\Delta x.\Delta y.\Delta z.\end{aligned}$$

Aslo mass of controlled volume so chosen as $\rho.\Delta V = \rho.\Delta x.\Delta y.\Delta z$.

So equation (1) becomes

$$\left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z}\right).\Delta x.\Delta y.\Delta z + \rho.B_y.\Delta x.\Delta y.\Delta z = \rho.\Delta x.\Delta y.\Delta z\left(\frac{\partial v}{\partial t} + u.\frac{\partial v}{\partial x} + v.\frac{\partial v}{\partial y} + w.\frac{\partial v}{\partial z}\right)$$

We devide both sides by factor Δx.Δy.Δz we get,

$$\left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z}\right) + \rho.B_y = \rho.\left(\frac{\partial v}{\partial t} + u.\frac{\partial v}{\partial x} + v.\frac{\partial v}{\partial y} + w.\frac{\partial v}{\partial z}\right) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

On the left hand side first 3 terms represent surface force per unit volume while the fourth term represents body force of fluid per unit volume[4]. On the right hand side the terms is inertial force per unit volume. Similarly we can derive the equation of motion along x and z direction. So we get total 3 sets of equations are called Navier-Stokes equations.

By conservation of mass, momentum and energy and using Newton's II law we can derive Navier-Stokes Equations in R^3 as

$$\frac{\partial(\rho u_x)}{\partial t} + \frac{\partial(\rho u_x^2)}{\partial x} + \frac{\partial(\rho u_x u_y)}{\partial y} + \frac{\partial(\rho u_x u_z)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right] \quad (\text{along x-axis})$$

$$\frac{\partial(\rho u_y)}{\partial t} + \frac{\partial(\rho u_x u_y))}{\partial x} + \frac{\partial(\rho u_y^2)}{\partial y} + \frac{\partial(\rho u_y u_z)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right] \quad (\text{along y-axis})$$

$$\frac{\partial(\rho u_z)}{\partial t} + \frac{\partial(\rho u_x u_z)}{\partial x} + \frac{\partial(\rho u_y u_z)}{\partial y} + \frac{\partial(\rho u_z^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right] \quad (\text{along z-axis}).$$

These are three equations and unknown are more so for unique and smooth solution, add two more constraints 1) continuity of conservation of momentum and 2) conservation of energy equation given as below

$$\frac{\partial(\rho)}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} + + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} = 0 \text{ i.e. } \nabla.(\rho.U) = 0. \quad (\text{Continuity equation})$$

And $$\frac{\partial(E_t)}{\partial t} + \frac{\partial(\rho u_x E_t)}{\partial x} + + \frac{\partial(\rho u_y E_t)}{\partial y} + \frac{\partial(\rho u_z E_t)}{\partial z} = -\frac{\partial(u_x p)}{\partial x} - \frac{\partial(u_y p)}{\partial y} - \frac{\partial(u_z p)}{\partial z} - \frac{1}{R_{e_r} P_{r_r}}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] + \frac{1}{R_{e_r}}\left[\frac{\partial(u_x\tau_{xx} + u_y\tau_{xy} + u_z\tau_{xz})}{\partial x} + \frac{\partial(u_x\tau_{xy} + u_y\tau_{yy} + u_z\tau_{yz})}{\partial y} + \frac{\partial(u_x\tau_{xz} + u_y\tau_{yz} + u_z\tau_{zz})}{\partial z}\right] \quad (\text{Energy Equation})$$

The Naiver-Stokes equations are second ordered non-linear partial differential equations. Let U =(u_x, u_y, u_z) represents the velocity of particle S and p denotes pressure exerted by fluid on S due to it's motion. Here E_t denotes total energy, Re is Reynolds number, ρ denote the density of fluid, P_r is Prandtl number, and τ denote amount of flux transport through unit area having nine components along all three directions. Here x, y, z and t are free variables and six unknown quantities as P = pressure, ρ = density, T= temperature, u_x = velocity component along x direction, u_y = velocity component along y direction, u_z = velocity component along z direction.

