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A Bulk Queuing Model of Optional Second Phase Service with Short and Long Vacations

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ABSTRACT

We study a single server queue, two phases of services with optional second phase service following a general service time distribution. Customers arrive in Batches of variable size. Arrival follows a Poisson distribution. The first phase service is essential. Other phase is optional. After service completion, the server has the option to take long or short vacations with probability m_1 and m_2 respectively. The system subject to breakdown at random. Repair time follows exponential distribution. We derive the system size distribution and other performance indices such as average number of customers and the average waiting time in the queue and the system by employing generating function and supplementary variable techniques.

Keywords: Bulk Arrival, Optional Services, Short Vacations, Long Vacations

I. INTRODUCTION

In Queueing literature many remarkable and excellent surveys on the earlier works of vacation models have been reported by Doshi [1], Takagi [2]. Vacation queues with different vacation policies have studied by Igaki[4], Levi and Yechilai[7], Madan and Abu-Dayyeh [12]. Chodhury.G[3]studied a non Markovian queue with second optional vacation. In this paper we study a batch arrival queueing system in which two phases of service are provided. First service is essential and the second phase of service is optional. Based on vacation after the completion of service, the server is allowed to take a vacation. Here, vacation is given in two types viz. a short vacation and a long vacation. Server takes a short vacation with probability m_1 or a long vacation with probability m_2.Vacation follows general distribution. Moreover the server may subject to breakdown and a repair process is taken into consideration. By employing generating function approach and supplementary variable technique the system size distribution and other performance of the model has been derived. The organization of this paper is as follows. In Section 2,mathematical description of the model is given. In Section3, Equations governing the system for this model is framed. In section 4, Performance measures for this model is derived and in section 5, Conclusion is obtained. In section 6, References are given.

II. MATHEMATICAL DESCRIPTION OF THE MODEL

a) Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a 'first come'-first served basis. Let $\lambda c_i (i = 1,2,3 \ldots .)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t + dt)$, where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches.

b) Each customer undergoes two phases of service provided by a single server on a first come first served basis. The service time follows general(arbitrary)distribution with distribution function $S_i(s)$ and density function $s_i(s)$. Let $\mu_i(x)dx$ be the conditional probability density of service completion during the interval $(x, x + dx)$,given that the elapsed time is x,so that

$$\mu_i(x) = \frac{s_i(x)}{1 - S_i(x)} \qquad (a)$$

and therefore

$$s_i(s) = \mu_i(s) e^{-\int_0^s \mu_i(x)dx} \quad i = 1,2 \qquad (b)$$

c) As soon as a service is completed, the server may go for a short or long vacation with probability m_1 and m_2 respectively. Server's vacation time follows a general(arbitrary) distribution with distribution function K_i (x) and density function k_i(x). Let $\gamma_i(x)$be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$,so that

$$\gamma_i(x) = \frac{k_i(x)}{1 - K_i(x)} \qquad (c)$$

And, therefore

$$k_i(s) = \gamma_i(s) e^{-\int_0^s \gamma_i(x)dx} \qquad i = 1,2 \qquad (d)$$

e) On returning from vacation the server instantly starts serving the customer at the head of the queue, if any.
f) The system may breakdown at random and breakdowns are assumed to occur according to a Poisson stream with mean breakdown rate $\alpha > 0$.
g) The repair time of the server is exponentially distributed with mean 1/β .
g) Various stochastic process involved in the system are assumed to be independent of each other.

III. EQUATIONS GOVERNING THE SYSTEM

$P_n^{(i)}(x,t)$: Probability that at time t,the server is active providing service and there are $n (n \geq 0)$ customers in the queue excluding the one being served in the ' i' th phase and the elapsed service time for this customer is x.

${V_n}^e(x,t)$:Probability that at time t, the server is on short vacation with elapsed vacation time x and there are $n (n > 0)$ customers waiting in the queue for service.

${V_n}^f(x,t)$:Probability that at time t, the server is on long vacation with elapsed vacation time x and there are $n (n > 0)$ customers waiting in the queue for service.

