

Demand Response for Residential Loads in Smart Grid Under Normal and Abnormal Conditions

Dr. Shaharam Karimi^a, Raul Alwan^a

^aDepartment of electrical Engineering, Faculty of Engineering, Razi University, Kermanshah, Iran

ABSTRACT

This paper proposes a price-based demand response program by the nonlinear control method. The demand response program is formulated as a nonlinear power management system with price feedback. We give the conditions of the price parameters for both the global asymptotic stability of the system and the social welfare optimality of the equilibrium point. Furthermore, the system is shown to be input-to-state (ISS) stable when there are additive disturbances on the power measurements and the price, and the discrete-time implementation of the power management system is given. Simulation results demonstrate the balance between supply and demand and the stability of the system with and without disturbances.

Keywords : Smart Grid, Demand Response, Power Management System, Nonlinear Control, Disturbances

I. INTRODUCTION

With the increasing population and the improving living standards, electrical energy use keeps rapidly increasing in the last several decades. The rapid increase of electricity demand has imposed great stress on electricity grid especially for maintaining grid power balance. The power supply and demand of a grid must always balance and such real time balance is a critical system requirement. Any power imbalance or mismatch will cause severe consequences in the reliability and quality of power supply (e.g. power outages, voltage fluctuations). In order to maintain the real time power balance, great efforts have been made from power demand side (e.g. demand response control).

In general, there are two categories of demand response programs: incentive-based programs and price-based programs. The incentive-based programs include the direct load control program, the emergency demand response program, and the ancillary services market. For the price-based programs, the utilities can hanged the power consumption of customers by pricing, such as time of use (TOU), critical peak pricing (CPP), extreme day CPP (ED-CPP), extreme day pricing (EDP), and real-time pricing (RTP) [1]. Smart grid increases the opportunities for demand response by providing real-

time data to providers and customers. In smart grid, the price can be provided to the customers in real time. For example, the electricity provider announces electricity prices on a rolling basis in the RTP program, and the price for a given time period (e.g., an hour) is determined and published before the start of the period (e.g., 15 min beforehand).

II. METHODS AND MATERIAL

A. Related Work

There exist a number of literatures on the price-based demand response programs. Different demand response programs were developed based on game theory [2–4], stochastic optimization [5,6], intelligent optimization [7], and dual decomposition method [8,9]. The social welfare maximization was achieved by optimizing the individual utilities of the customers in the demand response program based on dual decomposition. Then, a distributed power control algorithm was proposed for demand response with communication loss [10]. The works mentioned above assumed that the price is adjusted according to a pricing algorithm instead of an explicit pricing function. Recently, a linear pricing function was developed to achieve the balance between supply and demand for smart grid [11, 12], and a

nonlinear pricing function was used to design a distributed demand response algorithm [13]. Nevertheless, few works are devoted to the social optimality of the distributed power control under nonlinear pricing function and the influence of the disturbances on the power control algorithm.

In this study, we use a quadratic pricing function and establish the conditions on the social optimality of the distributed power control algorithm. Due to the unavoidable disturbances on power systems, we further consider the distributed power control with additive disturbances on the power measurements and the price.

In [14], user preferences are taken into account with the concept of discomfort level and an optimization problem is formulated to balance the load and minimize the user inconvenience caused by demand scheduling. Several ideas from the distributed computing area such as makespan have been introduced to energy consumption optimization. Similarly, in [15], an energy consumption scheduling problem is established to minimize the overall energy cost. Techniques similar to those used in wireless network resource allocation have been applied here to solve the underlying optimization problem. In both works, the user demands are known beforehand and the optimization problem is solved in numerical iterations.

While the idea in [16], the load control in a multiple-residence setup. The utility company adopts a cost function representing the cost of providing energy to end-users. Each residential end-user has a base load, two types of adjustable loads, and possibly a storage device. The first load type must consume a specified amount of energy over the scheduling horizon, but the consumption can be adjusted across different slots. The second type does not entail a total energy requirement, but operation away from a user-specified level results in user dissatisfaction. The research issue amounts to minimizing the electricity provider cost plus the total user dissatisfaction, subject to the individual constraints of the loads. The problem can be solved by a distributed sub gradient method. The utility company and the end-users exchange information through the Advanced Metering Infrastructure (AMI)-a two-way communication network-in order to converge to the optimal amount of electricity production and the optimal power consumption schedule. The algorithm finds near-

optimal schedules even when AMI messages are lost, which can happen in the presence of malfunctions or noise in the communications network. The algorithm amounts to a sub gradient iteration with outdated Lagrange multipliers, for which convergence results of wide scope are established.

