© 2019 IJSRST | Volume 6 | Issue 4 | Print ISSN: 2395-6011 | Online ISSN: 2395-602X
Themed Section: Science and Technology
DOI : https://doi.org/10.32628/IJSRST196437

Transition Analysis Using Markov Chains In Denim Manufacturing

S. Sathyapriya[1], S. Sweka[2], S. Vishalakshi[3]
[1]Assistant Professor, Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu, India
[2,3] UG Scholar Department of Mathematics, Sri Krishna Arts and Science College, Tamil Nadu, India

ABSTRACT

The main aim of this paper is to show how markov chains models is used in the real life. Markov chains has been used in denim manufacturing, finance, marketing and in many fields. This paper particularly explores the use of markov chains in the denim manufacturing. Markov chains plays a very important role in determining transition probabilities in the products, retention probabilities in dyeing, optimum acid level in washing denim.
Keywords : Markov Chains, Probability, Equilibrium

I. INTRODUCTION

Markov chain models are a particular class of probabilities models known as Stochastic processes, in which the current state of a system depends on all of its previous states. But, in many problems, a sequence of events or experimental outcomes is considered to be independent. That is, any given event or outcome does not depend on any of the preceding events or outcomes. A more general model assumes that there is a one stage dependence of events, with each event depending on the immediately preceding event, but not on other prior events. A process (sequence) of the type is said to be a Markov chain process, Markov chain or Markov process.

The major application of Markov analysis are on the following areas

1. PERSONNEL Determining future manpower requirements of an organization taking consideration retirements, deaths, resignations, etc.
2. FINANCE Customer accounts receivable customers buying behaviour in terms of loyalty to a particular product brand, switching patterns to other brands, and market share of the company versus its competitors behaviour.
3. PRODUCTION Helpful in evaluating alternative maintenance policies, certain classes of inventory and queuing problems, inspection and replacement analysis
4. MARKETING Useful in analysing and predicting

II. METHODS AND MATERIAL

ALGORITHM IN MARKOV CHAINS

METHOD 1

PROCEDURE FOR DETERMINING TRASITION PROBABILITY

STEP 1 - Multi – Period transition probabilities

One of the purposes of Markov chain is to predict the future. Given vector of state probabilities, $R_0 = [\ P_{11}, P_{12}, \ldots P_{1n}\ ]$ and the matrix of transition probabilities, at time period n = 0 , we can determine the state probabilities at a future time. For convenience, let R_1 represents the state probabilities at time (or state) n=1 .After one execution of the experiment it can be written in terms of row matrix as :

$$R_1 = R_0 * P$$

To compute the vector of state probabilities at time (or state) 2,3,...,n, multiply the system state at time 0 with the transition matrix (P) , that is ,

$$R_2 = R_1 * P = R_0 * P^2$$
$$R_n = R_{n-1} * P = R_0 * P^n$$

The elements of the n step transition matrix $P^n = [P_{ij}^{\,n}]_{m*m}$ are obtained by repeatedly multiplying the transition matrix P by itself. In general P^n is equal to

$$P^n = P^{n-1} * P$$

Where each row i of P^n represents the state probability distribution after n transitions, given that the process starts out in state i.

STEP 2 - Transition Diagram

The transition probabilities can be represented by the transition diagram where arrows from each state indicate the possible transitions to start and their corresponding probabilities.

STEP 3 - Probability Diagram

These diagrams are used to illustrate only a limited number of transitions of a Markov chain. The number in circle represents the state at the beginning of a transition. These trees can also be used to evaluate and determine the probability that the given system will be in any particular state at any particular time, given the current state. The diagram represents two possible outcomes from an experiment with their assumed probabilities of occurrence from one step to another, along with branches (or paths) that may connect them over a period of time.

METHOD 2

STEPS OF CONSTRUCTING MATRIX OF TRANSITION PROBABILITIES

In order to illustrate the Markov chain, a problem is presented in which the states of activities are brands of products and transition probabilities represent the likelihood of customers moving from one brands to another. The steps of constructing a matrix of transition probabilities may be summarized as follows :

STEP 1 – Determine Retention Probabilities

To determine the retention Probabilities, divide the number of customers retained for the period under review by the number of customers at the beginning of the period.

