

## A Critical Study of Quantum Phase Transition



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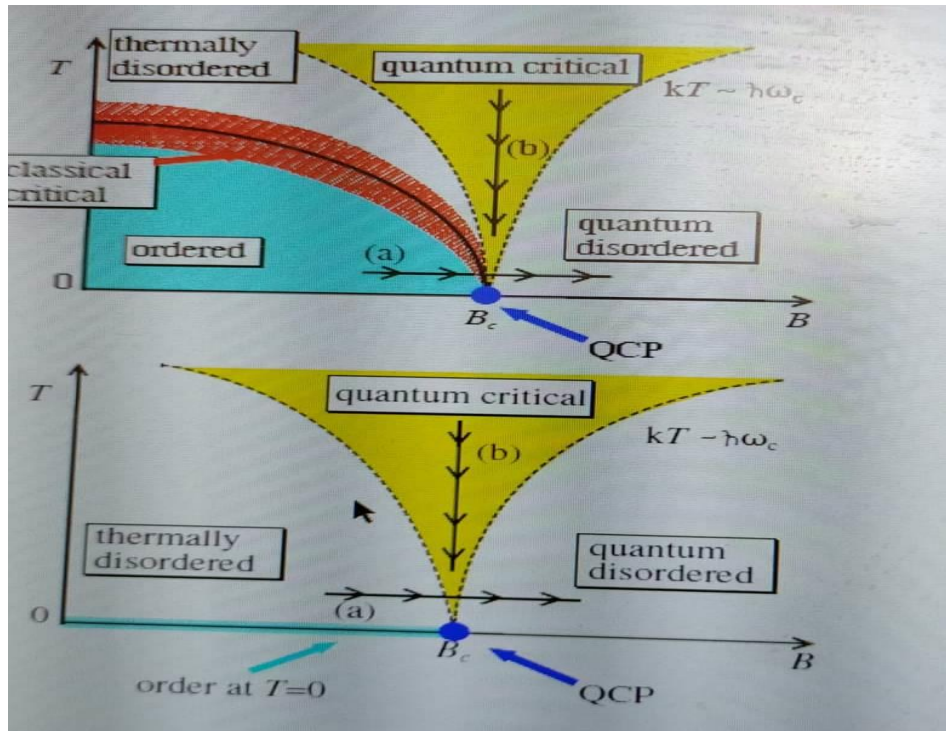
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We study the dynamical quantum phase transition of a critical quantum quench, in which the pre-quenched Hamiltonian, or the post quenched Hamiltonian, or both of them are set to be the critical points of equilibrium quantum phase transitions. We find a half-quantized or unquantized dynamical topological order parameter and dynamical Chern number; these results and also the existence of dynamical quantum phase transition are all closely related to the singularity of the Bogoliubov angle at the gap-closing momentum. The effects of the singularity may also be canceled out if both the prequenched and postquenched Hamiltonians are critical; then the dynamical topological order parameter and dynamical Chern number restore to integer ones. Our findings show that the widely accepted definitions of the dynamical topological order parameter and dynamical Chern number are problematic for the critical quenches in the perspective of topology, which call for new definitions of them.

The past decade has seen a substantial rejuvenation of interest in the study of quantum phase transitions, driven by experiments on the cuprate superconductors, the heavy fermion materials, organic conductors and related compounds. Although quantum phase transitions in simple spin systems, like the Ising model in a transverse field, were studied in the early 70's, much of the subsequent theoretical work examined a particular example: the metal-insulator transition. While this is a subject of considerable experimental importance, the greatest theoretical progress was made for the case of the Anderson transition of non-interacting electrons, which is driven by the localization of the electronic states in the presence of a random potential. The critical properties of this transition of non-interacting electrons constituted the primary basis upon which most condensed matter physicists have formed their intuition on the behavior of the systems near a quantum phase transition. On the other hand, it is clear that strong electronic interactions play a crucial in the systems of current interest noted earlier, and simple paradigms for the behavior of such systems near quantum critical points are not widely known.

