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Analysis of a M/EK/1 Queueing System with Two-Phase, N-Policy, Server Failure and Second Optional Batch Service with Customers Impatient Behaviour

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ABSTRACT

In this paper an M/EK/1 queueing system with server vacation, Startup, breakdowns and second optional service with impatient customer behavior is considered. All the customers will be given unique type of individual service in the first phase and then second phase service will be provided on option of the customer. Each service consists of K-phases.We obtain some important system characteristics, such as the number of customers in the system, the probability that the server is idle, busy and broken down states, the expected waiting time in the system. Sensitivity analysis is also conducted with various parameters on system's performance measures.

Keywords : Two-phase, optional service, Balking, N-Policy

I. INTRODUCTION

In the Present scenario of globalization the queuing models have gained a lot of significance. Some of the areas where queueing models have valuable applications are traffic flow (vehicles, communications, people), Scheduling (patients in hospitals, jobs n machines, programs on a computer), and facility design (banks, post offices, food courts).The current work deal with the transient analysis of a $M/E_K/1$ Queueing System with two phases of service, N-Policy, Server Failure, Customers' impatient behavior where customers may opt a second service in addition to the first essential service.

Vacation queueing models have attracted great attention of researchers and became an active research area. Miller [11] was the first to study an M/G/1 queueing system where the server is unavailable called as vacation for some random length of time.To optimize the length of the vacation period that can minimize the cost by deciding proper rule for switching the server on and off, various types of control policies like N-Policy, (M,N)-policy, T-policy, (N, T)-policy, min (N,T)-policy, D-policy, (p,T)-policy, Q-policy are generated among which more research is done on N-Policy when compared with other control policies. Yadin and Naor [16] were first to study the technique of N-policy and have obtained the optimal value of the queue size at which to start on a single server, assuming that the form of the policy is to turn on the server when the queue size reaches a certain number N and to turn him off when the system size is empty.

Krishna and Lee [9] have first studied two-phase queueing system by considering the exhaustive service for the M/M/1 queueing system with and without gating and derived the sojourn time distribution and its mean for an arbitrary customer. Anantha Lakshmi et .al and Vasanta Kumar et al. [1,15] studied the optimal control policy of two-phase, N-policy $M^X/M/1$ and $M^X/E_k/1$ queueing systems with server startup and breakdowns, respectively.

Madan [10] has first studied second optional service in an M/G/1 queueing system in which the distributions of first essential and second optional service times follow General and Exponential respectively. He also mentioned some important applications in day-to-day life conditions. His work is being extended by many other researchers where their work includes inclusion of N-Policy, breakdowns and optimization. Jau-chung-ke et al. [7] aimed to optimize a finite capacity M/M/1 queueing model with F-policy where some customers may request a second service in addition to the first essential service. They derived some important performance measures and optimized the cost function.

In many real systems, the server may meet unpredictable breakdowns or any other interruptions. Rama Devi et al. [14] have studied the impact of server breakdowns in two phase queueing systems and also derived optimum cost with N-Policy.

These days customers are busy entities. An assumption which is often attached to the analysis of many queuing model is the customers are willing to wait as long as it is necessary to obtain service. A customer is said to be impatient if he tends to join the queue only when a short wait is expected and tends to remain in the line if his wait has been sufficiently small. Impatience generally takes three forms. The first is balking, the unwillingness of a customer to join a queue upon arrival, the second reneging, the unwillingness to remain in line after joining and waiting, and the third jockeying between lines when each of a number of parallel lines has its own queue. Haight [4] was the first who introduced concept of customer impatience in the queuing theory and still people are exploiting this concept in various applications. Haight has analyzed the queue where the individual customer upon arrival measures the queue by its length. Recently, Altman and Yechiali [3] have proved that customers become annoyed only when the server will be on vacation by making a comprehensive study on some queueing models such as $M/M/1$, $M/G/1$ and $M/M/c$ queue with server vacations and customer impatience. Adan et al. [5] have worked on queueing models with vacations and synchronized reneging.