STEP 2 – Determine Gains and Losses Probabilities

For those customers who switch brands, show gains and losses among the brands for completing the matrix of transition.

To convert the customers switching of brands so that all gains and losses take the form of transition probabilities, divide the number of customers that each entity has gained (or lost) by the original number of customers it served.

STEP 3 – Develop matrix of Transition Probabilities

In a matrix of transition probabilities, retentions (as calculated in step 1) are shown in as values on the main diagonal. The rows in the matrix show the retention and loss of customers while the columns show the retention and gain of customers.

METHOD 3

STEADY STATE (EQULIBRIUM) CONDITION

The Markov chain reaches the steady state condition only when following conditions are met:

The transition matrix elements remain positive from one period to the next. This is often referred to as the regular property of a Markov chain.

It is possible to go from one state to another in a finite number of steps, regardless of the present state. This is often referred to as the ergodic (or absorbing states) property of a Markov chain.

III. RESULTS AND DISCUSSION

PROCEDURE FOR DETERMINIG STEADY STATE CONDITION

STEP 1 – Formulate a state transition matrix

Develop a state transition matrix by first calculating probabilities with retentions and their gains and losses.

STEP 2 – Calculate future probable market share

The market shares for any period n is determined by using the following equation

[Market shares in period 2] = [Market share in period 1] [Transition matrix]

[Market shares in period 3] = [Market share in period 2] [Transition Matrix]

[Market share in period n] = [Market share in period n-1] [Transition Matrix]

In general, once a steady state is reached, multiplication of a state condition by the transition probabilities does not change the state condition . That is

$$p^n = p^{n-1} * p$$

For any value of n after a steady state is reached.

STEP – 3 Determine steady state condition

The steady state condition can be determined by the use of matrix algebra and the solution of a set of simultaneous equations obtained using the above equation

PROBLEM 1

The company manufactures two types of products A and B which are competing in the market.

A product – Weaved Denim

B product – Knitted Denim

The customers using A product have high degree loyalty and reliable price use A product **70 percent** of time .But , customers using B product because of it's high price and maintenance cost switches to A product **40 percent** of time.

a. Construct state transition matrix of

1. Retention and loss
2. Retention and gain

b. Calculate the probability of market share of A and B products.

SOLUTION

a) Let us consider,

The transition probabilities can be arranged in the matrix.

The probability of a customer's purchase at next step n=1 (next purchase) depends on the products having now at step n=0 (present purchase).

Each probability in the matrix be a conditional probability for going from one product to another.

Next purchase (n=1)

$$\text{Present purchase } (n=0) \quad \begin{pmatrix} 0.70 & 0.30 \\ 0.40 & 0.60 \end{pmatrix}$$

RETENTION AND GAIN

Algebrically, conditional probabilities in the transition matrix can be stated as

1. $P(A_0/A_1) = p_{21} = 0.70$

Probability of customers using A product in n=0 (present purchase) will buy A product in n=1 (next purchase).

It is retention to A product.

2. $P(B_0/A_1) = p_{21} = 0.40$

Probability of customers using B product in n=0 (present purchase) will buy A product in n=1(next purchase)

It is loss to B product .

3. $P(A_0/B_1) = p_{12} = 0.30$

Probability of customers using A product in n=0(present purchase) will buy B product in n=1(next purchase)

It is loss to A product.

4. $P(B_0/B_1) = p_{22} = 0.60$

Probability of customers using B product in n=0(present purchase) will buy B product in n=1(next purchase)

It is retention to B product.

b) The possible outcomes at n=1 is given by The transition probability from one state s_i = A at n=0 to another state s_j = B at n=2.

