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Cartan Like Recurrent Connection

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ABSTRACT -

In this paper we shall Construct Cartan like Recurrent Connection.

Keywords: Cartan like Recurrent Connection, Riemannian Christoffel symbols.

INTRODUCTION :

Let M^n be an n-dimensional differentiable Manifold (M^n, a_{ij} (x)) be Riemannian Space equipped with fundamentals metric tensor $a_{ij}(x)$. From a Physical view poin S. Ikeda [i] Introduced line element space of M^n with a metric tensor $g_{ij}(x,y) = e^{2\rho}\, a_{ij}$ (x), (ρ = ρ (y), $y^i = dx^i$) It is however easily seen that this form of $g_{ij}(x,y)$ depends on a Particular choice of Co-ordinate (x^i). So generalizing the form, we shall introduce a metric $g_{ij}(x,y) = e^{2\rho}\, a_{ij}$ (x), where ρ = ρ (y), is positively homogenous of degree 0 in y^i. The metric thus defined in a kind of so called conformally Riemannian. Let $\overbrace{\Gamma}^{(r)}$ be Riemannian Christoffel symbols obtained from a_{ij} (x) and h and v covariant derivative $x_{i;j}$, $x_{i||j}$ of x_i with respect to connection

$$C\Gamma = (\overbrace{\Gamma^h_{jk}}^{(r)},\ \overbrace{\Gamma^h_{ok}}^{(r)},\ \overbrace{C^i_{jk}}^{(r)} = 0) \text{ are defined by}$$

$$x_{i;j} = \frac{\partial x^i}{\partial x^j} - \left(\frac{\partial x^i}{\partial y^S}\right) \overbrace{\Gamma^S_{oj}}^{(r)} - x_S\ \overbrace{\Gamma^S_{ij}}^{(r)}$$

$x_{i||j} = \frac{\partial x^i}{\partial y^j}$ We shall construct Cartan like recurrent connection $RC\Gamma = (F^i_{jk},\ N^i_k,\ C^i_{jk})$ by the following five oxioms referring to $g_{ij}\ (x,y)$ which are similar to those of Cartan Connection of a Finsler.

$$F_1: g_{ij|k} = \frac{\partial g_{ij}}{\partial x^k} - \left(\frac{\partial g_{ij}}{\partial y^r}\right) N^r_k - g_{rj}\, F^r_{ik} - g_{ir}\, F^r_{jk} = a_k\, g_{ij}$$

$$F_2: g_{ij|k} = \frac{\partial g_{ij}}{\partial y^k} - g_{rj}\, C^r_{ik} - g_{ir}\, C^r_{jk} = b_k\, g_{ij}$$

$$F_3: T^i_{jk} = F^i_{jk} - F^i_{kj} = 0$$

$$F_4: S^i_{jk} = C^i_{jk} - C^i_{kj} = 0$$

$F_5: Y^i_{|_j} = -N^i_j + F^i_{oj} = 0$

Where ($|$) and ($|$) denote h and v covariant derivatives with respect to $RC\Gamma$ respectively.

In this paper we show the existence and uniqueness of Cartan like recurrent Connection $RC\Gamma$ which such Finsler Connection for which metric tensor is h and V recurrent

CARTAN LIKE RECURRENT CONNECTION :-

We shall construct Cartran like recurrent Connection

$RC\Gamma = (F^i_{jk}, N^i_k, C^i_{jk})$ by considering five axioms as given in introduction Firstly from F_2 F_4 and

(2.1) $g_{ij||_k} = 2g_{ij}\,\rho_{||_k}$

The (h) hv torsion tensor C^i_{jk} of $RC\Gamma$ is given by

(2.2) $C^i_{jk} = (\rho_{||_j} - \frac{1}{2} b_j)\,\delta^i_k + (\rho_{||_k} - \frac{1}{2} b_k)\,\delta^i_j - (\rho^i_{||_j} - \frac{1}{2} b^i)\,g_{jk}$

Where $\rho^i_{||_j} = g^{ki}\ \rho_{||_k}$, $b^i = g^{ki} b_k$ and g^{ij} is reciprocal tensor of g_{ij} so we get

(2.3) $C^j_{ij} = \quad C_i = (\rho_{|_i} - \frac{1}{2} b_i)$ n

Now (2.1) is rewritten as

(2.4) $g_{ij||_k} = 2g_{ij}\,(\frac{1}{n}\,C_k + \frac{1}{2}\,b_k)$

PROPOSITION 1 :-

If generalized metric tensor g_{ij} is conformally Riemannian then equation (2.4) holds. The Christoffel Process [2] with respect to i, j, k applied on F_1 together with condition F_3 gives.