For practical significance, queueing models are to studied with respect to time. Many methods are available to solve the transient state of equations. A time-dependent solution for the number in a single-server queueing system with Poisson arrivals and exponential service times is derived in a direct way by P. R. Parthasarathy[12]. Jacob.M.J. and Madhusoodanan.T.P[6] examined the behaviour of the infinite capacity M/G/1 model with batch arrivals and server vacations in transient state. Dong-Yuh Yang and Ying-Yi Wu [2] presented Transient Behavior Analysis of a Finite Capacity Queue with Working Breakdowns and Server Vacations. Kalidass et al.[8] have presented the transient behavior of an $M/M/1$ multiple vacation queue and the possibilities of catastrophes. Sudhesh and Francis Raj [13] have derived the time dependent system size probabilities for an $M/M/1$ model with working vacation.

However, to the best of our knowledge, there is no literature which takes time dependent probabilities for M/M/1 queueing systems with N-Policy, second optional service, server failure and reneging. This motivates us to present the current work. The main objective of this paper is to calculate various system parameters with numerical illustrations by using Runge-Kutta method of order 4.

This paper is systematized in V sections. Section II details the mathematical model and includes the set of governing differential equations of the model. In Section III, some performance measures are provided. Numerical results are given in Section IV. Section V concludes the paper.

The main objectives of the analysis carried out in this paper are:

i. To establish the Transient state equations and obtain the Transient state probability distribution of the number of customers in the system in each state.

ii. To derive values for the expected number of customers in the system when the server is in different states

iii. To carry out sensitivity analysis on the System performance measures for various system parameters through numerical experiments.

II. THE SYSTEM AND ASSUMPTIONS

We consider the $M/E_K/1$ queueing model with N-policy, unreliable server, customer's impatience and second service on optional basis in Transient state with the following assumptions:

1. Customers are assumed to arrive according to Poisson process with mean arrival rate λ Customers will get the service in the order in which they arrive. The customers who arrive during the first phase service are also allowed to join the queue which is in service.
2. Customers are given service such that Individual service times are assumed to be exponentially distributed with mean $1/\mu$ in exhaustive manner and batch service times in the second phase are also exponentially distributed with an average of $1/\boldsymbol{\beta}$.
3. Each customer will be given k-phases of service.
4. The server provides first essential service to all existing customers in individual manner. Then it proceeds to the second phase and if there is a batch of customers of at least b, it provides batch service. Then returns to first phase and continues the cycle.
5. The probability to opt second phase is p.
6. Whenever the system becomes empty, the server goes on vacation. As soon as the total number of arrivals in the queue reaches or exceeds the pre-determined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. The server startup time follows exponential distribution with mean 1/θ. As soon as the server finishes startup, it starts serving the waiting customers.
7. The breakdowns are generated by Poisson process with rates ξ_1 for the first phase of service and β_1 for the second phase of service. When the server fails it is immediately repaired at a repair rate ξ_2 in first phase and β_2 in second phase, where the repair times are exponentially distributed. After repair the server immediately resumes the concerned service.
8. Customers are assumed to be annoyed and renege the system. The probability that i th customer will renege is (i-1)* α

III. Notations

We use the following notations to represent transient probabilities for the system to be in various modes:

$p^1_{i,0}(t) = p(\text{i customers in the system when server is in Vacation});\ i = 1k, 2k, \dots (N-1)k$

$p^2_{i,0}(t) = p(\text{i customers in the system when server is in start} - \text{up mode});\ i = Nk, (N+1)k, \dots, sk$

$p^3_{i,0}(t) = p(\text{i customers in the system when srver is doing first essential service});\ i = 0, 1k, \dots . sk$

$p^4_{i,0}(t) = p(\text{i customers in the system when server is broken down during first essential service});$
$i = 0, 1k, \dots . sk$

$p^5_{i,j}(t) = p(\text{i customers in the system when server is doing second optional service});$
$i = 1k, 2k, \ldots.(s-b)k, j = bk, (b+1)k, \ldots, sk$
$p^6_{i,j}(t) = p(\text{i customers in the system when server is broken down during second optional service});$
$i = 1k, 2k, \ldots.(s-b)k, j = bk, (b+1)k, \ldots, sk$