Let us write the system of first four of these equations into matrix form as

$$R_{e_r}\left(\left[\frac{\partial}{\partial t} \quad \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}\right]\begin{bmatrix} \rho u_x & \rho u_y & \rho u_z & \rho \\ \rho u_x^2 + p & \rho u_x u_y & \rho u_x u_z & \rho u_x \\ \rho u_x u_y & \rho u_y^2 + p & \rho u_y u_z & \rho u_y \\ \rho u_x u_z & \rho u_y u_z & \rho u_z^2 + p & \rho u_z \end{bmatrix}\right) = \left(\left[\frac{\partial}{\partial t} \quad \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}\right]\begin{bmatrix} 0 & 0 & 0 & 0 \\ \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 \end{bmatrix}\right)$$

So the momentum matrix of the above equation is

$$\begin{bmatrix} \rho u_x & \rho u_y & \rho u_z & \rho \\ \rho u_x^2+p & \rho u_x u_y & \rho u_x u_z & \rho u_x \\ \rho u_x u_y & \rho u_y^2+p & \rho u_y u_z & \rho u_y \\ \rho u_x u_z & \rho u_y u_z & \rho u_z^2+p & \rho u_z \end{bmatrix} = \rho . \begin{bmatrix} u_x & u_y & u_z & 1 \\ u_x^2 & u_x u_y & u_x u_z & u_x \\ u_x u_y & u_y^2 & u_y u_z & u_y \\ u_x u_z & u_y u_z & u_z^2 & u_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \end{bmatrix}$$

So determinant of velocity matrix is = $\det(\rho . \begin{bmatrix} u_x & u_y & u_z & 1 \\ u_x^2 & u_x u_y & u_x u_z & u_x \\ u_x u_y & u_y^2 & u_y u_z & u_y \\ u_x u_z & u_y u_z & u_z^2 & u_z \end{bmatrix}) =$

$$\rho^4 . u_x . u_y . u_z \ .\det(\begin{bmatrix} u_x & u_y & u_z & 1 \\ u_x & u_y & u_z & 1 \\ u_x & u_y & u_z & 1 \\ u_x & u_y & u_z & 1 \end{bmatrix}) = 0$$

Rank of velocity matrix is 1. So the range of motion of incompressible fluid is 1 dimensional. Hence we conclude that the motion of fluid is 1 dimensional curve in R^3. If R_{e_r} is constant throughout the motion then we have

$$\left[\frac{\partial}{\partial t} \quad \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}\right] . R_{e_r} . \begin{bmatrix} \rho u_x & \rho u_y & \rho u_z & \rho \\ \rho u_x^2+p & \rho u_x u_y & \rho u_x u_z & \rho u_x \\ \rho u_x u_y & \rho u_y^2+p & \rho u_y u_z & \rho u_y \\ \rho u_x u_z & \rho u_y u_z & \rho u_z^2+p & \rho u_z \end{bmatrix} = (\left[\frac{\partial}{\partial t} \quad \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}\right] \begin{bmatrix} 0 & 0 & 0 & 0 \\ \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 \end{bmatrix}$$

$$\rightarrow \left[\frac{\partial}{\partial t} \quad \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}\right] (R_{e_r} . \begin{bmatrix} \rho u_x & \rho u_y & \rho u_z & \rho \\ \rho u_x^2+p & \rho u_x u_y & \rho u_x u_z & \rho u_x \\ \rho u_x u_y & \rho u_y^2+p & \rho u_y u_z & \rho u_y \\ \rho u_x u_z & \rho u_y u_z & \rho u_z^2+p & \rho u_z \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 \end{bmatrix}) = 0$$

$\rightarrow \nabla A = 0$ where

$$A = (R_{e_r} . \begin{bmatrix} \rho u_x & \rho u_y & \rho u_z & \rho \\ \rho u_x^2+p & \rho u_x u_y & \rho u_x u_z & \rho u_x \\ \rho u_x u_y & \rho u_y^2+p & \rho u_y u_z & \rho u_y \\ \rho u_x u_z & \rho u_y u_z & \rho u_z^2+p & \rho u_z \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 \end{bmatrix})$$

Here we also find that all the points where fluid motion takes placed are critical points at constant temperature. So the turbulence in motion can occur even if very small external force is applied. Also the divergence matrix is applied on the velocity matrix to get acceleration. So there is scope to study entire description motion of fluid i. e. extreme values, change of nature of path using Eigen values and trace of the acceleration matrix. Hassian matrix can be used to find extreme values of motion.

III. CONCLUSION

The Nevier-Stokes equations are converted into matrix form. Also all points of the motion through which the fluid is passing are critical points. So the turbulence for motion can occur even if very small force is applied. Velocity matrix has rank 1. So the motion of fluid is 1 dimensional curve in 3-space.

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