

Time Dependence of Various Cosmological Parameters in the Framework of Kaluza-Klein Space-Time

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ABSTRACT

A theoretical model, regarding the time dependence of several cosmological parameters, has been constructed in the present study, in the framework of Kaluza-Klein theory, using its field equations for a spatially flat metric. Time dependent empirical expressions of the cosmological constant and the equation of state (EoS) parameter have been substituted into the field equations to determine the time dependence of various cosmological parameters. Time variations of these parameters have been shown graphically. The cosmological features obtained from this model are found to be in agreement with the observed characteristics of the accelerating universe. Interestingly, the signature flipping of the deceleration parameter, from positive to negative, is predicted by this model, indicating a transformation of the universe from a state of decelerated expansion to accelerated expansion, as obtained from astrophysical observations. Time dependence of the gravitational constant (G), energy density (ρ), cosmological constant (Λ) and the EoS parameter (ω) have been determined and depicted graphically in the present study.

Keywords: General Theory of Relativity, Kaluza-Klein Space-time, Cosmological Parameters, Dark Energy, Cosmic Acceleration, Gravitational Constant.

I. INTRODUCTION

The universe is found to be highly homogeneous and isotropic on large scales, as obtained from observations on large scale structures and Cosmic Microwave Background Radiation (CMBR) [1, 2]. On the basis of a large number of recent astrophysical observations, it has been very firmly established that the universe is presently undergoing an accelerated expansion [3-7]. An exotic form of energy, with a negative pressure, has been found to be responsible for the accelerated expansion of the universe. This energy is known as dark energy (DE). The nature of DE is yet to be determined and, despite several attempts, there is still

no satisfactory explanation of the origin of dark energy [8]. A parameter, known as cosmological constant (Λ) in General Relativity (GR), has very often been used to represent DE in theoretical calculations. It has its own shortcomings, although it has explained several experimental observations satisfactorily [9]. DE has been conventionally characterized by the equation of state (EoS) parameter ($\omega = P/\rho$), which should not be regarded as a constant. On account of insufficient observational evidence to estimate the time variation of ω , the EoS parameter has been regarded as a constant in many theoretical studies, with values -1, 0, 1/3 and +1 for vacuum fluid, dust fluid, radiation and stiff fluid dominated universe respectively [9]. In

general, the EoS parameter is a function of time or redshift [10]. Recent years have witnessed the emergence of various models on the time dependence of ω [11]. Yadav et al and Pradhan have recently studied the characteristics of a time varying EoS parameter on the basis of generalized models of DE [12, 13].

The expansion of the universe was initially believed to be governed solely by gravitational attraction among celestial bodies, which is capable of causing only decelerated expansion. The observation of the accelerated expansion of the universe became evident from the negative value of the deceleration parameter (q). In the third decade of the previous century, Kaluza and Klein attempted to unify electromagnetic force with gravitational force which resulted in the development of Kaluza-Klein (KK) theory. In KK approach, an extra dimension, i.e., a fifth dimension was introduced for coupling these two forces. Kaluza has demonstrated that the general theory of relativity (GR), when constructed as a five-dimensional theory, contains four-dimensional GR along with the existence of electromagnetic field, consistent with Maxwell's electromagnetism [14]. Kaluza proposed that GR was not actually modified, it was just extended to five-dimensions, and there is no physical dependence on the fifth dimension. Klein recommended the compactification of the fifth dimension [15]. Chodos and Detwiler have shown in their five dimensional models that the extra dimension contracts due to cosmic evolution [16]. According to Guth and the group of Alvarez and Gravela, production of a huge entropy due to the presence of an extra dimension can solve the flatness and horizon problems without invoking the idea of inflation [17, 18]. So, a five-dimensional model in the framework of KK theory has been successful in addressing some of the problematic issues of Big Bang Cosmology and the other realms of physics.

A number of works are available in scientific literature where one or both of the cosmological quantities, Λ and G , are assumed to be variables. Ray et al. have constructed models in FRW space-time with ansatzes of $\Lambda \sim H^2$ and $\Lambda \sim \ddot{a}/a$ and showed the time variation of various cosmological parameters [19]. Khadekar et al and Ozel et al have worked in the framework of KK cosmology with variable Λ and constant G [20, 21]. Sharif and Khanum and more recently Oli have worked with KK cosmological models by considering both of Λ and G to be variables [22, 23]. Mukhopadhyay et al have recently formulated a model with KK space-time having $\Lambda \sim \ddot{a}/a$. They have determined the time variations of both Λ and G [24].