$R_n(t)$: Probability that at time t, the server is inactive due to system breadown and the system is under repair, while there are $n (n \geq 0)$ customers in the queue.

$Q(t)$: Probability that at time t, there are no customers in the system and the server is idle but available in the system.

The queueing model is then, governed by the following set of differential-difference equations:

$$\frac{d}{dx} P_n^{(1)}(x) + (\lambda + \mu_1(x) + \alpha) P_n^{(1)}(x) = \lambda \sum_{i=1}^{n-1} C_i \, P_{n-i}(x) \qquad (1)$$

$$\frac{d}{dx} P_0^{(1)}(x) + (\lambda + \mu_1(x) + \alpha) P_n^{(1)}(x) = 0 \qquad (2)$$

$$\frac{d}{dx} P_n^{(2)}(x) + (\lambda + \mu_1(x) + \alpha) P_n^{(2)}(x) = \lambda \sum_{i=1}^{n-1} C_i \, P_{n-i}(x) \qquad (3)$$

$$\frac{d}{dx} P_0^{(2)}(x) + (\lambda + \mu_1(x) + \alpha) P_n^{(2)}(x) = 0 \qquad (4)$$

$$\frac{d}{dx} V_n^{(e)}(x) + \big(\lambda + \gamma_1(x)\big) V_n^{(e)}(x) = \lambda \sum_{i=1}^{n-1} C_i \, V_{n-i}^e(x) \qquad (5)$$

$$\frac{d}{dx} V_0^{(e)}(x) + \big(\lambda + \gamma_1(x)\big) V_n^{(e)}(x) = 0 \qquad (6)$$

$$\frac{d}{dx} V_n^{(f)}(x) + \big(\lambda + \gamma_2(x)\big) V_n^{(f)}(x) = \lambda \sum_{i=1}^{n-1} C_i \, V_{n-i}^f(x) \qquad (7)$$

$$\frac{d}{dx} V_0^{(f)}(x) + \big(\lambda + \gamma_1(x)\big) V_n^{(f)}(x) = 0 \qquad (8)$$

$$(\lambda + \beta) R_n = \lambda \sum_{i=1}^{n-1} C_i \, R_{n-i} + \alpha \int_0^\infty P_{n-1}^{(1)}(x)dx + \alpha \int_0^\infty P_{n-1}^{(2)}(x)dx \qquad (9)$$

$$(\lambda + \beta) R_0 = 0 \qquad (10)$$

$$\lambda Q = \beta R_0 + \int_0^\infty V_0^e(x)\gamma_1(x)dx + \int_0^\infty V_0^f(x)\gamma_2(x)dx + m_3 P_0^{(2)}(x)\mu_2(x)dx \qquad (11)$$

Boundary Conditions are

$$P_n^{(1)}(0) = m_3 \int_0^\infty P_{n+1}^{(2)}(x)\mu_2(x)dx + \int_0^\infty V_{n+1}^e(x)\gamma_1(x)dx + \int_0^\infty V_{n+1}^f(x)\gamma_2(x)dx + \beta R_{n+1} + \lambda C_{n+1} Q \quad n \geq 0 \qquad (12)$$

$$P_n^{(2)}(0) = s \int_0^\infty P_n^{(1)}(0)\mu_1 \qquad (12a)$$

$$V_n^e(0) = m_1 \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx \qquad (13)$$

$$V_n^f(0) = m_2 \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx \qquad (13a)$$

Queue size distribution at random epoch Probability Generating Function is defined as

$$\left.\begin{aligned} P_q^{(i)}(x,z) &= \sum_{n=0}^{\infty} z^n P_n^{(i)}(x), P_q^{(i)}(z) = \sum_{n=0}^{\infty} z^n P_n^{(i)} \quad i = 1,2 \\ V_q^e(x,z) &= \sum_{n=0}^{\infty} z^n V_n^e(x), V_q^e(x,z) = \sum_{n=0}^{\infty} z^n V_n^e \\ V_q^f(x,z) &= \sum_{n=0}^{\infty} z^n V_n^f(x), V_q^f(x,z) = \sum_{n=0}^{\infty} z^n V_n^f \\ R_q(z) &= \sum_{n=0}^{\infty} z^n R_n \\ C(z) &= \sum_{i=1}^{n=0} C_i z^i \end{aligned}\right\} \qquad (14)$$