(2.5) $e^{2\rho}\ \overset{(r)}{\Gamma}_{ijk} + \rho_{|_k} + g_{jk}\rho_{|_i} - F_{ijk} = \frac{1}{2}\,(a_k\,g_{ij} + a_i\,g_{jk} - a_j\,g_{ki})$ F_5 we get :-

(2.6) $\overset{(r)}{\Gamma}{}^h_{ok} + y^h\,\rho_{|_k} + \delta^h_k\,\rho_{|_o} - y_k\,\rho^h_{|} - N^h_k = \frac{1}{2}\,(a_k\,y^h + a_o\,\delta^h_k - a^h\,y_k)$

Contracting (2.6) by $\rho_{||_h}$ and paying attention to $\rho_{||_o} = 0$ we get

(2.7) $\rho_{|_k} + \rho_{||_k}\left(\rho_{|_o} - \frac{1}{2}\,a_o\right) - y_k\left(\rho_{||_\rho} - \frac{1}{2}\,\rho_{||_a}\right) = \rho_{;k}$

Now Contacting (2.7) by y^k and $\rho_{||_k}$ respectively gives

(2.8) $\rho_{|_o} - 2F\,\left(\rho_{||_\rho} - \frac{1}{2}\,\rho_{||_a}\right) = \rho_{;o}$

(2.9) $\rho_{||_\rho} + \rho^2\left(\rho_{||_\rho} - \frac{1}{2}\,a_o\right) = \rho_{;\rho}$

From (2.8) and (2.9) we get –

$\rho\,|_o = [\rho_{;o} + 2F\ \rho_{;\rho} - F\rho\,||_a + F\rho^2\ a_o]/D$

$\rho\,||_\rho = [\rho_{;\rho} - \rho^2(\rho_{;o} - F\rho\,||_a) + \frac{1}{2}\,\rho^2 a_o\,]/D$

Where $D = \begin{vmatrix} 1 & -2F \\ \rho^2 & 1 \end{vmatrix}$

Pulling above values in (2.7) we get

(2.10) $\rho\,|_k = \rho_{;k} + \frac{1}{2}\,a_o\rho\,||_k\ y_k - \frac{1}{2}\,\rho\,||_a\,y_k - \rho\,||_k\{\rho;_o + 2F\,\rho_{;\rho} + F\rho^2\ a_o\,\}/D$

$+\,y_k\,\left\{\rho;_\rho - \rho^2(\rho_{;o} - F\rho\,||_a) + \frac{1}{2}\rho^2\,a_o\,\right\}/D$

Substituting (2.11) in (2.6) and (2.11)

$\overset{(r)}{\Gamma}{}^h_{ik} + \delta^h_i\,\rho\,|_k + \delta^h_k\,\rho\,|_i - g_{ik}\,\rho^h_i - F^h_{ik} = \frac{1}{2}\,(a_k\,\delta^h_i + a_i\,\delta^h_k - a^h\,g_{ki})$

the connection coefficients N^h_k and F^h_{ik} are uniquely determined Which are as follows

(2.11) $N^h_k = \overset{(r)}{\Gamma}{}^h_{ok} + y^h\ \rho_{;k} - y_k\ e^{-2\rho}\ \rho^h_i + (\rho_{;o} + 2F\,\rho_{;\rho} - F\rho\,||_a + F\rho^2\,a_o)\,(\delta^h_k - y^h\,\rho\,||_k + y_k\,\rho^h_{||})/D + \frac{1}{2}a_o\,(y^h\,\rho\,||_k - y_k\,\rho^h_{||}) - \frac{1}{2}\{a_k\ y^h + a_0\,\delta^h_k - a^h\,y_k\}$

(2.12) $F^h_{ik} = \overset{(r)}{\Gamma}{}^h_{ik} + \delta^h_i\,\rho_{;k} + \delta^h_k\,\rho_{;i}\,g_{ik}\,e^{-2\rho}\,\rho^h_{;} - (\rho_{;o} + 2F\,\rho_{;\rho} - F\rho\,||_a + \rho^2\ a_o)$

$(\delta^h_i\,\rho\,||_k + \delta^h_k\,\rho\,||_i - g_{ik}\,\rho^h_{||})/D + (\rho_{;\rho} - \rho^2\,(\rho_{;o} - \rho\,||_a) + \frac{1}{2}\,\rho^2\,a_o)\,(\delta^h_i\,y_k + \delta^h_k\,y_i - g_{ik}\ y^h\,)/D$

$+\,(\frac{1}{2}a_o - \rho\,||_a - 1)\Big[\delta^h_i\,(\rho\,||_k + y_k + a_k) + \delta^h_k(\rho\,||_i + y_i + a_i) - g_{ik}\,(\rho^h_{||} + y^h + a^h\Big].$

Conclusion :-

In this paper we have obtained Proposition I and Cartan Like recurrent connection which are given by (2.11), (2.12) and (2.3)

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