Our works the price-based demand response program is formulated as a nonlinear power management system. The condition is established for the equivalence of the equilibrium point of the system and the optimal solution of a social welfare maximization problem. The proof of the stability is given for the power management system with and without disturbances on the power measurements and the price.

B. System Design

As shown in Fig (1), we consider a smart power system consisting of one electricity provider and N customers. The operation cycle of the power system is divided into several time slots. In each time slot, the electricity provider decides the electricity price and announces it to the customers. Then, the customers manage their power consumption according to the announced price. We employ the utility functions to characterize the profits of the customers [17].

A quadratic utility function with linear decreasing marginal benefit is defined as:

$$U_i(X_i) = \begin{cases} w_i x_i - x_i^2 / 2, & \text{if } 0 \leq x_i \leq \frac{w_i}{a} \\ \frac{w_i^2}{2a} & \text{if } x_i > \frac{w_i}{a} \end{cases} \rightarrow (1)$$

Where:

- X_i is the power consumption of customer ($i \in \{1, 2, \dots, N\}$).
- W_i denoted the willingness to increase the power consumption.
- W_i/a denoted the maximum demand of customer I for instance.

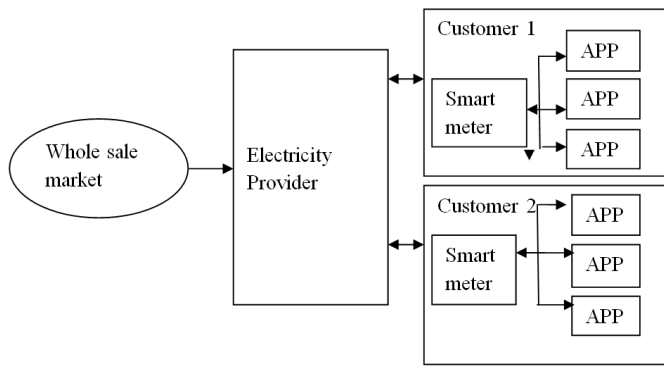


Figure 1. Smart Power System

The utility functions with different willingness parameter are shown in table (1).

Willingness parameters	Power consumption	Utility
$Wi=6, a=1$	1	5
	2	10
	3	12.5
	4-12	15
$Wi=8, a=1$	1	7.5
	2	12.5
	3	17.5
	4	22.5
	5	26.5
	6	28.5
	7-12	30
$Wi=10, a=1$	1	10
	2	17.5
	3	25
	4	31.5
	5	37.5
	6	42.5
	7	45
	8	47.5
	9-12	50

Table (1) utility value with different willingness parameters

The quadratic utility function indicates that a customer is willing to choose larger power consumption with Wi/a as the saturation value. In general, the objective of demand response is to maximize the social welfare [18], which can be formulated as the following optimization problem:

$$(p_1) = \max \sum_{i \in N} U_i(x_i) \\ s. t \sum_{i \in N} X_i = Q$$

Where Q denotes the power supply. The constraint in (P1) indicates that the total power consumption should match with the power supply. The optimization problem (P1) is a convex optimization problem and can be solved by the following primal algorithm [19]:

$$\dot{x}_i = k_i(w_i - ax_i(m) - p(m)), i \in N \rightarrow (2)$$

Where $(k1)$ is the control gain, $p(x)$ is the pricing function of the electricity provider, and $x = (x_1, x_2, \dots, x_N)^T$ denotes the set of power consumption of all the customers.

C. Problem Function

1-Stability function:

Definition 1 (Stability [7]). Let $x = 0$ be an equilibrium point for $\dot{x} = f(x)$ with $x(0)=x_0$. The equilibrium point $x = 0$ of $\dot{x} = f(x)$ is said to be globally asymptotically stable if $\lim_{t \rightarrow \infty} \|x(t)\| = 0$ for all initial conditions x_0 .