Now multiply Equation (5) by Z^n & summing over n from 1 to ∞, we get adding to Equation (6) and using Generating function we have

$$\frac{d}{dx} P_q^{(1)}(x,z) + (\lambda - \lambda C(z) + \mu_1(x) + \alpha) P_q^{(1)}(x,z) = 0 \qquad (15)$$

Similarly,

$$\frac{d}{dx} P_q^{(2)}(x,z) + (\lambda - \lambda C(z) + \mu_2(x) + \alpha) P_q^{(2)}(x,z) = 0 \qquad (15a)$$

$$\frac{d}{dx} V_q^{(e)}(x,z) + (\lambda - \lambda C(z) + \gamma_1(x) + \alpha) V_q^{(e)}(x,z) = 0 \qquad (16)$$

$$\frac{d}{dx} V_q^{(f)}(x,z) + (\lambda - \lambda C(z) + \gamma_2(x) + \alpha) V_q^{(f)}(x,z) = 0 \qquad (16a)$$

$$(\lambda - \lambda C(z) + \beta) R_q(z) = \alpha z \left[\int_0^\infty P_q^{(1)}(x,z)dx + \int_0^\infty P_q^{(2)}(x,z)dx\right] \quad (17)$$

Following the same process for boundary conditions, we get

$$Z P_q^{(1)}(0,z) = \int_0^\infty P_q^{(2)}(x,z)\,\mu_2(x)dx + \int_0^\infty V_q^{(e)}(x,z)\,\gamma_1(x)dx + \int_0^\infty V_q^{(f)}(x,z)\,\gamma_2(x)dx - \left([1 - m_1]\int_0^\infty V_0^e(x)\gamma_1(x)dx + [1 - m_2]\int_0^\infty V_0^f(x)\gamma_2(x)dx + m_3 \int_0^\infty P_0^{(2)}\mu_2(x)dx\right) + \beta R_q(z) + \lambda C(z) Q$$

Using Equation (11), we have

$$Z P_q^{(1)}(0,z) = m_3 \int_0^\infty P_q^{(2)}(0,z)\,\mu(x)dx + \int_0^\infty V_q^e(x,z)\gamma_1(x)dx + \int_0^\infty V_q^f(x,z)\gamma_2(x)dx + \beta R_q(z) + \lambda(C(z) - 1)Q \qquad (18)$$

Similarly

$$P_q^{(2)}(0,z) = s \int_0^\infty P_q^{(1)}(0,z)\,\mu_1(x)dx \qquad (18a)$$

$$V_q^{(e)}(0,z) = m_1 \int_0^\infty P_q^{(2)}(0,z)\mu_2(x)dx \qquad (19)$$

$$V_q^{(f)}(0,z) = m_2 \int_0^\infty P_q^{(2)}(0,z)\mu_2(x)dx \qquad (19a)$$

Integrating Equation (15) from 0 to x

$$P_q^{(1)}(x,z) = P_q^{(1)}(0,z)e^{-(\lambda-\lambda C(z)+\alpha)x-\int_0^x \mu_1(t)dt}$$

Again integrating by parts, we get

$$P_q^{(1)}(z) = P_q^{(1)}(0,z)\left[\frac{1-\bar{S}_1(\lambda-\lambda C(z)+\alpha)}{\lambda-\lambda C(z)+\alpha}\right]$$

Where $\bar{S}_1$the Laplace Stieltjes is transform of the service time

Multiply both side of Equation (2) by $\mu_1(x)$ & integrating over x, we get

$$\int_0^\infty P_q^{(1)}(x,z)\mu_1(x)dx = P_q^{(1)}(0,z)\bar{S}_1(\lambda-\lambda C(z)+\alpha) \qquad (20)$$