In the present study we have solved the field equations obtained from Kaluza-Klein metric for a space with spatial curvature zero. For this purpose, we have used time-dependent empirical expressions for Λ and ω . The constants, connected to these expressions, have been determined from field equations. One of these constants, denoted by m , is a free parameter whose value has been varied to find the time dependence of various cosmological quantities such as ρ , G , Λ , ω etc. The nature of time variations of these quantities is in accordance with that obtained from other cosmological models that have accounted for astrophysical observations.

II. METHODS OF FORMULATION

1. Kaluza-Klein Geometry and the Field Equations

The line element for a non-flat universe in the framework of the Kaluza-Klein cosmology can be expressed as [24],

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 + dX^2 \right] \quad (1)$$

Here, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, $dX^2 = (1 - kr^2)d\psi^2$, $a(t)$ is the scale factor, k ($= -1, 0, 1$) is the curvature

parameter for spatially open, flat and closed universe respectively.

Einstein's field equations are given by,

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi GT_{ij} \quad (2)$$

Here R_{ij} , g_{ij} , R , Λ are the Ricci tensor, metric tensor, Ricci scalar and the cosmological constant respectively. The energy-momentum tensor (T_{ij}) for an universe filled with perfect fluid is given by,

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \quad (3)$$

Combining equations (1), (2) and (3) one gets the following equations.

$$8\pi G\rho + \Lambda = 6\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) \quad (4)$$

$$8\pi Gp - \Lambda = -3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) \quad (5)$$

For a flat universe (i.e., $k = 0$), equations (4) and (5) take the following forms.

$$8\pi G\rho + \Lambda = 6\frac{\dot{a}^2}{a^2} = 6H^2 \quad (6)$$

$$8\pi Gp - \Lambda = -3\dot{H} - 6H^2 \quad (7)$$

Here $H = \frac{\dot{a}}{a}$ is the Hubble parameter, p = pressure of the cosmic fluid.

In the present formulation, G and Λ are regarded as functions of time (t).

Combining equations (6) and (7) and using the equation of state, $p = \omega\rho$ we get,

$$\frac{1}{\omega} = \frac{6H^2 - \Lambda}{-3\dot{H} - 6H^2 + \Lambda} \quad (8)$$

Here ω denotes the equation of state (EoS) parameter.

2. Theoretical Model

For the present study we have used the following ansatz for the cosmological constant (Λ).

$$\Lambda = 6H^2 + \beta t^n \quad (9)$$

Where β and n are constant parameters.

Putting this expression into equation (8) and simplifying we get,

$$\frac{dH}{dt} = \frac{\beta(1+\omega)}{3} t^n \quad (10)$$

For the EoS parameter (ω) we have chosen the following ansatz.

$$\omega = At^m - 1 \quad (11)$$

Here A and m are constant parameters.

Using equation (11) in (10) we get,

$$\frac{dH}{dt} = \frac{B}{3} t^k \quad (12)$$

where, $B = A\beta$ and $k = m + n$

Solving equation (12) by putting $H = \frac{\dot{a}}{a}$ we get,

$$\ln a = \frac{B}{3(k+1)(k+2)} t^{k+2} + C_1 t + C_2 \quad (13)$$

Here C_1 and C_2 are the constants of integration.

For simplicity we have taken $C_1 = C_2 = 0$ in the present study.

Thus, from equation (13), we have obtained the following scale factor.

$$a(t) = \text{Exp} \left\{ \frac{B}{3(k+1)(k+2)} t^{k+2} \right\} \quad (14)$$

Taking $a_{t=t_0} = a_0$, we get the following expression for $a(t)$ from equation (14).

$$a(t) = a_0 \text{Exp} \left[\frac{B}{3(k+1)(k+2)} (t^{k+2} - t_0^{k+2}) \right] \quad (15)$$

The deceleration parameter $q = \frac{-\ddot{a}a}{\dot{a}^2}$, obtained from equation (15), is given by,

$$q(t) = -\left[1 + \frac{3(k+1)^2}{Bt^{k+2}}\right] \quad (16)$$

Using equations (9) and (12), H and Λ can be expressed as,

$$H(t) = \frac{B}{3(k+1)} t^{k+1} \quad (17)$$

$$\Lambda(t) = \frac{2B^2}{3(k+1)^2} t^{2k+2} + \beta t^n \quad (18)$$

In deriving equation (17) from (12), we have taken the integration constant to be zero.