Proof. Let $\varphi(x) = x^\circ$, where $\varphi(x) = (\varphi_1(x); \dots; \varphi_N(x))$ and $\dot{X} = \langle x_1 | \dots | x_N \rangle$ define a Lyapunov candidate function as :

$$V(X) = \frac{1}{2} \varphi^T(x) \varphi(x) \rightarrow (3)$$

Where $V(x)$ is strictly positive for all x , except for $x = x^\circ$. The time derivative of $V(x)$ is obtained as

$$\begin{aligned} \dot{V}(x) &= \sum_{i \in N} (\varphi_i(x) \cdot \dot{\varphi}_i(x)) \\ &= \sum_{i \in N} (\varphi_i(x) \cdot \sum_{j \in N} \frac{\partial \varphi_i}{\partial x_j} \cdot \dot{x}_j) \\ &= \sum_{i \in N} (\varphi_i(x) \cdot \sum_{j \in N} \frac{\partial \varphi_i}{\partial x_j} \cdot \varphi_j(x)) = \varphi^T(x) J \varphi(x) \rightarrow (4) \end{aligned}$$

Where J is the Jacobin matrix of $Q(x)$ and can be defined as:

$$J = \begin{bmatrix} -a - 2b\sigma - c & -2b\sigma - c & \dots & -2b\sigma - c \\ -2b\sigma - c & -a - 2b\sigma - c & \dots & -2b\sigma - c \\ \vdots & \vdots & \ddots & \vdots \\ -2b\sigma - c & -2b\sigma - c & \dots & -a - 2b\sigma - c \end{bmatrix}$$

→ (5)

Where $\sigma = \sum_{i \in N} x_i$

2- Pricing and power consumption calculation:

In this study, we select the quadratic pricing function

$$p(x) = b \left(\sum_{i \in N} x_i \right)^2 + c \sum_{i \in N} x_i,$$

Where b and c are positive price parameters, b=0.01 in the case of disturbance conditions we select these equations:

$$\dot{x}_i = k_i(w_i - ax_i(m) - p(m) + d_1), i \in N \rightarrow (6)$$

$$p(x) = b(\sum_{i \in N} X_i + d_2)^2 + c(\sum_{i \in N} X_i + d_2) \rightarrow (7)$$

While equations with the discrete -time control algorithm without Disturbances:

$$X_i(m+1) = X_i(m) + \mu(w_i - aX_i(m) - p(m)) \rightarrow (8)$$

$$p(m+1) = b(\sum_{i \in N} X_i(m))^2 + c(\sum_{i \in N} X_i(m)) \rightarrow (9)$$

Equations with the discrete -time control algorithm with disturbances:

$$X_i(m+1) = X_i(m) + \mu(w_i - aX_i(m) - p(m) + d_1) \rightarrow (10)$$

$$p(m+1) = b(\sum_{i \in N} X_i(m) + d_2)^2 + c(\sum_{i \in N} X_i(m) + d_2) \rightarrow (11)$$

d1 and d2 denote the additive disturbances on the price and the total power consumption, respectively and μ is the step size =0.07 .

D. Proposed Algorithm

The first step: the electricity provider sets the initial electricity price according to the Forecast demand and then announces it to the customers.

The second step: the customers adjust their power consumption according to (10) defining a all positive scalar d, the demand response program is turned to step 3 if $|x_i(m+1)-x_i(m)| > \delta$ for any $i = 1; 2; \dots ;N$. Otherwise, the demand response program is terminated.

The third step: the electricity provider updates the electricity price according to (11) and then announces the updated price to the customers. Then, the demand response program is turned to step 2.