Therefore we have

$$P_q^{(2)}(x,z) = sP_q^{(1)}(x,z)\bar{S}_1(\lambda-\lambda C(z)+\alpha)$$

Similarly applying the same process for Equation (16),

$$\int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx = s\,P_q^{(1)}(0,z)\bar{S}_1(\lambda-\lambda C(z)+\alpha)\bar{S}_2(\lambda-\lambda C(z)+\alpha) \qquad (21)$$

Also we have

$$\int_0^\infty V_q^e\ (x,z)\gamma_1(x)dx = m_1\bar{K}_1\big(\lambda-\lambda C(z)\big)s\,P_q^{(1)}(0,z)\bar{S}_1(\lambda-\lambda C(z)+\alpha)\bar{S}_2(\lambda-\lambda C(z)+\alpha) \qquad (22)$$

$$\int_0^\infty V_q^f\ (x,z)\gamma_2(x)dx = m_2sP_q^{(1)}(0,z)\bar{K}_2\big(\lambda-\lambda C(z)\big)\bar{S}_1(\lambda-\lambda C(z)+\alpha)\bar{S}_2(\lambda-\lambda C(z)+\alpha) \qquad (23)$$

Now from Equation (17), we have

$$R_q(z) = \frac{\alpha z\,P_q^{(1)}(0,z)}{(\lambda-\lambda C(z)+\alpha)}\left[\frac{1-\bar{S}_1(\lambda-\lambda C(z)+\alpha)}{\bar{S}_1(\lambda-\lambda C(z)+\alpha)}\right] + [1 - \bar{S}_2(\lambda-\lambda C(z)+\alpha)] \qquad (24)$$

Using Equation (20),(21),(22),(23) & (24) in (18) we get

$$P_q^{(1)}(0,z) = \frac{\lambda(C(z)-1)Q}{\begin{array}{c} Z-m_3s\bar{S}_1(\lambda-\lambda C(z)+\alpha)\bar{S}_2(\lambda-\lambda C(z)+\alpha) \\ -m_1\bar{K}_1\big(\lambda-\lambda C(z)\big)s\bar{S}_1(\lambda-\lambda C(z)+\alpha)\bar{S}_2(\lambda-\lambda C(z)+\alpha) \\ -m_2s\bar{K}_2\big(\lambda-\lambda C(z)\big)\bar{S}_1(\lambda-\lambda C(z)+\alpha)\bar{S}_2(\lambda-\lambda C(z)+\alpha) \\ -\alpha z\beta\frac{\left[\left(1-\bar{S}_1(\lambda-\lambda C(z)+\alpha)\right)+1-\bar{S}_2(\lambda-\lambda C(z)+\alpha)\right]}{(\lambda-\lambda C(z)+\beta)(\lambda-\lambda C(z)+\alpha)} \end{array}}$$

$$\frac{f_1(z)f_2(z)\lambda(C(z)-1)Q}{\begin{array}{c} f_1(z)f_2(z)[Z-s\bar{S}_1(f_1(z))\bar{S}_2(f_2(z))][m_3+m_1\bar{K}_1(\lambda-\lambda C(z))+m_2\bar{K}_2(\lambda-\lambda C(z))] \\ -\alpha z\beta[2-\bar{S}_1(f_1(z))-\bar{S}_2(f_1(z))] \end{array}} \qquad (25)$$