It is the purpose of this book to move interactions to center stage by describing and classifying the physical properties of the simplest interacting systems undergoing a quantum phase transition. The effects of disorder will be neglected for the most part, but will be considered in the concluding chapters. Our focus will be on the dynamical properties of such systems at non-zero temperature, and it shall become apparent that these differ substantially from the non-interacting case. We shall also be considering inelastic collision-dominated quantum dynamics and transport: our results will apply to clean physical systems whose inelastic scattering time is much *shorter* than their disorder-induced elastic scattering time. This is the converse of the usual theoretical situation in Anderson localization or mesoscopic system theory, where inelastic collision times are conventionally taken to be much *larger* than all other time scales.

One of the most interesting and significant regimes of the systems we shall study is one in which the inelastic scattering and phase coherence times are of order  $\hbar/k_B T$ , where  $T$  is the absolute temperature. The importance of such a regime was pointed out by Varma *et al.* [66,67] by an analysis of transport and optical data on the cuprate superconductors. Neutron scattering measurements of Hayden *et al.* [30] and Keimer *et al.* [37] also supported such an interpretation in the low doping region. It was subsequently realized [59,12,56] that the inelastic rates are in fact a *universal number* times  $k_B T/\hbar$ , and are a robust property of the high temperature limit of renormalizable, interacting quantum field theories which are not asymptotically free at high energies. In the Wilsonian picture, such a field theory is defined by renormalization group flows away from a critical point describing a second order quantum phase transition. It is not essential for this critical point to be in an experimentally accessible regime of the phase diagram: the quantum field theory it defines may still be an appropriate description of the physics over a substantial intermediate energy and temperature scale. Among the implications of such an interpretation of the experiments was the requirement that response functions should have prefactors of anomalous powers of  $T$  and a singular dependence on the wavevector; recent observations of Aeppli *et al.* [1], at somewhat higher dopings, appear to be consistent with this. These recent experiments also suggest that the appropriate quantum critical points involve competition between an insulating state in which the holes have crystallized into a striped arrangement, and a  $d$ -wave superconductor. There is no theory yet for such quantum transitions, but we shall discuss numerous simpler models here which capture some of the basic features.



The particular quantum phase transitions that are examined in this book are undoubtedly heavily influenced by my own research. However, I do believe that my choices can also be justified on pedagogical grounds, and lead to a logical development of the main physical concepts in the simplest possible contexts. Throughout, I have also attempted to provide experimental motivations for the models considered: this is mainly in the form of a guide to the literature, rather than in-depth discussion of the experimental issues. I have also highlighted some

especially interesting experiments in a recent popular introduction to quantum phase transitions [57]. An experimentally oriented introduction to the subject of quantum phase transitions can also be found in the excellent review article of Sondhi, Girvin, Carini and Shahar [62]. Readers may also be interested in a recent introductory article [70], intended for a general science audience.

A separate motivation for the study of quantum phase transitions is simply the value in having another perspective on the physics of an interacting many body system. A traditional analysis of such a system would begin from either a weak coupling Hamiltonian, and then build in interactions among the nearly free excitations, or from a strong-coupling limit, where the local interactions are well accounted for, but their coherent propagation through the system is not fully described. In contrast, a quantum critical point begins from an intermediate coupling regime which straddles these limiting cases. One can then use the powerful technology of scaling to set up a systematic expansion of physical properties away from the special critical point. For many low-dimensional strongly correlated systems, I believe that such an approach holds the most promise for a comprehensive understanding. Many of the vexing open problems are related to phenomena at intermediate temperatures, and this is precisely the region over which the influence of a quantum critical point is dominant.

## Conclusion

- ✚ Quantum phase transitions occur at zero temperature as a function of a parameter like pressure, chemical composition, disorder, magnetic field
- ✚ quantum phase transitions are driven by quantum fluctuations rather than thermal actuations
- ✚ quantum critical points control behavior in the quantum critical region at nonzero temperatures
- ✚ quantum phase transitions in metals have fascinating consequences: non-Fermi liquid behavior and exotic superconductivity
- ✚ quantum phase transitions are very sensitive to disorder

## References

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