The continuity equation for the cosmic constituents is given by,

$$\dot{\rho} + 4H(p + \rho) = 0 \quad (19)$$

Using equation (11), the barotropic equation of state is given by,

$$p = \omega\rho = \rho(At^m - 1) \quad (20)$$

Where, p denotes the pressure of the cosmic fluid, ρ is the energy density and ω denotes the equation of state (EoS) parameter.

Using equation (20) in (19) we get the following differential equation.

$$\dot{\rho} + 4H\rho At^m = 0 \quad (21)$$

Solving the above differential equation we get,

$$\rho = C \text{Exp}\left[\frac{-4AB}{3(k+1)(m+k+2)} t^{m+k+2}\right] \quad (22)$$

Here C is the constant of integration.

Using the condition that $\rho = \rho_0$ for $t = t_0$ we can write,

$$\rho = \rho_0 \text{Exp}\left[D(t^{m+k+2} - t_0^{m+k+2})\right] \quad (23)$$

where $D = \frac{-4AB}{3(k+1)(m+k+2)}$

Equation (23) represents mathematically the time dependence of the energy density (ρ).

Using the present model, we have also studied the time dependence of the gravitational constant (G) in the following way.

Using equations (6) and (9) we get,

$$G = \frac{6H^2 - \Lambda}{8\pi\rho} = \frac{-\beta t^n}{6\pi\rho} \quad (24)$$

Using equation (23) in (24) we get,

$$G(t) = -\frac{\beta t^{k-m}}{8\pi\rho_0} \text{Exp}\left[D(t_0^{m+k+2} - t^{m+k+2})\right] \quad (25)$$

where $D = \frac{-4AB}{3(k+1)(m+k+2)}$

Here we have used the relation $k = m + n$. Equation (25) determines the time dependence of the gravitational constant. Let G_0 denote the present value of the gravitational constant. Using $G_{t=t_0} = G_0$ in equation (25), we get,

$$\beta = -8\pi G_0 \rho_0 t_0^{(m-k)} \quad (26)$$

Equation (26) clearly indicates that the constant parameter β has negative values.

Using the relations $q_{t=t_0} = q_0$ and $H_{t=t_0} = H_0$ in equations (16) and (17) respectively, we have obtained the following expression for the constant parameter k from them.

$$k = m + n = -1 - H_0 t_0 (1 + q_0) \quad (27)$$

Using equation (27) in (26), one obtains,

$$\beta = -8\pi G_0 \rho_0 t_0^{1+m+H_0 t_0(1+q_0)} \quad (28)$$

$$\frac{\dot{G}}{G} = \frac{3(1+k)(k-m)+4ABt^{2+k+m}}{3(1+k)t} \quad (35)$$

Using equation (17) we get,

$$B = 3H_0(k+1)t_0^{-k-1} \quad (29)$$

Using equations (26) and (29), the expression for the constant A is written as,

$$A = \frac{B}{\beta} = -\frac{3H_0(k+1)t_0^{-1-m}}{8\pi G_0 \rho_0} \quad (30)$$

Using equations (11) and (30), the expression for the EoS parameter takes the following form.

$$\omega = -\frac{3H_0(k+1)}{8\pi G_0 \rho_0 t_0} \left(\frac{t}{t_0}\right)^m - 1 \quad (31)$$

Its value at the present epoch (i.e. at $t = t_0$) is therefore given by,

$$\omega_0 = -\frac{3H_0(k+1)}{8\pi G_0 \rho_0 t_0} - 1 \quad (32)$$

Combining equations (31) and (32) we can express ω as,

$$\omega = (\omega_0 + 1) \left(\frac{t}{t_0}\right)^m - 1 \quad (33)$$

Using equation (11) in (18), we get,

$$\Lambda = \frac{2B^2}{3(k+1)^2} \left(\frac{\omega+1}{A}\right)^{\frac{2(k+1)}{m}} + \beta \left(\frac{\omega+1}{A}\right)^{\frac{k-m}{m}} \quad (34)$$

Equation (34) is an expression for the cosmological constant (Λ), written as a function of the EoS parameter (ω). Here we have used the relation $k = m + n$.