Where $f_1(z) = s+\lambda-\lambda C(z)+\alpha$; $\quad f_2(z) = \lambda-\lambda C(z)+\beta$

$$P_q^{(1)}(z) = \frac{f_2(z)\lambda(C(z)-1)Q[1-\bar{S}_1(f_1(z))]}{Dr} \qquad (26)$$

$$P_q^{(2)}(z) = \frac{s\bar{S}_1(f_1(z))f_2(z)\lambda(C(z)-1)Q[1-\bar{S}_2(f_1(z))]}{Dr} \qquad (27)$$

$$V_q^{(e)}(z) = \frac{m_1\bar{K}_1(\lambda-\lambda C(z))\bar{S}_1(f_1(z))\bar{S}_2f_2(z)sf_1(z)f_2(z)}{Dr} \qquad (28)$$

$$V_q^{(f)}(z) = \frac{m_2sQf_1(z)f_2(z)\bar{S}_1(f_1(z))\bar{S}_2f_2(z)[\bar{K}_2(\lambda-\lambda C(z))-1]}{Dr} \qquad (29)$$

$$R_q(z) = \frac{\alpha z\lambda(c(z)-1)Q[2-\bar{S}_1(f_1(z))-\bar{S}_2(f_1(z))]}{Dr} \qquad (30)$$

Let $W_q(z)$ denote the probability generating function of the queue size

$$W_q(z) = P_q^{(1)}(z)+P_q^{(2)}(z)+V_q^{(e)}(z)+V_q^{(f)}(z) + R_q(z)$$

$$\begin{aligned} W_q(1) = \lambda E(I)Q\big[\beta\big(1-\bar{S}_1(\alpha)\big)\big] & + s\beta\bar{S}_1(\alpha)[1-\bar{S}_2(\alpha)] \\ & +[m_1\alpha\beta s\bar{S}_1(\alpha)\bar{S}_2(\alpha)]E(K_1)] \\ & +[m_2\alpha\beta s\bar{S}_1(\alpha)\bar{S}_2(\alpha)]E(K_2)] \\ & +\alpha[2-\bar{S}_1(\alpha)\bar{S}_2(\alpha)] \\ & -\lambda E(I)[1-s\bar{S}_1(\alpha)\bar{S}_2(\alpha)][\alpha+\beta] \\ & +\alpha\beta[1+s\lambda E(I)[E(s_1)\bar{S}_2(\alpha)+E(s_2)\bar{S}_1(\alpha)+[m_1E(K_1)+m_2E(K_2)][\bar{S}_1(\alpha)\bar{S}_2(\alpha)]] \\ & -\alpha\beta\left\{2-[\bar{S}_1(\alpha)\bar{S}_2(\alpha)]+\lambda E(I)\begin{bmatrix} E(S_1) \\ +E(S_2)\end{bmatrix}\right\} \end{aligned} \qquad (31)$$

To find Q, we use the normalizing condition $W_q(1) + Q = 1$ for $Z = 1$ $W_q(z)$ is indeterminate. Applying L'Hopital;s rule we get

$$Q = 1 - W_q(1)$$

And hence the Utilization factor ρ can be derived. $\rho < 1$ is the stability condition for the existence of the steady state.

IV PERFORMANCE MEASURES

Let L_q denote the mean queue size.

Then $L_q = \frac{d}{dz} P_q(z)\Big|_{z=1}$

$$= \frac{D'(1)N''(1) - N'(1)D''(1)}{2(D'(1))^2} \quad (32)$$

$$D'(1) = -\lambda E(I)[1 - s\bar{S}_1(\alpha)\bar{S}_2(\alpha)][\alpha + \beta] + \alpha\beta[1 + s\lambda E(I)[\bar{S}_2(\alpha)E(S_1) + \bar{S}_1(\alpha)E(S_2)] + m_1E(K_1)\bar{S}_1(\alpha) + m_2E(K_2)\bar{S}_2(\alpha)] - \alpha\beta\{2 - [\bar{S}_1(\alpha)\bar{S}_2(\alpha)] + \lambda E(I)[E(S_1) + E(S_2)]\} \quad (33)$$

$$D''(1) = -\lambda E(I(I-1))(\alpha+\beta)\left(1 - s\bar{S}_1(\alpha)\bar{S}_2(\alpha)\right) - \lambda E(I)[-\lambda E(I) + \beta][1 - s\bar{S}_1(\alpha)\bar{S}_2(\alpha)]$$