The Transient state equations governing the system size probabilities at any arbitrary point of time are given by the following Differential Equations:

$$\frac{dp^1_{0,0}(t)}{dt} = -\lambda p^1_{0,0}(t) + \mu(1-p)p^3_{1,0}(t) + \beta p^5_{0,j}(t); bk \le j \le sk \ (1)$$

$$\frac{dp^1_{i,0}(t)+}{dt} = -(\lambda+(i-k)\alpha)\ p^1_{i,0}(t) + \lambda p^1_{i-k,0}(t) + \mu(1-p)p^3_{i+1,0}(t) + \beta p^5_{i,j}(t); b \le j \le s, 1k \le i \le (N-1)\text{k} \ (2)$$

$$\frac{dp^1_{Nk,0}(t)+}{dt} = -(\lambda+((N-1)k)\alpha)\ p^1_{i,0}(t) + \lambda p^1_{(N-1)k,0}(t) \ (3)$$

$$\frac{dp^2_{i,0}(t)+}{dt} = -\big(\lambda+\big((i-1)k\big)\alpha + \theta)\ p^2_{i,0}(t) + \lambda p^1_{i-k,0}(t); (N+1\big)k \le i \le (s-1)k \ (4)$$

$$\frac{dp^2_{sk,0}(t)+}{dt} = -\big((s-1)k\big)\alpha + \theta)\ p^2_{sk,0}(t) + \lambda p^1_{(s-1)k,0}(t) \ (5)$$

$$\frac{dp^3_{i,0}(t)+}{dt} = -\big(\lambda + \mu + \xi_1 + (i-k)\big)\alpha + \theta)\ p^3_{i,0}(t) + \mu(1-p)p^3_{i+k,0}(t) + \lambda p^1_{(i-1)k,0}(t) + \beta\ p^5_{i,j}(t) + \xi_2\ p^4_{i,0}(t);\ bk \le j \le (s-1)k, k \le i \le (N-1)k \ (6)$$

$$\frac{dp^3_{i,0}(t)}{dt} = -\big(\lambda + \mu + \xi_1 + (i-k)\big)\alpha + \theta)\ p^3_{i,0}(t) + \mu p p^3_{i+k,0}(t) + \lambda p^1_{(i-1)k,0}(t) + \beta\ p^5_{i,j}(t) + \xi_2\ p^4_{i,0}(t) + \theta p^2_{i,0}(t);\ bk \le j \le (s-1)k, k \le i \le (N-1)k \ (7)$$

$$\frac{dp^3_{sk,0}(t)}{dt} = -\big(\mu + \xi_1 + (sk-k)\big)\alpha + \theta)\ p^3_{sk,0}(t) + \lambda p^1_{(s-1)k,0}(t) + \xi_2\ p^4_{sk,0}(t) + \theta p^2_{sk,0}(t) \ (8)$$

$$\frac{dp^4_{i,0}(t)+}{dt} = -\big(\lambda + \xi_2 + (i-k)\big)\alpha)\ p^4_{i,0}(t) + \lambda p^4_{(i-1)k,0}(t) + \xi_1\ p^3_{i,0}(t);\ k \le i \le (s-1)k \ (9)$$

$$\frac{dp^4_{sk,0}(t)+}{dt} = -\big(\xi_2 + (sk-k)\big)\alpha)\ p^4_{i,0}(t) + \lambda p^4_{(s-1)k,0}(t) + \xi_1\ p^3_{sk,0}(t) \ (10)$$

$$\frac{dp^5_{0,j}(t)+}{dt} = -(\lambda + \beta + \beta_1))\ p^5_{0,j}(t) + \mu(1-p)p^3_{k,j}(t) + \beta_2 p^6_{0,j}(t)\ ;\ bk \le j \le (s-1)k \ (11)$$

$$\frac{dp^5_{0,sk}(t)+}{dt} = -(\beta + \beta_1))\ p^5_{0,sk}(t) + \beta_2 p^6_{0,sk}(t) \quad (12)$$

$$\frac{dp^5_{i,j}(t)}{dt} = -\big(\lambda + \beta + \beta_1 + (j-k)\big)\alpha)\ p^5_{i,j}(t) + \lambda p^5_{(i-1)k,j}(t) + \beta_2\ p^6_{i,j}(t);\ bk \le j \le (s-1)k, k \le i \le (s-1)k \ and\ i + J < sk \ (13)$$

$$\frac{dp^5_{i,j}(t)}{dt} = -\big(\beta + \beta_1 + (j-k)\big)\alpha)\ p^5_{i,j}(t) + \lambda p^5_{(i-1)k,j}(t) + \beta_2\ p^6_{i,j}(t); bk \le j \le (s-1)k, k \le i \le (s-1)k \ and\ i + J = sk \ (14)$$