Using equation (25) we get the following expression for \dot{G}/G .

Using equation (35), one gets following expression for the value of \dot{G}/G at the present time (i.e., $t = t_0$).

$$\left(\frac{\dot{G}}{G}\right)_{t=t_0} = \frac{3(1+k)(k-m)+4ABt_0^{2+k+m}}{3(1+k)t_0} \quad (36)$$

The value of the parameter k is obtained from equation (27). Thus, the time dependence of all cosmological quantities (except a, H & q) discussed above, depends on the parameter m .

For the present study, we have used the following values of cosmological parameters.

$$H_0 = 72 \text{ km sec}^{-1} \text{ Mpc}^{-1} = 2.33 \times 10^{-18} \text{ s}^{-1}, \quad t_0 = 14 \text{ Gyr} = 4.415 \times 10^{17} \text{ s}, \quad q_0 = -0.55, \\ \rho_0 = 9.9 \times 10^{-27} \text{ Kg m}^{-3}, \quad G_0 = 6.674 \times 10^{-11} \text{ Nm}^2 \text{ Kg}^{-2}.$$

Using these values we have found that $k = -1.463$, $B = -4.765 \times 10^{-10}$ and $\omega_0 = -0.558$.

III. RESULTS AND DISCUSSION

FIGURES

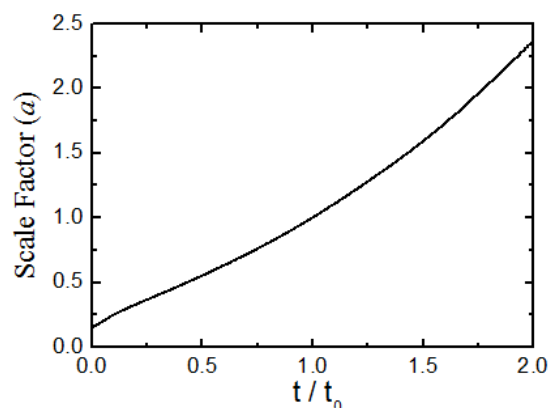


Figure 1: Variation of the scale factor as a function of time.

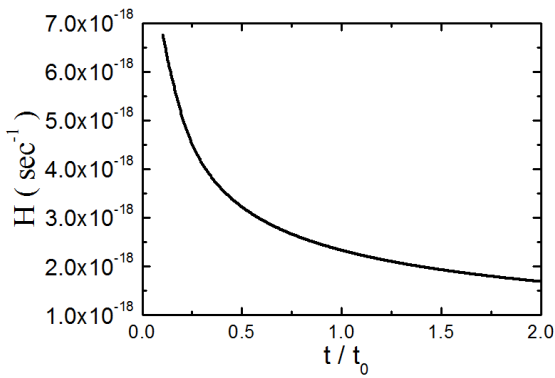


Figure 2: Variation of the Hubble parameter as a function of time

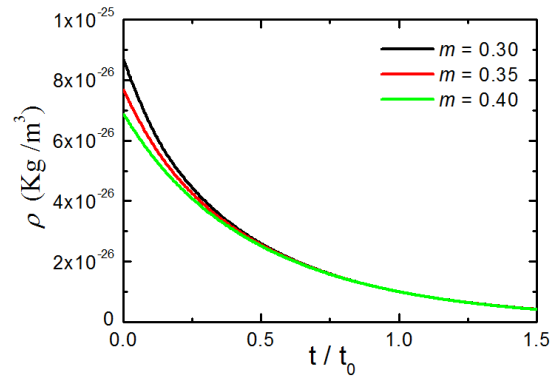


Figure 5: Variation of the energy density as a function of time for three positive values of the parameter m .