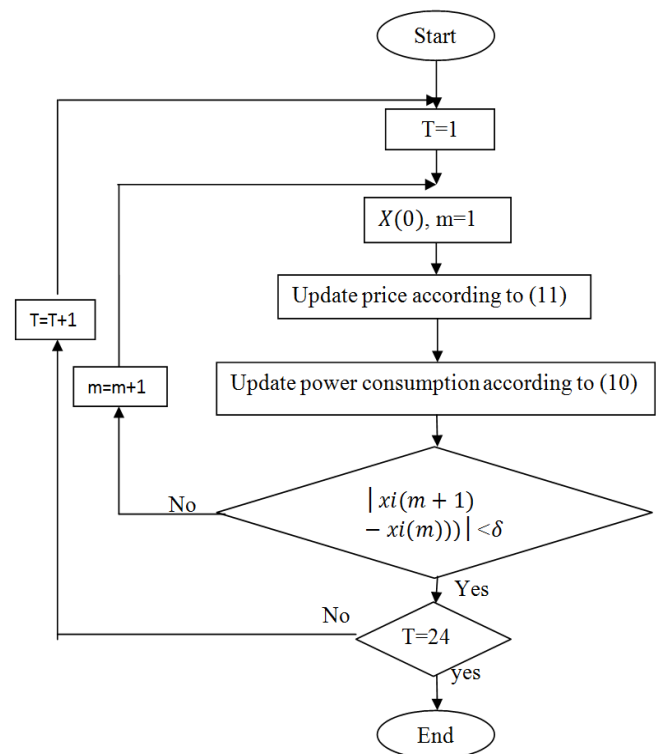


Figure 2. Flow chart of the DR Program

III. RESULTS AND DISCUSSION

In the simulations, we consider a residential power system composed of ten customers and one electricity provider. Supply Q is varying from 10 kW to 42 kW .The parameters (a) is set to 3.3.The willingness parameter wi is randomly selected from [13, 20].

Table (2) the electric price in for different time slots in a day

Time	1am	2am	3am	4am	5am	6am	7am	8am	9am	10am	11am	12am
P	14.5	14.25	13.75	13.5	13	13.75	14.5	15	15.25	15.75	16.25	16.75
PWD	14.9	14.25	13.75	13.4	12.8	13.95	14.35	14.9	15.25	15.7	16.45	16.65
Time	13bm	14bm	15bm	16am	17bm	18bm	19bm	20bm	21bm	22bm	23am	24bm
P	17.25	16.75	16.25	15.75	16.75	17	17.25	18.25	18.5	18.25	17.5	15.75
PWD	17.4	16.5	16.35	15.6	16.85	17.6	17.37	18.13	18.7	18.97	17.63	15.6

Table (3) the total power consumption across different time slots in a day

Time	1am	2am	3am	4am	5am	6am	7am	8am	9am	10am	11am	12am
TPC	15	14	13	12	11	13	15	17	18	20	22	25
TPCWD	14.25	14	12.5	11.5	10.5	13	16.25	16	18	20.5	23	24.25
Time	13bm	14bm	15bm	16am	17bm	18bm	19bm	20bm	21bm	22bm	23am	24bm
TPC	28	25	22	20	25	26	28	36	39	36	30	20
TPCWD	29	26.5	23.75	19.5	24.5	27	29	36	38.25	36	30	19

Where: P denotes the price in (cents/kwh) ,PWD denotes the price with disturbances in(cents/kwh),TPC denotes total power consumption in (kw)and PCWD denotes total power consumption with disturbances in (kw).From table(3) we observe that the disturbances cause errors to the electricity price, the total power consumption (TPC) matches exactly with the power supply across different time slots in a day when there are no disturbances. However, the TPC will deviate from the power supply when there exist some disturbances on the power measurements and the electricity price. The average deviations of the (TPC) and (DPC) in a day are defining as:

$$\Delta c = \frac{\sum_{t=1}^{24} |Q^t - \sum_{i \in N} x_i^t|}{24} \rightarrow (12)$$

$$\Delta p = \frac{\sum_{t=1}^{24} |p_o^t - p_d^t|}{24} \rightarrow (13)$$

Where t denotes the time slot in a day, x_i^t is the power consumption of customer i in time slot t; p_o^t is the electricity price without disturbances, and p_d^t is the electricity price with disturbances; $\Delta c = 0.64$ and $\Delta p = 0.21$.

Table (4) performance of power consumption control with and without disturbances

	DPC(kwh)	DAP(cents/kwh)	PAR
Disturbances	531.5	16.5	1.8
No disturbances	529.5	16.5	1.7

Where: DPC denotes the daily power consumption, DAP denotes the daily average price and PAR denotes the peak to average ratio is defined as [9]:

$$PAR = \frac{24 \max_{t \in (1, \dots, 24)} \sum_{i \in N} x_i^t}{\sum_{t=1}^{24} \sum_{i \in N} x_i^t} \rightarrow (14)$$

And the (DAP) is defined as:

$$p = \frac{\sum_{t=1}^{24} \bar{p}^t (\sum_{i \in N} x_i^t)}{\sum_{t=1}^{24} \sum_{i \in N} x_i^t} \rightarrow (15)$$

The social welfare ($\sum_{i \in N} U_i(x_i)$) obtained from the power management system is given in table below shown that the social welfare obtained from the power management system achieves the optimal value of (p_1), and the disturbances will result in the deviations of the social welfare from the optimal value.