$$\frac{dp^6_{0,j}(t)}{dt} = -(\lambda + \beta_2))\ p^6_{0,j}(t) + \beta_1\ p^5_{0,j}(t);\ k \le i \le (s-1)k \ (15)$$

$$\frac{dp^6_{0,sk}(t)}{dt} = -(\beta_2))\ p^6_{0,sk}(t) + \beta_1\ p^5_{0,sk}(t) \ (16)$$

$$\frac{dp_{i,j}^{6}(t)}{dt} = -\big(\lambda + \beta + \beta_2 + (i-k)\big)\alpha)\, p_{i,j}^{6}(t) + \lambda p_{(i-1)k,j}^{6}(t) + \beta_1\, p_{i,j}^{5}(t);\; bk \le j \le (s-1)k, k \le i \le (s-1)k \text{ and } i + J < sk \quad (17)$$

$$\frac{dp_{i,j}^{6}(t)}{dt} = -\big(\beta + \beta_2 + (i-k)\big)\alpha)\, p_{i,j}^{6}(t) + \lambda p_{(i-1)k,j}^{6}(t) + \beta_1\, p_{i,j}^{5}(t);\; bk \le j \le (s-1)k, k \le i \le (s-1)k \text{ and } i + j = sk$$

IV. Performance measures

Some performance measures are calculated to predict the system behaviour using the probabilities obtained through Runge-Kutta method:

1. $P(server\ being\ idle\ at\ time\ t) = I(t) = \sum p_{i,0}^{1}(t) + \sum p_{i,0}^{2}(t)$
2. $P(server\ being\ busy\ at\ time\ t) = S(t) = \sum p_{i,0}^{3}(t) + \sum\sum p_{i,j}^{5}(t)$
3. $P(server\ being\ broken\ down\ at\ time\ t) = B(t) = \sum p_{i,0}^{4}(t) + \sum\sum p_{i,j}^{6}(t)$
4. $L(t) = Expected\ number\ of\ customers\ in\ the\ systemat\ time\ t = \sum n * p_n$
5. $W(t) = waiting\ time\ in\ the\ system\ at\ time\ t = L(t) \big/ (\lambda * (1 - p_{max.customers}\ at\ time\ t))$

V. Numerical results

MATLAB software is used to develop the computational program to find out system performance measures by giving numeric values to all the parameters. And also the effect of various parameters on the system performance measures is studied. The effect of different parameters in the system on performance measures(length and waiting time) is summarized in Tables 1-10.

In all numerical computations, the model parameters are taken as

$$N = 4, s = 6, \lambda = 0.4, \mu = 0.8, \theta = .001, \beta = 2, \beta_1 = 0.001, \beta_2 = 0.002, \xi_1 = 0.002, \xi_2 = 0.003, p = .4 \text{ and } k = 2$$

VI. Conclusions and further scope of study

In this paper we have detailed transient analysis of a M/E$_k$/1 Queueing System with Two-Phase, N-Policy, Server Failure and Second Optional Batch Service with Customers impatient behaviour. Sensitivity analysis is also performed to know the influence of various parameters on system performance measures. This study can be extended as Steady State analysis by considering the general distribution for service times with cost analysis can be done for optimum solution.

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Appendix

TABLE 1: Effect of λ

Parameter (λ)	t	0.5	1	1.5	2	2.5
0.4	$L_S^{(t)}$	0.4	0.8	1.199999	1.599996	1.999985
	$W_S^{(t)}$	1.001066	2.014221	3.058966	4.151799	5.300224
0.41	$L_S^{(t)}$	0.41	0.82	1.229999	1.639995	2.049984
	$W_S^{(t)}$	1.001141	2.015157	3.06253	4.160094	5.314761
0.42	$L_S^{(t)}$	0.42	0.84	1.259999	1.68	2.1
	$W_S^{(t)}$	1.001219	2.016126	3.06619	4.17	5.33
0.43	$L_S^{(t)}$	0.43	0.86	1.289999	1.719994	2.149981
	$W_S^{(t)}$	1.0013	2.017128	3.069943	4.177105	5.344059