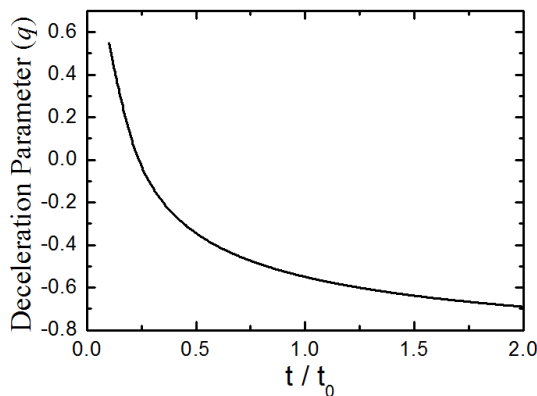


Figure 3: Variation of the deceleration parameter as a function of time.

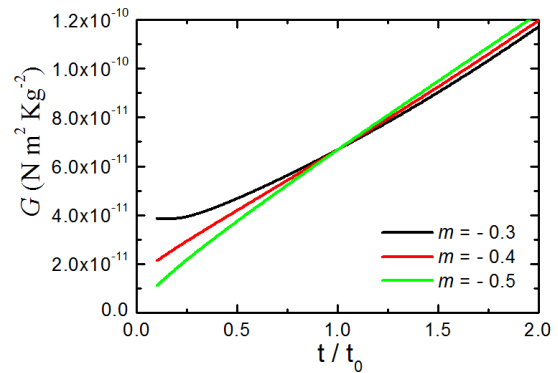


Figure 6: Variation of the gravitational constant as a function of time for three negative values of the parameter m .

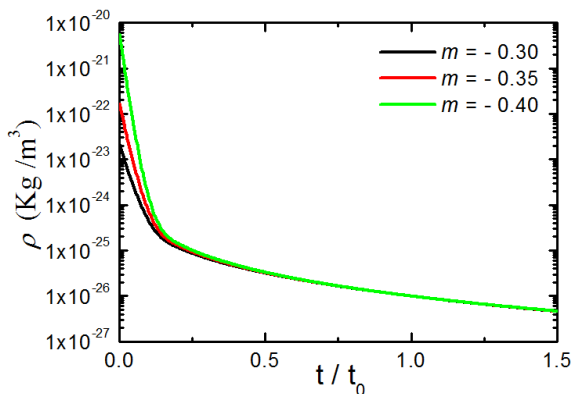


Figure 4: Variation of the energy density as a function of time for three negative values of the parameter m .

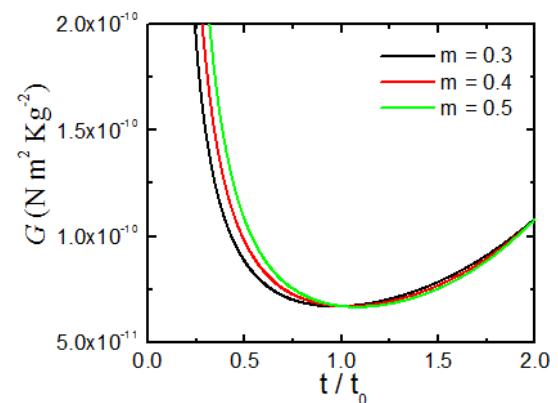


Figure 7: Variation of the gravitational constant as a function of time for three positive values of the parameter m .

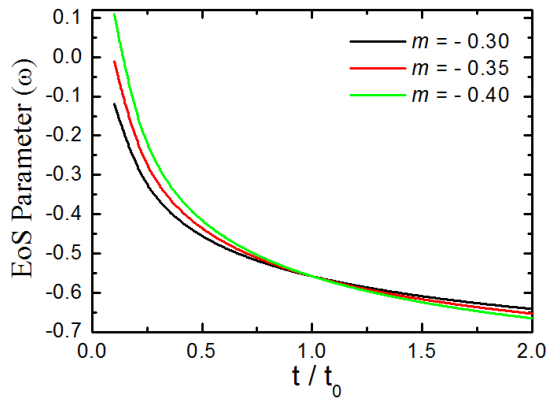


Figure 8: Variation of the EoS parameter as a function of time for three negative values of the parameter m .