PROBABILITY CALCULATIONS

If we start with a customer's purchase of A product in state s_1 at n=0,then,

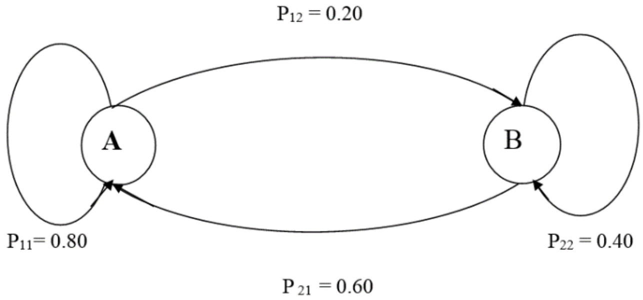
P_{11} = 1 and P_{12} = 0,

So, the value will be $R_0 = [1 \quad 0]$

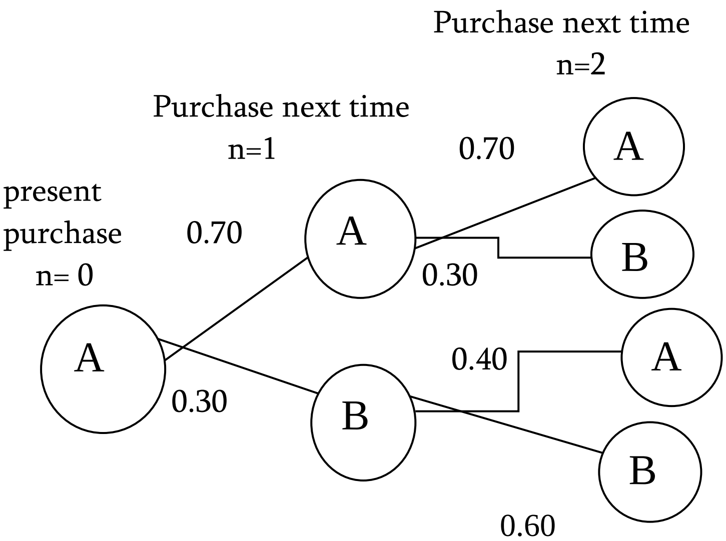
After the first transition, the state probabilities R which describes all

$$R_1 = R_0 * P = [1 \quad 0] \begin{pmatrix} 0.70 & 0.30 \\ 0.40 & 060 \end{pmatrix}$$

$R_1 = [0.70 \;\; 0.30]$



TRANSITION DIAGRAM

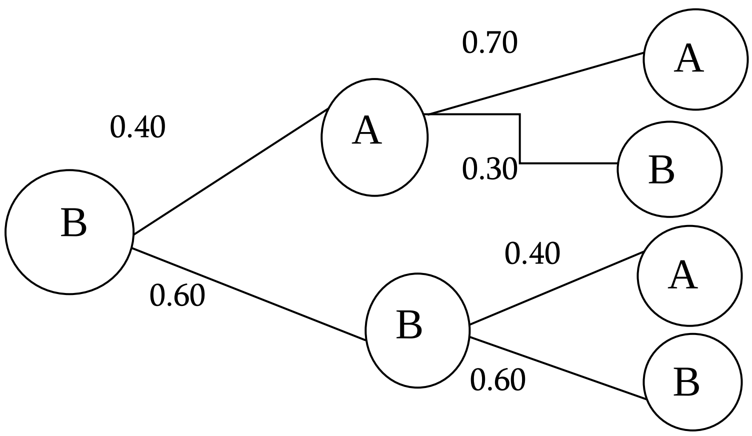


Joint probabilities

P_{11} = 0.70 (0.70) + 0.30 (0.40)
=0.49 + 0.12
=0.61

P_{12} = 0.70(0.30) + 0.30 (0.60)
= 0.21 + 0.18
= 0.39

PROBABILITY TREE DIAGRAM



P_{21} = 0.40 (0.70) + 0.60(0.40)
= 0.28 +0.24
= 0.52

P_{22} = 0.40(0.30) + 0.60(0.60)
= 0.12 + 0.36
= 0.48

The probability of the customers using A product at the end of state 1 is **70 percent** and there are **30 percent** chances that the customers will switch to B product at end of period 1.