TABLE 2: Effect of μ

Parameter (μ_1)	t	0.5	1	1.5	2	2.5
0.8	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066	2.014220795	3.058965688	4.151798812	5.300223566
0.81	$L_S^{(t)}$	0.4	0.799999924	1.199999114	1.5999955	1.999984733
	$W_S^{(t)}$	1.001066382	2.014220793	3.058965664	4.151798698	5.300223198
0.82	$L_S^{(t)}$	0.4	0.799999923	1.199999104	1.599995455	1.999984586
	$W_S^{(t)}$	1.001066382	2.014220791	3.058965641	4.151798584	5.300222832
0.83	$L_S^{(t)}$	0.4	0.799999922	1.199999095	1.599995409	1.99998444
	$W_S^{(t)}$	1.001066382	2.014220789	3.058965617	4.15179847	5.300222467

TABLE 3: Effect of θ						
Parameter (θ)	t	0.5	1	1.5	2	2.5
0.001	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
0.0011	$L_S^{(t)}$	0.4	0.799999918	1.199999035	1.5999951	1.999983368
	$W_S^{(t)}$	1.001066382	2.014220776	3.058965469	4.151797697	5.300219786
0.0012	$L_S^{(t)}$	0.4	0.79999991	1.199998948	1.599994654	1.999981855
	$W_S^{(t)}$	1.001066382	2.014220758	3.058965249	4.151796582	5.300216005
0.0013	$L_S^{(t)}$	0.4	0.799999903	1.19999886	1.599994208	1.999980342
	$W_S^{(t)}$	1.001066382	2.014220739	3.058965029	4.151795467	5.300212223

TABLE 4: Effect of β_1						
Parameter (β_1)	t	0.5	1	1.5	2	2.5
0.001	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
0.0011	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223567
0.0012	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.999984881
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798813	5.300223568
0.0013	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995547	1.999984881
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798813	5.30022357

TABLE 5: Effect of β_2						
Parameter (β_2)	t	0.5	1	1.5	2	2.5
.002	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
.0021	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
.0022	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
0.0023	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566

TABLE 6: Effect of ξ_1						
Parameter (ξ_1)	t	0.5	1	1.5	2	2.5
.002	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
.0021	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
.0022	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
.0023	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223567

TABLE 7: Effect of ξ_2						
Parameter (ξ_2)	t	0.5	1	1.5	2	2.5
.003	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488

	$W_S^{(t)}$	1.001066	2.014220795	3.058965688	4.151798812	5.300223566
.0031	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066	2.014220795	3.058965688	4.151798812	5.300223566
.0032	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066	2.014220795	3.058965688	4.151798812	5.300223566
.0033	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066	2.014220795	3.058965688	4.151798812	5.300223566

TABLE 8: Effect of p

Parameter (p)	t	0.5	1	1.5	2	2.5
.4	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
.41	$L_S^{(t)}$	0.4	0.799999925	1.199999124	1.59999555	1.999984896
	$W_S^{(t)}$	1.001066382	2.014220795	3.05896569	4.151798822	5.300223605
.42	$L_S^{(t)}$	0.4	0.799999925	1.199999125	1.599995554	1.999984911
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965691	4.151798832	5.300223644
.43	$L_S^{(t)}$	0.4	0.799999925	1.199999125	1.599995558	1.999984927
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965693	4.151798842	5.300223683

TABLE 9: Effect of β

Parameter (β)	t	0.5	1	1.5	2	2.5
2	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
3	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
4	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
5	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566

TABLE 10: Effect of k

Parameter (k)	t	.5	1	1.5	2	2.5
2	$L_S^{(t)}$	0.4	0.799999925	1.199999123	1.599995546	1.99998488
	$W_S^{(t)}$	1.001066382	2.014220795	3.058965688	4.151798812	5.300223566
3	$L_S^{(t)}$	0.6	1.199999833	1.799998061	2.399990236	2.999967179
	$W_S^{(t)}$	1.501599573	3.021331056	4.588446973	6.227690511	7.950310011
4	$L_S^{(t)}$	0.8	1.599999705	2.399996619	3.199983147	3.999944003
	$W_S^{(t)}$	2.002132764	4.028441228	6.117927307	8.303577767	10.60038279
5	$L_S^{(t)}$	1	1.999999543	2.999994847	3.999974602	4.999916578
	$W_S^{(t)}$	2.502665955	5.035551313	7.647406819	10.37946138	13.25044497