$$\left(-\lambda E(I)\right)(\alpha+\beta)\begin{bmatrix} 1 - s[-\lambda E(I)] \\ \left(\bar{S}_1(\alpha)\bar{S}_2(\alpha) + \bar{S}_1(\alpha)E(S_2)\right) \\ -\lambda E(I)\bar{S}_1(\alpha)\bar{S}_2(\alpha) \\ \left(m_1E(K_1) + m_2E(K_2)\right) \end{bmatrix}$$

$$-\left(\lambda E(I)\right)^2 s(\alpha+\beta)\left[\left(\lambda E(I)\right)^2\alpha\beta s - \alpha\beta\left(\lambda E(I)\right)^2\right]$$

$$\left[\left(m_1E(K_1) + m_2E(K_2)\right)\right]$$

$$-\lambda E(I)(\alpha+\beta) - s\left[\left(\lambda E(I)\right)^2\right](\alpha+\beta)\bar{S}_1(\alpha)\bar{S}_2(\alpha)$$

$$\left[\left(m_1E(K_1) + m_2E(K_2)\right)\right]$$

$$+s\lambda E(I(I-1)\alpha\beta\left[\left(m_1E(K_1) + m_2E(K_2)\right)\right]\bar{S}_1(\alpha)\bar{S}_2(\alpha) - s(\lambda E(I))^2\alpha\beta\bar{S}_1(\alpha)\bar{S}_2(\alpha)\left(m_1E(K_1^2) + m_2E(K_2^2)\right)$$

$$-s(\lambda E(I))^2\alpha\beta\left[\left(m_1E(K_1) + m_2E(K_2)\right)\right][\bar{S}_1(\alpha)E(S_2) + E(S_1)\bar{S}_2(\alpha)] \quad (34)$$

$$N'(1) = sQ\begin{Bmatrix}\bar{S}_1(\alpha)\beta[1 - \bar{S}_2(\alpha)] \\ +E(I)[\bar{S}_1(\alpha)\bar{S}_2(\alpha)]\end{Bmatrix}\{\left[\left(m_1E(K_1) + m_2E(K_2)\right)\right] + \left[(m_1 + m_2)(\alpha+\beta)[\bar{S}_2(\alpha)E(S_1) + \bar{S}_1(\alpha)E(S_2)]\right]\}\} \quad (35)$$

$$N''(1) = -\left(\lambda E(I)\right)^2 Q\left(1 - \bar{S}_2(\alpha)\right) - \left(-\lambda E(I)\right)QE(S_1)\alpha - \left(-\lambda E(I)\right)SQ\bar{S}_1(\alpha)\left(1 - \bar{S}_2(\alpha)\right)$$

$$-2(\lambda E(I))^2E(S_1)\alpha sQ\left(1 - \bar{S}_2(\alpha)\right) + 2(\lambda E(I))^2\alpha sQE(S_2)\bar{S}_1(\alpha) + Q\alpha\lambda E(I)[2 - \bar{S}_1(\alpha) - \bar{S}_2(\alpha)] - E(I)\alpha Q[1 - \bar{S}_1(\alpha)] - \lambda E(I)Q[1 - \bar{S}_1(\alpha)]$$

$$+\lambda E(I)E(S_1)\alpha Q - (\lambda E(I))^2 sQ\bar{S}_1(\alpha)[1 - \bar{S}_2(\alpha)] + sQ\alpha\lambda E(I(I-1)\bar{S}_1(\alpha)\left(1 - \bar{S}_2(\alpha)\right) \quad (36)$$

The other performance measures W_q ,W and L are given by the usage of Little's formulae L = L_q+ρ , W_q= L_q/ ƛ and W = L/ ƛ

IV. CONCLUSION

In this paper, we have proposed a queueing model of two phases of service in which second phase is optional. After the completion of service, based on the need the server is allowed to take long vacation or short vacation. Server may get into a breakdown which is followed by a repair process. We derive the probability generating queue size and the other performance measures of the model defined.

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