The probability of customer using A product in state s at n=0 also use A product in the state s at n=2 can be obtained by calculating row vector of state probabilities in step n=2.

$$R_2 = R_1 * P$$

$$= [0.70 \quad 0.30] \begin{pmatrix} 0.70 & 0.30 \\ 0.40 & 0.60 \end{pmatrix}$$

$$= [0.61 \quad 0.39]$$

Hence , the probability of market share at the end of two periods

A product – **61 percent**

B product – **39 percent**

PROBLEM 2

The three types of cloth samples are send through the dyeing machines. Assume that the total quantity of cotton is **2000** which is distributed to the cloth samples A,B,C.

A – Denim B – Poly Denim C – Non Denim

It is seen that the dye used for the samples for the betterment of color, in which some are observed by the samples few are left over. The following table illustrates the pigmentation of the samples before and after dyeing.

SAMPLES	BEFORE DYEING	CHANGES DURING THE PERIOD		AFTER DYEING
		GAIN	LOSS	
A	85	33	21	97
B	45	26	33	38
C	70	16	21	65
TOTAL	200	75	75	200

Analyze the tansition probabilities in dyeing process. Determine the retention probabilities and gain and loss probabilities for the dyeing process.

SOLUTION

DETERMINE RETENTION PROBABILITIES

To determine the retention probability of dye retained in the cloth samples.

RETENTION PROBABILITIES

CLOTH SAMPLES	BEFORE DYEING	NUMBER LOST	NUMBER RETAINED	PROBABILITY OF RETENTION
A	85	21	64	64/85 =0.75
B	45	33	12	12/45=0.26
C	70	21	49	49/70=0.70

DETERMINE GAINS AND LOSSES PROBABILITIES

In order to calculate the rate at which the three samples gains dye during each period, not only net gain or net loss but the interrelationships among the gains and losses of dyes by each cloth samples.

CLOTH SAMPLES	BEFORE DYEING	GAINS FROM			LOSSES TO			AFTER DYEING
		A	B	C	A	B	C	
A	85	0	17	16	0	9	12	97
B	45	12	0	14	16	0	17	38
C	70	4	4	0	10	11	0	65

In the matrix of transition probabilities we have included for each sample the retention probability and the probability of loss of dyes.

The rows in the matrix show the retention and loss of customers while the columns show the retention and gain of customers.

MATRIX OF TRANSITION PROBABILITIES

$$\begin{pmatrix} 64/85=0.75 & 9/85=0.10 & 12/85=0.14 \\ 16/45=0.35 & 12/45=0.26 & 17/45=0.37 \\ 10/70=0.14 & 11/70=0.15 & 49/70=0.70 \end{pmatrix}$$

The row of matrix are interpreted as follows:

The first row shows the sample A retains 75% of its own dye, loses 10% of dye to sample B and loses 14% of dye to sample C.

The second row shows the sample B loses 35% of its dye to sample A, retains 26% of its own dye and loses 37% of its dye to sample C.

The third row show the sample C loses 14% of its dye to sample A loses 15% of its dye to sample B and retains 70% of its own dye.

The column of matrix are interpreted as follows :

The first column shows the sample A retains 75% of its own dye and gains 35% of sample B and gains 14% of sample C.

The second column shows the sample B gains 10% dye of sample A, retains 26% of its own dye and gains 15% of sample C

The third row shows the sample C gains 14% dye of sample A ,gains 37% of sample B and retains 70% of its own dye.

PROBLEM 3

The company has various effects on washing process. The table shows the acid level in the washing process. This acid leads to pollution.

BOD	COD	TDS
0.40	0.30	0.28
0.30	0.25	0.38
0.25	0.38	0.24

The present proportion of pollution are 4.0 , 4.3, 4.5

a) what will be the proportion of pollution after 3 years

b) Determine the steady state solution or equilibrium solution

SOLUTION

DETERMINE PROPORTION

CALCULATION OF FIRST YEAR

$$[4.0 \quad 4.3 \quad 4.5] \quad 0.40 \begin{pmatrix} 0.30 & 0.28 & \\ 0.30 & 0.25 & 0.38 \\ 0.25 & 0.38 & 0.26 \end{pmatrix} = [\,3.9 \quad 3.9 \quad 3.9\,]$$