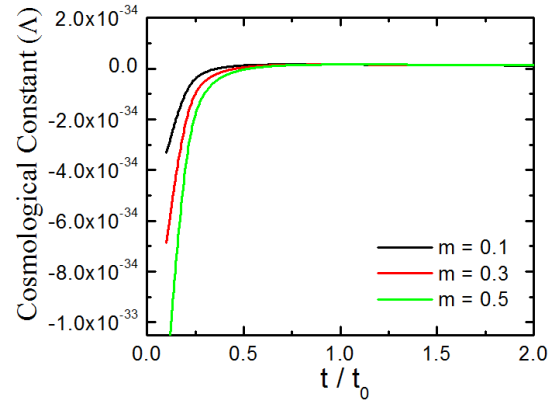


Figure 11: Variation of the cosmological constant as a function of time for three positive values of the parameter m .

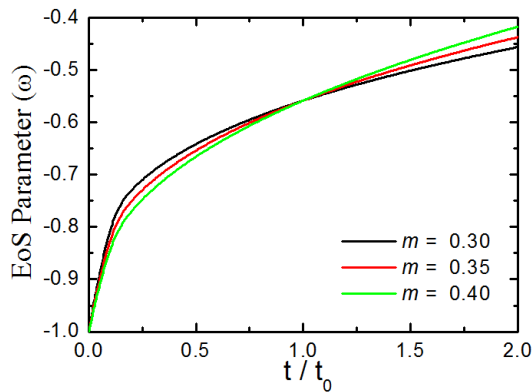


Figure 9: Variation of the EoS parameter as a function of time for three positive values of the parameter m .

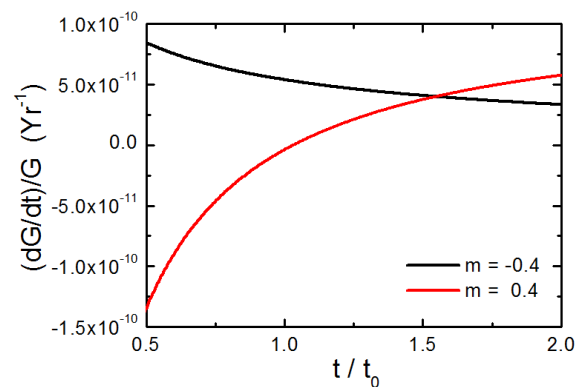


Figure 12: Variation of \dot{G}/G as a function of time for a negative and a positive value of the parameter m .

Graphical Depiction of Theoretical Findings

Figure 1 shows the variation of the scale factor as a function of time. It increases with time, indicating clearly the expansion of the universe with time.

Figure 2 shows the variation of the Hubble parameter as a function of time. It decreases with time with a gradually slower rate of change.

Figure 3 shows the variation of the deceleration parameter as a function of time. It shows a change of sign from positive to negative, indicating a transition from a phase of decelerated expansion to accelerated expansion. This transition took place around the time of $0.25t_0$, as evident from this plot.

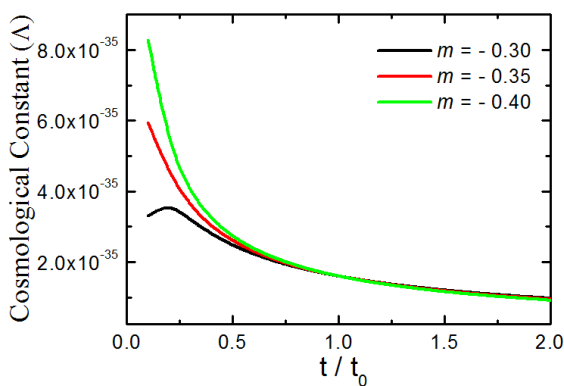


Figure 10: Variation of the cosmological constant as a function of time for three negative values of the parameter m .

Figure 4 shows the variation of the energy density as a function of time for three negative values of the parameter m . It decreases with time with a gradually decreasing slope. Figure 5 shows the variation of the energy density as a function of time for three positive values of the parameter m . It decreases with time at a gradually decreasing rate. Time variations of energy density, as evident from the Figures 4 and 5, are similar to those obtained from some other recent studies [12, 13, 24-28].

Figure 6 shows the variation of the gravitational constant as a function of time for three negative values of the parameter m . It increases with time. Figure 7 shows the variation of the gravitational constant as a function of time for three positive values of the parameter m . Here, G initially decreases and subsequently increases with time. There are cosmological models where G has been shown to be increasing with time [28, 29]. There are models where this parameter is found to be a decreasing function of time [19, 26].