Table (5) Social welfare obtained from the power management system

Time	1am	2am	3am	4am	5am	6am	7am	8am	9am	10am	11am	12am
Optimal S.W	261	250	238	220	205	238	260	282	289	310	330	355
S.W	261	250	238	220	205	238	260	282	289	310	330	355
S.W.D	250	244	242	230	210	212	262	272	300	319	338	358
Time	1am	2am	3am	4am	5am	6am	7am	8am	9am	10am	11am	12am
Optimal S.W	370	358	330	310	358	360.5	365	382	372	382	380	315
S.W	370	358	330	310	358	360.5	365	382	372	382	380	315
S.W.D	376	374	331	310	360	360.5	365	382	371	382	380	317

Where (S.W) denotes the social welfare and (S.W.D) denotes the social welfare with disturbances. The convergence of the power control algorithms without and with disturbances is shown in table (6) and (7), respectively. The convergence of price in such two cases is shown in table (8) it is observed that both the power consumption and the price can converge within 30 iterations. Typically, the disturbances incur longer settling time and larger overshoot in the adjustment of the power Consumption and the price.

Table (6) convergence of customer power consumption in (kw) without disturbances

Customer	1	2	3	4	5	6	7	8	9	10
Iteration										
1	1.19	1.25	1.3	1.37	1.42	1.49	1.55	1.6	1.67	1.72
2	1.17	1.21	1.28	1.32	1.4	1.45	1.52	1.58	1.65	1.7
3	1.2	1.26	1.31	1.375	1.43	1.5	1.57	1.61	1.68	1.73
4	1.18	1.22	1.285	1.33	1.41	1.46	1.56	1.59	1.67	1.71
5	1.185	1.23	1.29	1.335	1.415	1.47	1.565	1.595	1.675	1.715
6	1.8	1.22	1.285	1.33	1.41	1.46	1.56	1.59	1.67	1.71
7-30	1.8	1.22	1.285	1.33	1.41	1.46	1.56	1.59	1.67	1.71

Table (7) convergence of customer power consumption in(kw) without disturbances

Customer	1	2	3	4	5	6	7	8	9	10
Iteration										
1	1.19	1.25	1.3	1.37	1.42	1.49	1.55	1.6	1.67	1.72
2	1.1	1.18	1.23	1.4	1.48	1.52	1.48	1.54	1.59	1.66
3	1.21	1.29	1.27	1.45	1.49	1.45	1.53	1.65	1.71	1.78
4	1.15	1.2	1.3	1.32	1.47	1.51	1.57	1.69	1.65	1.7
5	1.2	1.25	1.28	1.36	1.45	1.47	1.55	1.62	1.69	1.75
6	1.19	1.22	1.29	1.34	1.42	1.48	1.56	1.61	1.68	1.71
7	1.195	1.23	1.28	1.35	1.43	1.475	1.555	1.615	1.685	1.72
8	1.19	1.22	1.285	1.345	1.425	1.479	1.56	1.61	1.68	1.71
9	1.19	1.225	1.28	1.35	1.45	1.479	1.56	1.612	1.682	1.715
10-30	1.19	1.22	1.28	1.35	1.45	1.479	1.56	1.612	1.681	1.71

Table (8) Convergence of electricity price in (*cents/kwh*) with and without disturbances

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
P	14	14.2	14.5	14.2	13.6	13.65	13.8	14.2	14	13.9	14.1	14.2	14.1	14	13.9
PWD	14.4	15	15.5	15.2	14.5	14	13.7	13.5	13.7	14	14.2	14.5	14.7	14.5	14.2
Iteration	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30-60
P	14	14.05	14	14.05	14.15	14.1	14.05	14.02	14.03	14	14.03	14.02	14.01	14.015	14
PWD	14	14.2	14.15	14.1	14.05	14.1	14.05	14.01	14.03	14	14.04	14.02	14.01	14.015	14

IV. CONCLUSION

This paper uses a nonlinear control method to generate a price-based demand response program. The demand response program is formulated as a nonlinear power management system, and the stability is shown for the system with and without disturbances. It is shown that the power management system can match supply with demand when there are no disturbances, and the disturbances will result in the errors in electricity price and the matching errors between supply and demand. This further degrades the transient performance of the system. In the future, we will consider the demand response program with renewable energy supplies, which will generate a stochastic power management system. Further results should be given for the stability of the stochastic system with and without disturbances.

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