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Pairwise Regular in Bitopological Spaces

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First of all we present definitions of different types of pairwise regular spaces.

Definition (1) : In a bitopological space (X, P, Q), P is said to be regular with respect to Q if for every point $x \in X$ and for every Q- closed set F not containing x, there exist two disjoint sets G and H such that G is P – open and H is Q – open and satisfy condition.

$X \in G$ and $F \subseteq H$, , ... (1)

If P is regular with respect to Q and Q is regular with respect to P then (X, P, Q) is called pairwise regular.

Definition (2) : A bitopological space (X, P, Q) is said to be weak pairwise regular if given any point $x \in X$ and either a P- closed or Q- closed set F not containing x there exist disjoint sets U and V which are open either in P or in Q such that

$X \in U, F \subset V$, ... (2)

Definition (3) : A bitopological space (X, P, Q) is said to be quasi pairwise regular if closed and open set in the definition (2) are replaced by quasi-closed and quasi open sets.

How we improve the definition (1) in term of 12- pre open (pre closed) and 21- pre open (pre closed).

Definition (4) : Let (X, T_1, T_2) be a bitopological space. Then T_1 is said to be *- regular with respect to T_2 if for every point $x \in X$ and for every 21- pre closed set F not containing X_1 there exist two disjoint sets G and H such that G is 12- pre open, H is 21- Pre open and satisfy the condition (1).

The bitopological space (X, T_1, T_2) is said to be regular if a T_1 is *- regular with respect to T_2 and T_2 is *- regular with respect to T_1.

Similarly, definition (2) can be improved as follows :

Definition (5) : A bitopological space (X, T_1, T_2) is said to be weak pairwise - regular if given any point $x \in X$ and either a 12- pre closed or 21- pre closed set F not Containing x there exist disjoint sets U and V which are either 12- pre open or 21- pre open such that condition (2) is satisfied.

Now we improve theorem (Sinha, 1998) as follows :

Theorem (1) : Let (X, T_1, T_2) be a bitopological space. Then T_1 is *-regular with respect to T_2 iff for each $x \in X$ and each 21- pre open set V containing x there exist 12- pre open set U and 21- pre closed set L such that

$X \in U \subset L \subset V$,................................(3)

Proof. Suppose that for a bitopological space (X, T_1, T_2), T_1 is *- regular with respect to T_2. Let $x \in X$ and let V be a 21- pre open set containing x.

Then V^c is 21- pre closed and $X \notin V^C$. Since (X, T_1, T_2) is T_1 *- regular with respect to T_2, there exist 12- pre open set U and 21- pre open set W such that

$x \varepsilon U, V^C \subset W$ and $U \cap W = \phi$:

This means that $x \in U \subset W^C \subset V$. If we put $W^C = L$ then L is 21- pre closed and $x \in U \subset L \subset V$. So condition (3) is necessary condition.

Conversely, suppose that the condition given in theorem is satisfied. Let $x \in X$ and let B be a 21- pre closed set not containing x. Then B^C is 21- pre open and $x \in B^C$. So according to our given condition there exist 12- pre open set U and 21- pre closed set L such that-

$x \in U \subset L \subset B^C$.

This means that $x \in U$, $B \subset L^C$ and $U \cap L^C = \phi$. This proves that T_1 is *- regular with respect to T_2.

Remark : Similar result can be obtained for a bitopological space to be T_2 *- regular with respect to T_1.

Thus we have the following result :

Theorem (2) : Let (X, T_1, T_2) be a bitopological space. Then it is pairwise *- regular iff the following conditions are satisfied:

(1) For each $x \in X$ and for each 12- pre open set V with $x \in V$ there exist 21- pre, open set U and a 12- pre closed set L such that $x \in U \subset L \subset V$, and (ii) For each $x \in X$ and for each 21- pre open set V with $x \in V$ there exist a 12- pre open set U and a 21- pre closed set L such that $x \in U \subset L \subset V$.

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