Figure 8 shows the variation of the EoS parameter as a function of time for three negative values of the parameter m . It becomes more negative with time. Figure 9 shows the variation of the EoS parameter as a function of time for three positive values of the parameter m . It becomes less negative with time. These behaviours are similar to those obtained from some recent studies mentioned in the list of references [12, 13, 27, 30].

Figure 10 shows the variation of the cosmological constant as a function of time for three negative values of the parameter m . It decreases with time, remaining positive throughout the span we have considered. Figure 11 shows the variation of the cosmological constant as a function of time for three positive values of the parameter m . It increases with time. Rising sharply from a negative value, it becomes asymptotic to a value closer to zero. Such behaviours are similar to those obtained from some recent studies mentioned in our list of references [13, 24, 26, 28].

Figure 12 shows the variation of \dot{G}/G as a function of time for a negative and a positive value of the parameter m . For these two cases, it respectively decreases and increases with time. Its value at the present epoch (i.e., $t = t_0$) is found to be $5.39 \times 10^{-11} \text{ Yr}^{-1}$ and $-2.23 \times 10^{-12} \text{ Yr}^{-1}$ for these two cases respectively. These values are quite consistent with those obtained by Ray et al from their model and several other studies [19].

IV. CONCLUDING REMARKS

The present study is based on a spatially flat, homogeneous and isotropic universe, in the framework of Kaluza-Klein space-time. Here we have studied the time evolution of various cosmological parameters such as the scale factor, Hubble parameter, deceleration parameter, energy density, gravitational constant, equation of state (EoS) parameter, cosmological constant etc. The time dependence of \dot{G}/G has been determined and shown graphically. This formulation is based on two ansatzes, expressed by equations (9) and (11), regarding the cosmological constant (Λ) and the equation of state (EoS) parameter (ω) respectively. The solution of the field equations has led to a deceleration parameters that shows a change of sign from positive to negative, indicating a transition from deceleration to acceleration, in accordance with astrophysical observations [3, 4]. It has been found here that the characteristics of time variation of some cosmological parameters depend solely upon a constant parameter, denoted by m . The time evolution of the scale factor, Hubble parameter and deceleration parameter has no dependence upon this constant. A completely new feature, regarding the time dependence of G , has been observed for positive values of the parameter m . For these values, G shows an initial fall with time, followed by a rise with a smaller rate of change. Time variations of all cosmological quantities have been shown graphically by plotting them with respect to a dimensionless cosmological

time (t/t_0). As a future plan for an extension of this study, several types of ansatzes can be chosen for Λ and ω , to formulate new models from which one can get the time dependence of the cosmological parameters which are sufficiently consistent with recent astrophysical observations.

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VI. REFERENCES

- [1]. M. Tegmark et al., 2004. Cosmological parameters from SDSS and WMAP. *Phys. Rev. D*, 69: 103501, 1-26.
- [2]. D. N. Spergel et al., 2003. First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological parameters. *Astrophys. J. Suppl. Ser.*, 148: 175-194.
- [3]. A. G. Riess et al., 1998. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astron. J.*, 116: 1009-1038.
- [4]. A. G. Riess et al., 2001. The farthest known supernova: support for an accelerating universe and a glimpse of the epoch of deceleration. *Astrophys. J.*, 560(1): 49-71.
- [5]. C. L. Bennett et al., 2003. First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results. *Astrophys. J. Suppl.*, 148: 1-27.
- [6]. E. Komatsu et al., 2011. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. *Astrophys. J. Suppl.*, 192(18): 1-47.
- [7]. W. J. Percival et al., 2010. Baryon Acoustic Oscillations in the Sloan Digital Sky Survey Data Release 7 Galaxy Sample. *Mon. Not. Roy. Astron. Soc.*, 401: 2148-2168.
- [8]. T. Padmanabhan and T. R. Choudhury, 2003. A theoretician's analysis of the supernova data and the limitations in determining the nature of dark energy. *Mon. Not. R. Astron. Soc.*, 344: 823-834.
- [9]. V. Sahni and A. Starobinsky, 2000. The case for a positive cosmological Λ term. *Int. J. Mod. Phys. D*, 9(4): 373-443.
- [10]. R. Jimenez, 2003. The value of the equation of state of dark energy. *New. Astron. Rev.*, 47: 761-767.
- [11]. U. Mukhopadhyay et al., 2008. Lambda-CDM universe: A Phenomenological Approach with Many Possibilities. *Int. J. Mod. Phys. D*, 17(2): 301-309.
- [12]. Yadav et al., 2011. Dark Energy Models with Variable Equation of State Parameter. *Int. J. Theor. Phys.*, 50: 871-881.
- [13]. A. Pradhan, 2013. Accelerating Dark Energy Models with Anisotropic Fluid in Bianchi Type-VI₀ Space-Time. *Research in Astron. Astrophys.*, 13(2): 139-158.
- [14]. T. Kaluza, 2018. On the Unification problem in Physics. *Int. J. Mod. Phys. D*, 27 (14): 1870001, 1-7.
- [15]. O. Klein, 1926. Quantentheorie und fünfdimensionale Relativitätstheorie. *Zeitschrift für Physik*, 37 (12): 895-906.
- [16]. A. Chodos and S. Detweiler, 1980. Where has the fifth dimension gone? *Physical Review D*, 21(8): 2167-2170.
- [17]. A. H. Guth, 1981. Inflationary Universe: A possible solution to the horizon and flatness problems. *Phys. Rev. D*, 23(2): 347-356.