Present proportion transition matrix Proportion after one year

CALCULATION OF SECOND YEAR

$$[3.9 \quad 3.9 \quad 3.9] \begin{pmatrix} 0.40 & 0.30 & 0.28 \\ 0.30 & 0.25 & 0.38 \\ 0.25 & 0.38 & 0.26 \end{pmatrix} = [\,3.5 \quad 3.3 \quad 3.3\,]$$

Proportion after one year transition matrix Proportion after second year

CALCULATION OF THIRD YEAR

$$[3.5 \quad 3.3 \quad 3.3] \begin{pmatrix} 0.40 & 0.30 & 0.28 \\ 0.30 & 0.25 & 0.38 \\ 0.25 & 0.38 & 0.26 \end{pmatrix} = [\,3.2 \quad 3.0 \quad 3.0\,]$$

Proportion after second year transition matrix Proportion after third year

DETERMINE STEADY STATE SOLUTION In order to calculate the percentage of acid level in each washing process at equilibrium, consider again state transition matrix .The steady- state condition for acid level in washing process may be expressed as :

$$[\,S_A \quad S_B \quad S_C\,] = [\,S_A \quad S_B \quad S_C\,] \begin{pmatrix} 0.40 & 0.30 & 0.20 \\ 0.30 & 0.25 & 0.38 \\ 0.25 & 0.38 & 0.24 \end{pmatrix}$$

Period n period n-1 transition matrix

The acid level in the period n-1 are the same as the shares in the period n because a steady- state condition has been reached.

$S_A = 0.40\, S_A + 0.30\, S_B + 0.20\, S_C$

$S_B = 0.30\, S_A + 0.25\, S_B + 0.38\, S_C$

$S_C = 0.25\, S_A + 0.38\, S_B + 0.24\, S_C$

$S_A + S_B + S_C = 1$

The last equation shows that the acid level must total unity.

$S_A = 0.23$ $S_B = 0.31$ $S_C = 0.39$

Hence , the solution for equilibrium or steady state acid level are

BOD = 23 % COD = 31 % TOS = 39 %

IV. CONCLUSION

This article bases on the Markov property of the denim manufacturing process and uses Markov model to predict the production and processing details in denim manufacturing. Thus, predicting the future production and processing of denim yearly forecast data are used. But, by Markov models we have predicated the future only by considering the present condition of the company and the market.

V. REFERENCES

[1]. W.Jun and W.Juan, "Stochastic Process and Its Financial Application", Beijing Tsinghua University Press and Beijing Jiatong University Press, (2007), pp. 96-97

[2]. P.Zhi – Hang, X. Le- Tian and S. Yong – Mei, " Application of Weighted Markov Chain in the Prediction of Year's Harvest of Crops", Mathematics in Practice and Theory, (2005), pp. 30-35

[3]. H.E. Roman and M. Porto, " Fractional Brownian motion with stochastic variance: modeling absolute returns in stock markets", International Journal of Modern Physics, C. Physics and computers, vol 19, no.8 (2008), pp.1221-1242

[4]. Q.L. Song and C. Song, "Application of Markov Chain in the Prediction of Market Economy", Journal of Business Research, vol 2, (2009) pp. 46-69

[5]. J. Yue- Zhi and S. Lei, " Markov Chain and its Application in Economic Prediction", Journal of ChangChun University of Technology, vol. 24, no. 2 (2003), pp. 26-28

[6]. J-J. Sun, "Application of Markov Method in Teaching Evaluation and Educational Prediction", Educational Research and Experiment, vol. 1,(1991), pp.369-371

Cite this article as :

S. Sathyapriya, S. Sweka, S. Vishalakshi, "Transition Analysis Using Markov Chains In Denim Manufacturing", International Journal of Scientific Research in Science and Technology (IJSRST), Online ISSN : 2395-602X, Print ISSN : 2395-6011, Volume 6 Issue 4, pp. 243-250, July-August 2019. Available at doi : https://doi.org/10.32628/IJSRST196437 Journal URL : http://ijsrst.com/IJSRST196437