- [18]. E. Alvarez and M. B. Gravela, 1983. Entropy from extra dimensions. *Phys Rev. Lett.*, 51 (10): 931-934.
- [19]. S. Ray, U. Mukhopadhyay and S. B. Dutta Choudhury, 2007. Dark Energy Models with a Time-Dependent Gravitational Constant. *Int. J. Mod. Phys. D* 16 (11): 1791-1802.
- [20]. G. S. Khadekar and V. Patki, 2008. Kaluza-Klein type Friedmann-Robertson Walker cosmological models with dynamical cosmological term Λ . *Int. J. Theor. Phys.*, 47 (6): 1751-1763.
- [21]. C. Ozel, H. Kayhan and G. S. Khadekar, 2010. Kaluza-Klein Type Cosmological Model with Strange Quark Matter. *Adv. Studies Theor. Phys.*, 4(3): 117-128.
- [22]. M. Sharif and F. Khanum, 2011. Kaluza-Klein cosmology with varying G and Λ . *Astrophys. Space Sc.*, 334 (1): 209-214.
- [23]. S. Oli, 2014. Five-Dimensional Space-Times with a Variable Gravitational and Cosmological Constant. *J. Gravit., Art. ID 874739*: 1-4.
- [24]. U. Mukhopadhyay, I. Chakraborty, S. Ray and A. A. Usmani, 2016. A Dark Energy Model in Kaluza-Klein Cosmology, *Int. J. Theor. Phys.*, 55(1): 388-395.
- [25]. D. R. K. Reddy and Y. Aditya, 2018. Kaluza-Klein FRW type Perfect Fluid Cosmological Models with Linearly varying Deceleration Parameter in a Modified Gravity. *Int J Phys Stud Res.*, 1(1): 42-46.
- [26]. M. A. Hossain, M. M. Alam and A. H. M. M. Rahman, 2017. Kaluza-Klein Cosmological Models with Barotropic Fluid Distribution. *Phys Astron Int J*, 1(3): 00018, 1-7.
- [27]. U. Mukhopadhyay, S. Ray and F. Rahaman, 2010. Dark Energy Models with Variable Equation of State Parameter. *Int. J. Mod. Phys. D*, 19: 475-487.
- [28]. B. Saha, V. Rikhvitsky and A. Pradhan, 2015. Bianchi type-1 cosmological models with time dependent gravitational and cosmological constants: An alternative approach. *Rom. Journ. Phys.*, 60(1-2): 3-14.
- [29]. S. Roy, 2019. Time evolution of the matter content of the expanding universe in the framework of Brans-Dicke gravity, *Research in Astronomy and Astrophysics*, 19(4): 61-74.
- [30]. A. K. Yadav, 2011. Some Anisotropic Dark Energy Models in Bianchi type-V Space-time. *Astrophys. Space Sci.*, 335: 565-575.

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